A Complicated Continuous Model of Supply Chain

Chun-Hsiung Lan[1], Hai-Ming Chen[2], Chih-Wei Chiu[2]* and Mei-Hsiu Chen[2]

[1]Department of Business Administration, Hsing Wu Institute of Technology,
Taipei, Taiwan 244, R.O.C.
[2]Graduate Institute of Management Sciences, Tamkang University,
Tamsui, Taiwan 251, R.O.C.

Abstract

The issue of supply chain management has focused on practical operations and their theoretical applications for comparison with the traditional business aim, which merely pursues the maximal benefit. A business should simultaneously consider the benefits, resources and facilities for all stages in the supply chain system; i.e., consider the creation of an overall value for the system instead of the traditional goal of pursuing maximal benefits in certain focused stages.

This study aims to provide a referenced decision-making tool for a decision-maker in the current complex environment and also function as a decision-making tool for focusing on the overall and real-time analyses of multi-interval planning for a supply chain system. This system will investigate the purchase, production, inventory, distribution, and product shortage to deal with customers' demands for pursuing the maximal profit under a constrained production/inventory and finite distribution capacity. This research not only considers the multi-product order, multi-factory production, multi-material purchase, multi-distribution transportation, quantity discounts, diverse-customer demand, product inventory cost and limited inventory space but also provides the punishment cost of the demand shortage to construct an Integer Nonlinear Programming (INLP) mathematical model for achieving the maximum profit. Moreover, the proposed mathematical model is constructed by using the syntax of Lingo 9.0, wherein the built-in "global solver" is selected as the solution method. A numerical example then follows. This study creates a highly repetitive characteristic because of the application of the packaged software (Lingo 9.0); therefore, the proposed model and the solution method can be treated as valuable.

Key Words: Optimization, Supply Chain Management, Lingo 9.0, Punishment Cost

1. Introduction

Among the conventional microeconomic theories, the "theory of the firm" assumes that the pursuit of profits is the fundamental motivation of producers. However, in the real world, firms pursue not simply maximization of self profits. In the classification of market structures, in addition to number of firms and product differentiation, maximization of profit for the entire supply chain should also be considered. To expand productivity and extend business goals, all firms, no matter conventional or IT-based, domestic or foreign, will attempt to collect capital by issuing stocks and set up subsidiary firms to engage in various types of commercial behaviors. Factories and warehouses are also major parts of corporate operations. Therefore, management of suppliers, factories, distributors, and customers has become an important issue in recent years.

Supply chain is a very complicated network. It consists of various logistic facilities and organizations. According to the Association for Operations Management (APICS), Supply Chain Management (SCM) encom-

*Corresponding author. E-mail: 896620092@s96.tku.edu.tw

passes the design, planning, execution, control, and monitoring of supply chain activities with the objective of creating net value, building a competitive infrastructure, leveraging worldwide logistics, synchronizing supply with demand, and measuring performance globally.

For the complicated issues in supply chain management, many researchers have proposed different perspectives and solutions to certain problems. Newhart et al. applied mathematical programming and a heuristic algorithm to construct an optimal supply chain system with an objective of minimizing the inventory [1]. Robinson and Satterfield employed the concept of supply chain to construct a mixed-integer linear programming model, with an objective to maximize the profit of distribution networks. This model allows enterprises to determine the optimal location to deploy facilities, estimate market share, and also find out the impact of market share on facility location [2]. Amit Gary developed a SC Modeling and Analysis Tool (SCMAT) for supply chain management (SCM) at large electronics product manufacturers. The objective of this study was to help managers solve problems for considering inventory, costs, and designing in SCM and reducing inventory cost by shortening production lead time according to capacity of each node in the SC [3]. Jolayemi develop an integrated mixed-integer linear programming model for production-distribution and transportation planning in three-stage supply chains [4].

Yung and Yang proposed to obtain combined solutions of supply chain formation using genetic algorithm (GA). Based on attributes considered in Yung and Yang's study, algorithm engineering was applied to upstream and downstream firms of an enterprise to obtain optimum solutions. However, this method was applicable to only exploration of an optimum relationship between upstream and downstream firms. It could not be used to address problems involving multi-level firm cooperation [5]. Min and Melachrinoudis used production and inventory control as criteria and applied analytic hierarchy process (AHP) to evaluate the logistic problem involving multi-hierarchical supply chain connections among supplier, manufacturers, retailers, and customers [6]. With maximum production capacity and minimum operating cost considered as the goal, and profitability, technical capability, capacity, and cost as constraints, Zhou et al. applied goal programming (GP) and AHP to obtain the optimum decision for SCM with social, economic, resource, and environmental considerations [7]. Moncayo-Martı′nez and Zhang proposes an algorithm based on Pareto Ant Colony Optimization as an effective meta-heuristic method for solving multi-objective supply chain design problems [8].

Jayaraman and Pirkul focused on suppliers, factories, warehouses, and customers. Considering the constraints of factories and large-scale retailers and the condition that the total cost (fixed cost plus variable cost) should be minimized, they developed a Mixed-Integer Linear Programming (MILP) model to plan multi-stage productions [9]. Wang and Liang set the goals in the proposed model as minimizing production cost, shortage cost, and changes in the workforce level; inventory, workforce, and capacity as constraints to develop a multi-product programming model and search solutions using fuzzy multi-objective linear programming model [10]. Silva et al., used the discrete optimization model based on ant colony optimization to distributed supply chain management problem [11].

In practice, enterprises usually offer quantity discounts to encourage customers to purchase more. Weng studied the effect of quantity discounts on the reduction of operating cost and increase of customers' needs [12]. Fazel et al. proposed that quantity discounts present a decreasing function of order quantity and unit price, which can be the third kind of quantity discount [13]. Vismanathan and Wang evaluated the effects of volume discount and quantity discount as a coordination mechanism on price-sensitive demands in a single-vendor and single-retailer distribution channel [14]. Tsai solved a nonlinear SCM model capable of treating various quantity discount functions simultaneously, including linear, single breakpoint, step, and multiple breakpoint functions. By utilizing the presented linearization techniques, such a nonlinear model is approximated to a linear mixed 0–1 program solvable to obtain a global optimum [15].

The above studies mainly focused on the production stage of a supply chain to optimize the production system. In the aspect of quantity discounts, only the consumer and vendor were discussed. In fact, only a partial stage of a supply chain was discussed in these studies. From a more comprehensive perspective, this study is aimed to explore the integrated relationship in each level of a supply chain with considerations of multiple factors, including supply, purchase, production, inventory, distribution, and shortage. In the aspect of supply and purchase, in addition to the constraints of supplier capacity and inventory volume, supply quantity discount is also discussed. In the production stage, combinations of raw materials in different ratios needed in multi-product production and limit of raw material supply are considered. In the distributors, storage volume, quantity discount for customers, and punishment for product shortage are discussed. In addition, for the aspect of quantity discount, the focal view is placed on all-units discount. Considering multi-interval, multi-product, multi-material, multi-supplier, multi-plant, and multi-distributor inventory and delivery, we will construct an integer nonlinear programming (INLP) model with an objective of profit maximization of the entire supply chain. Later, this model will be applied to find the optimum quantity of purchase of each raw material, optimum quantity of production of each product, optimum quantity of each product delivered to each distributor, and optimum quantity of each raw material or each product that should be held in stock in each time interval. In addition, the optimum quantity of each product that each distributor should hold in inventory, optimum quantity of each product to be delivered to customers, and the optimum product shortage level for each product can also be obtained. The syntax of Lingo 9.0 extended version will be employed to build our model, and the its built-in Global Solver is selected to find out the optimal solution. In this paper, a practical model that integrates constraints and goals of both a firm and its upstream/downstream firms will be presented. This model is expected to serve as an important decision making tool for supply chain management.

2. Assumptions and Notations

All the assumptions and Notations used in this paper are explained as follows respectively:

2.1 Assumptions

The assumptions in this paper are listed as follows:

(1) Based on a four-level supply chain (supplier, factory, distributor, customer) structure, an Complicated Continuous Supply Chain Model (CCSCM) with an objective of profits optimization is constructed.

(2) All the members in the supply chain are fully cooperative, and all the factories and distributors are subsidiaries of the same corporate group. The customers of this corporate group are retailers.

(3) The length of production schedule is known.

(4) Suppliers should deliver all orders to customers by the deadline of each order.

(5) Product demand is estimated on the historic sales data.

(6) Inflation rate is fixed. The original price of each product is fixed and does not vary with time.

2.2 Notations

2.2.1 Notations in the Model

t: the code of the time interval, $t = 1, 2, \ldots, T$

d: the code of the supplier, $d = 1, 2, \ldots, D$

f: the code of the factory, $f = 1, 2, ..., F$

w: the code of the distributor, $w = 1, 2, ..., W$

c: the code of the customer, $c = 1, 2, ..., C$

g: the code of the product, $g = 1, 2, ..., G$

r: the code of the raw material, $r = 1, 2, ..., R$

2.2.2 Input Parameters

P_{cg}: the price for customer c to purchase product g

L_{tcg}: the quantity of product g that the demand of customer c at time interval t

IC_{tfr}: the cost for factory f to hold raw material r in inventory at time interval t

TC_{tdfr}: the cost for supplier d to transport raw material r to factory f at time interval t

PB_{tdr}: the maximum quantity of raw material r that supplier d can provide at time interval t

RA_{gr}: the ratio of raw material r to the materials needed for product g

MT_{tfg}: the time that factory f needs to produce one unit of product g at time interval t

IC_{tfg}: the cost for factory f to hold one unit of product g in inventory at time interval t

S_r: the inventory space needed to hold one unit of raw material r

S_g: the inventory space needed to hold one unit of product g

TC_{tfwg}: the cost of transporting one unit of product g from factory f to distributor w at time interval t

IC_{twg}: the cost for distributor w to hold one unit of product g in inventory at time interval t

TC_{twcg}: the cost for distributor w to transport one unit of product g to customer c at time interval t

UP_{tf}: the maximum working hours in factory f at time interval t

US_{tf}: the inventory space that factory f can provide at time interval t

US_{tw}: the inventory space that distributor w can provide at time interval t

2.2.3 Decision Variables

RA_{tdr}: the quantity of raw material r to be purchased from supplier d at time interval t

TA_{tdfr}: the quantity of raw material r that supplier d transports to factory f at time interval t

IA_{tfr}: the quantity of raw material r stored by factory f at the end of time interval t

MA_{tfg}: the quantity of product g produced by factory f at time interval t

IA_{tfg}: the quantity of product g stored by factory f at the end of time interval t

TA_{tfwg}: the quantity of product g transported from factory f to distributor w at time interval t

GA_{tfwg}: the shortage of product g for distributor w from factory f at time interval t

IA_{twg}: the quantity of product g stored by distributor w at time interval t

GA_{twcg}: the shortage quantity of product g for customer c from distributor w at time interval t

TA_{twcg}: the quantity of product g that distributor w needs to deliver to customer c at time interval t

TPC_{twg}: the punishment cost incurred to distributor w when the shortage of product g happens at time interval t

2.2.4 Input Functions

$F_p(MA_{tfg})$: the step function of unit production cost for factory f to produce product g at time interval t. It can be defined as $F_p(MA_{tfg}) = \sum_{\ell=1}^{n} \beta_g^l(1) f_g^l(MA_{tfg})$, where $\alpha_g^l \in \Re$ and $\begin{cases} \text{if } MA_{tfg} \in [\alpha_g^l, \alpha_g^{l+1}), f_g^l(MA_{tfg}) = 1 \\ \text{otherwise } f_g^l(MA_{tfg}) = 0 \end{cases}$; where α_g^l denotes the minimum quantity of product g that should be produced at ℓ step and $\beta_g^l(1)$ donates the unit production cost for factory f to produce product g under the ℓ step.

$F_r(RA_{tdr})$: the step function of unit purchase cost of raw material r at time interval t. It can be defined as $F_r(RA_{tdr}) = \sum_{\ell=1}^{n} \beta_r^l(2) f_r^l(RA_{tdr})$, where $\alpha_r^l \in \Re$ and $\begin{cases} \text{if } RA_{tdr} \in [\alpha_r^l, \alpha_r^{l+1}), f_r^l(RA_{tdr}) = 1 \\ \text{otherwise } f_r^l(RA_{tdr}) = 0 \end{cases}$; where α_r^l denotes the minimum quantity of raw material r that should be purchased at ℓ step and $\beta_g^l(2)$ donates the unit purchase cost of raw material r under the ℓ step.

$F_g(GA_{twcg})$: the step function of price of product g backordered from distributor w at time interval t. It can be defined as $F_g(GA_{twcg}) = \sum_{\ell=1}^{n} \alpha_g^l(3) f_g^l(GA_{twcg})$, where $\alpha_g^l \in \Re$ and $\begin{cases} \text{if } GA_{twcg} \in [\alpha_g^l, \alpha_g^{l+1}), f_g^l(GA_{twcg}) = 1 \\ \text{otherwise } f_g^l(GA_{twcg}) = 0 \end{cases}$; where α_g^l denotes the minimum quantity of product g that can be backordered at ℓ step and $\beta_g^l(3)$ donates the price of product g under the ℓ step.

3. Model Construction

Based on the proposed assumptions and notations, a CCSCM is constracted in the following section.

$$\begin{aligned} \text{Max } & \sum_g \sum_c \left[P_{cg} \times \left(\sum_t L_{tcg} - \sum_t \sum_w GA_{twcg} \right) \right] \\ & -\{\sum_t \sum_d \sum_r [F_r(RA_{tdr}) \times RA_{tdr}] \\ & + \sum_t \sum_d \sum_f \sum_r (TC_{tdfr} \times TA_{tdfr}) \\ & + \sum_t \sum_f \sum_r (IC_{tfr} \times IA_{tfr}) \\ & + \sum_t \sum_f \sum_g \{[F_p(MA_{tfg}) \times MA_{tfg}] + (IC_{tfg} \times IA_{tfg})\} \\ & + \sum_t \sum_f \sum_w \sum_g (TC_{tfwg} \times TA_{tfwg}) \\ & + \sum_t \sum_w \sum_g [(IC_{twg} \times IA_{twg}) + TPC_{twg}]\} \end{aligned} \tag{1}$$

s.t.

$$\sum_f \sum_g (MA_{tfg} \times RA_{gr}) \le \sum_d RA_{tdr} \le \sum_d PB_{tdr} \qquad \forall t, r \tag{2}$$

$$\sum_g (MA_{tfg} \times MT_{tfg}) \le UP_{tf} \qquad \forall t, f \tag{3}$$

$$\sum_g IA_{tfg} \times S_g + \sum_r IA_{tfr} \times S_r \le US_{tf} \qquad \forall t, f \tag{4}$$

$$\sum_g IA_{twg} \times S_g \le US_{tw} \qquad \forall t, w \tag{5}$$

$$\sum_f TA_{tdfr} = RA_{tdr} \qquad \forall t, d, r \tag{6}$$

$$IA_{(t-1)fr} + \sum_d TA_{tdfr} - \sum_g (MA_{tfg} \times RA_{gr}) = IA_{tfr} \qquad \forall t, f, r \tag{7}$$

$$IA_{(t-1)fg} + MA_{tfg} - \sum_w TA_{tfwg} = IA_{tfg} \qquad \forall t, f, g \tag{8}$$

$$IA_{(t-1)wg} + \sum_f TA_{tfwg} - \sum_c TA_{twcg} = IA_{twg} \qquad \forall t, w, g \tag{9}$$

$$\sum_c \left(L_{tcg} - \sum_w TA_{twcg} \right) = \sum_w \sum_c GA_{twcg} \qquad \forall t, g \tag{10}$$

$$TPC_{twg} = \sum_c [F_g(GA_{twcg}) \times GA_{twcg}] \qquad \forall t, w, g \tag{11}$$

RA_{tdr}, TA_{tdfr}, IA_{tfr}, MA_{tfg}, IA_{tfg}, TA_{tfwg}, GA_{twcg}, IA_{twg}, TA_{twcg}, $TPC_{twg} \ge 0$ and integer (12)

Formula (1) is the objective function of the model. It is to maximize the difference between total revenue and total cost (including cost of raw materials, transportation cost for suppliers, factories, and distributors, raw material and product inventory cost, inventory cost for distributors, production cost for factories, and stockout cost for distributors).

Formula (2) is the constraint of supply. The quantity of raw material r needed by all the factories cannot be greater than the quantity of raw material r supplied by all suppliers; the quantity of raw materials r supplied by all suppliers cannot be greater than the maximum quantity of raw material r that all suppliers can supply. Formula (3) is the constraint of factory production capacity at each time interval. It defines that the hours that factory f needs to produce all products is not greater than its maximum working hours of factory f at each time interval. Formula (4) is the constraint of factory inventory space at each time interval. The inventory space that factory f uses to store all products and raw materials at each time interval is not greater than the maximum inventory space of factory f. Formula (5) is the constraint of inventory space of distributor at interval t. It defines that the inventory space that distributor w uses to store all products cannot be greater than its maximum inventory space at each time interval. Formula (6) defines that the quantity of raw material r transported by supplier d to all factories is equal to the quantity of raw material r that all factories purchase from supplier d at each time interval.

Formula (7) means that the inventory of material r in factory f at the end of time interval t–1 plus the quantity of material r that all suppliers transport to factory f at time interval t should be equal to the quantity of mater r consumed by factory f for production at time t plus the inventory of material r in factory f at the end of time interval t. Formula (8) means that the inventory of product g in factory f at the end of time interval t–1 plus the quantity of product g produced by factory f at time interval t minus the quantity of product g transported from factory f to all distributors at time interval t should be equal to the inventory of product g in factory f at the end of time interval t. Formula (9) means that the inventory of product g in distributor w at the end of time interval t–1 plus the quantity of product g transported from all factories to distributor w at time interval t minus the quantity of product g transported from warehouse w to all customers at time interval t should be equal to the inventory of product g in distributor w at the end of time interval t. Formula (10) means that the quantity of product g demanded by customer c minus the quantity of product g transported from all distributors to customer c should be equal to the short quantity of product g demanded by customer c from distributor w. Formula (11) defines the stockout cost for distributor w to have insufficient inventory of product g at interval t. Formula (12) defines that all decision variables are non-negative and integer constraints. From Formula (1)~(12), it can be concluded that this model is an integer nonlinear programming (INLP) problem.

4. Analysis of a Numerical Example

For the entire supply chain, all the costs associated with purchase, production, inventory, distribution, and shortage may become key factors to affect of the entire supply chain. Therefore, in supply chain programming, all the factors that may be encountered in practical situations should be considered, so as to obtain optimum solution.

In the previous section, a CCSCM is constructed with an objective of profit maximization and provided a detailed explanation of the model. In this section, an example involving multiple suppliers, multiple manufacturers, multiple distributors, and multiple retailers is provided to find out how to plan for multi-interval, multi-factory production and inventory under constraints of limited capacity and inventory space; how to plan for purchase of raw materials from multiple suppliers with consideration of quantity discount; how to plan for transportation and inventory of multiple distributors with consideration of distribution capacity; and how to plan for distribution to multiple retailers and deal with punishment for shortage.

In this example, the supply chain system involves a corporate group, which owns three suppliers, two factories, and two distributors, and three retailers. In this system, each supplier can supply three raw materials to the two factories, and the factories can use different proportions of these three raw materials to produce two products. The duration of the time length can be divided into three time intervals, and each time interval is one month. The original price for product 1 is \$600, and the original price for product 2 is \$800. Table A.1 shows the demand of customer c for product g at each time interval, and such demand is increasing with time. Table A.2 shows the unit cost for factory f to hold raw material r in inventory at each time interval. Table A.3 shows the unit cost for supplier d to transport raw material r to factory f at each time interval. Because the distances between suppliers and factories are different, the transportation cost also varies with distance. Table A.4 shows the maximum supply of raw material r by supplier d at each time interval.

Table A.5 shows the ratio of raw material r to all materials needed for producing one unit of product g, ie Table A.5 is a BOM (bill of materials). Table A.6 shows the time that factory f needs to produce one unit of product g at each time interval. Table A.7 shows the cost for factory f to hold one unit of product g at each time interval. In this example, for all raw materials, the space needed to store each unit of material is 1 cubic unit; the space needed to store one unit of product 1 is 9 cubic units; and the space needed to store one unit of product 2 is 6 cubic units. Table A.8 shows the cost of transporting one unit of product g from factory f to distributor w at each time interval. Table A.9 shows the cost for distributor w to hold one unit of product g in inventory at each time interval. Table A.10 shows the cost of transporting one unit of product g from distributor w to customer c at each time interval. In this example, every factory can provide at maximum 7,000 working hours at maximum at each interval and has 7,000 cubic units of inventory space at each time interval. The inventory space of every distributor is 4,500 cubic units.

Quantity discount and punishment for product shortage are also considered. Table A.11 shows the quantity discount provided by suppliers for each raw material. The quantity discount is offered when 6,000 units of raw materials are purchased. Table A.12 shows the unit production cost for the factories. If more than 2,501 product units have been produced, an economy of scale will emerge, and the unit production cost will begin to decline. Table A.13 shows the cost of punishment for demand shortage. If more than 200 units are short, the stockout cost will increase.

In this model, there are 318 variables, 150 of which are nonlinear and 243 of which are integer variables. In addition of the 223 constraints, 88 are nonlinear. This model is classified as an integer nonlinear programming (INLP) model. The global optimum solution (maximum profit) is \$6,805,130. The result of optimum solution is shown in Tables 1~7.

Table 1. The quantities of raw materials purchased from suppliers (RA_{tdr})

t	dr								
	11	12	13	21	22	23	31	32	33
1	6,001	7,000	9,992	2,000	6,000	6,052	0	7,981	0
2	6,001	7,000	9,948	2,479	6,000	6,970	0	8,000	0
3	6,023	7,000	10,000	2,690	6,000	7,339	0	8,000	24

Note: t = 1, dr = 11 present the quantity of raw material 1 purchased from supplier 1 in time interval 1 is 6,001.

Table 1 shows the quantity of raw materials purchased from suppliers at each time interval. Table A.11 shows the unit supply cost is higher (Level 1) if the quantity supplied is below 6,000 units; the unit supply cost is lower (Level 2) if the quantity supplied is above 6,001 units. Under the condition that the quantity purchased is fixed, cost reduction is a good way to maximize profit. Table 2 shows the quantity of raw material *r* transported from supplier *d* to factory *f* at each time interval. From Tables 1 and 2, it can be discovered that supplier 1 supplies 6,001 units of raw material 1 at time intervals 1 and 2. The raw materials purchased from supplier 1 at time intervals 1 and 2 are transported to factory 1 and factory 2 respectively. This implies that the firm uses strategic purchase with combined orders to reduce the unit cost of purchasing raw materials.

Table 3 shows the quantities of raw material *r* and product *g* held by factory *f* at each time interval. Table 4 shows the quantity of product *g* produced by factory *f* at each time interval. The unit production cost is higher if the quantity is below 2,500 units (Level 1). The unit production cost is lower if the quantity exceeds 2,501 (Level 2). It can be discovered that the factories have done their best to achieve its economic scale of production. They adopted strategic production to reduce the unit product cost.

Table 5 shows the quantity of product *g* transported from factory *f* to distributor *w* at each time interval. Table 6 shows the quantities of products delivered to customers at each time interval. Table 7 shows the short quantity of product *g* demanded by customer *c* from distributor *w* at

Table 2. The quantities of raw materials transported from suppliers to factories (TA_{tdfr})

td	fr 11	12	13	21	22	23
11	4,975	7,000	9,992	1,026	0	0
12	0	0	0	2,000	6,000	6,052
13	0	4,926	0	0	3,055	0
21	5,450	7,000	9,948	551	0	0
22	0	0	910	2,479	6,000	6,060
23	0	4,872	0	0	3,128	0
31	6,023	7,000	10,000	0	0	0
32	0	0	1,983	2,690	6,000	5,356
33	0	5,981	0	0	2,019	24

Note: td = 11, fr = 11 present the quantity of raw material 1 transpored from supplier 1 to factory 1 in time interval 1 is 4,975.

Table 3. The quantities of raw materials and products stored in factories (IA_{tfr} and IA_{tfg})

t	fr/fg 11	12	13	21	22	23
1	0/0	0/376	42/NA	0/0	1/902	0/NA
2	0/0	0/0	0/NA	0/0	51/999	0/NA
3	38/0	0/0	13/NA	0/0	0/0	0/NA

Note: t = 1, fr = 11 present the quantity of raw material 1 stored in factory 1 in time interval 1 is 0.
t = 1, fg = 11 present the quantity of product 1 stored in factory 1 in time interval 1 is 0.

Table 4. The quantities of products produced by the factories (MA_{tfg})

t	fg 11	12	21	22
1	2,999	1,976	24	3,002
2	4,478	972	12	3,018
3	4,974	1,011	0	2,690

Note: t = 1, fg = 11 present the quantity of product 1 produced by the factory 1 in time interval 1 is 2,999.

Table 5. The quantities of products transported from factories to distributors (TA_{tfwg})

t	fwg 111	112	121	122	211	212	221	222
1	2,000	1,600	999	0	0	0	24	2,100
2	2,813	1,348	1,665	0	0	1,621	12	1,300
3	3,687	1,011	1,287	0	0	2,289	0	1,400

Note: t = 1, fwg = 111 present the quantity of product 1 transported from factory 1 to distributor 1 in time interval 1 is 2,000.

Table 6. The quantities of products transported from distributors to customers (TA_{twcg})

tw	cg 11	12	21	22	31	32
11	800	0	1,200	1,600	0	0
21	1,200	1,169	1,600	1,800	0	0
31	1,600	1,400	2,100	1,900	0	0
12	0	1,000	0	0	900	1,100
22	0	0	0	0	1,300	1,300
32	0	0	0	0	1,787	1,400

Note: tw = 11, cg = 11 present the quantity of product 1 transported from distributor 1 to customer 1 in time interval 1 is 800.

Table 7. The short quantities of products demanded by customers from distributors (GA_{twcg})

tw	*cg* 11	12	21	22	31	32
11	0	0	0	0	0	0
21	0	31	0	0	0	0
31	0	200	0	200	12	200
12	0	0	0	0	0	0
22	0	200	0	0	0	0
32	0	200	0	200	1	200

Note: tw = 11, cg = 11 present the short quantity of product 1 demanded by customer 1 from distributor 1 in time interval 1 is 0.

each time interval t. As shown in the table, there is no shortage at interval 1, and all distributors provide 100% service to each retailer. At interval 2, there is some shortage of products. As shown in Table 1, all suppliers have supplied the maximum quantity of raw material 2 in interval 2 and interval 3. It can be inferred that the shortage in time intervals 2 and 3 are caused by shortage of raw material 2. At interval 3, the total short quantity of product 2 demanded by retailers is 400 units. Because the suppliers are subsidiaries of the same corporate group, they will adopt a strategic stockout policy to let its two distributors share the short quantity. Thus, every distributor is short of 200 units of product 2, and the shortage can be controlled at the level of 200 units to keep the punishment at lower level and increase the profit of the entire supply chain.

5. Conclusion

In this study, all the situations that may occur in practical supply chain planning (e.g. unit purchase cost varies with quantity, stockout cost varies with the shortage quantity, and problems surrounding storage and distribution of products and raw materials) were comprehensively considered to derive an integer nonlinear programming model of SCM. Further, the time factor in multi-interval planning was also considered to conduct a thorough discussion of the supply chain system.

The major contributions of this paper are as follows: Firstly, we considered practical and integrate the time factor for developing the CCSCM (Complicated Continuous Supply Chain Model) with an objective of profit maximization of the entire supply chain is constructed practically, and thus CCSCM has become a complicated integer nonlinear programming (INLP) model.

Secondly, a computerized decision making tool was provided in this paper. This tool could allow enterprises to make decisions on how to plan for multi-interval, multi-factory production and inventory under constraints of limited capacity and inventory space; how to plan for purchase of raw materials from multiple suppliers with consideration of quantity discounts; how to plan for transportation and inventory of multiple distributors with consideration of distribution capacity; and how to plan for distribution to multiple retailers and deal with punishment for demand shortage, and thus the obtained solutions could be an important reference for enterprises in total supply chain planning.

Thirdly, a software package was applied to find solutions in this paper, so the proposed model featured a repeated characteristic. In other words, if all the parameters, including product price, order quantity, capacity, transportation cost, inventory cost, inventory space etc. are input in the model, a suggested solution can be obtained. Thus, this study is valuable for practical applications.

Appendix

Table A.1. The quantities of products demanded by customers (L_{tcg})

t	*cg* 11	12	21	22	31	32
1	800	1,000	1,200	1,600	900	1,100
2	1,200	1,400	1,600	1,800	1,300	1,300
3	1,600	1,800	2,100	2,300	1,800	1,800

Table A.2. The cost of holding one unit of raw materials at factory (IC_{tfr}, unit: dollar)

t	*fr* 11	12	13	21	22	23
1	2	3	4	1	2	3
2	2	3	4	1	2	3
3	2	3	4	1	2	3

Table A.3. The cost of transporting one unit of raw materials from supplier to factory (TC_{tdfr}, unit: dollar)

td	*fr* 11	12	13	21	22	23
11	30	60	20	40	70	60
12	40	70	30	30	60	50
13	45	65	35	35	65	55
21	30	60	20	40	70	60
22	40	70	30	30	60	50
23	45	65	35	35	65	55
31	30	60	20	40	70	60
32	40	70	30	30	60	50
33	45	65	35	35	65	55

Table A.4. The maximum quantities of raw materials that can be supplied (PB_{tdr})

t	dr								
	11	12	13	21	22	23	31	32	33
1	8,000	7,000	10,000	7,000	6,000	9,000	6,000	8,000	9,000
2	8,000	7,000	10,000	7,000	6,000	9,000	6,000	8,000	9,000
3	8,000	7,000	10,000	7,000	6,000	9,000	6,000	8,000	9,000

Table A.5. The ratio of raw material to all materials needed for one unit of product (RA_{gr})

r	g	
	1	2
1	1	1
2	2	3
3	2	2

Table A.6. The time needed to produce one unit of product (MT_{tfg}, unit: hour)

t	fg			
	11	12	21	22
1	1	2	1	2
2	1	2	1	2
3	1	2	1	2

Table A.7. The cost of holding one unit of product at factory (IC_{tfg}, unit: dollar)

t	fg			
	11	12	21	22
1	30	40	30	40
2	30	40	30	40
3	30	40	30	40

Table A.8. The cost of transporting one unit of product from factory to distributor (TC_{tfwg}, unit: dollar)

tf	wg			
	11	12	21	22
11	20	30	25	40
21	20	30	25	40
31	20	30	25	40
12	25	30	20	25
22	25	30	20	25
32	25	30	20	25

Table A.9. The cost of holding one unit of product by distributor (IC_{twg}, unit: dollar)

t	wg			
	11	12	21	22
1	20	50	20	50
2	20	50	20	50
3	20	50	20	50

Table A.10. The unit cost of transporting one unit of product from distributor to customer (TC_{twcg}, unit: dollar)

tw	cg					
	11	12	21	22	31	32
11	15	20	10	15	30	35
21	15	20	10	15	30	35
31	15	20	10	15	30	35
12	20	25	25	30	15	20
22	20	25	25	30	15	20
32	20	25	25	30	15	20

Table A.11. Levels of cost of supplying one unit of raw materials

r	Level	Quantity	Cost per unit (dollar)
1	1	6,000↓	11
	2	6,001↑	9
2	1	6,000↓	14
	2	6,001↑	12
3	1	6,000↓	17
	2	6,001↑	15

Table A.12. Levels of cost of producing one unit of products by factory

f	Level	Short quantity	Cost per unit (dollar)
1	1	2,500↓	50
	2	2,501↑	40
2	1	2,500↓	50
	2	2,501↑	40

Table A.13. Levels of cost of punishment for demand shortage

g	Level	Short quantity	Cost per unit (dollar)
1	1	200↓	300
	2	201↑	600
2	1	200↓	400
	2	201↑	800

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