

One-Dimensional Numerical Solution of the Maxwell-Minkowski Equations

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Abstract

In this paper the author demonstrates one-dimensional numerical simulation results of the propagation of electromagnetic pulses onto a dielectric slab that is moving with a constant speed. The Maxwell-Minkowski equations are numerically approximated using the characteristic-based method. The effects of the moving dielectric slab on the reflected, transmitted electric fields and that inside the dielectric slab for various velocities are illustrated and compared based on the stationary slab case. The frequency-domain results are obtained through Fourier transform of the time-domain data. The medium used in the numerical model is assumed to be infinite, homogeneous, isotropic, lossless, 50 cm thick, and have a dielectric constant of 4. The dielectric slab is set to move either toward or away from the incident electromagnetic pulse with a constant speed of 10 percent of the light speed.

Key Words: Characteristic-Based Method, Maxwell-Minkowski Equations, Moving Dielectric Slab

1. Introduction

Even though the study on electromagnetic fields inside moving medium can be dated back to as early as the beginning of the twentieth century, publications of such interest did not become popular until the period of 60s to 80s. Among them are the study of the reflection and transmission of plane electromagnetic waves by uniformly moving dielectric half-space which is homogeneous, isotropic, lossless, and linear [1], and the study of the guided waves in moving media [2–4]. Some proposed technique for solution of the fields inside various waveguides. However, the earliest analytical study of such problem was published in 1908 when Minkowski [5] exactly solved the problem. Though the finite-difference time-domain (FDTD) technique was used for numerically approximating electromagnetic fields scattering from moving surfaces [6], Harfoush et al. solved problems featured with moving boundary in free space yet not moving medium.

The chief objectives of this paper are as follows: to present numerical simulation results of the propagation of electromagnetic pulse onto a constantly moving infinite dielectric slab using characteristic-based method in one-dimensional model, and to show that the presented numerical method is suitable for solving such type of problems in which portion of grid cells are changing dimensions. Some verifications of the scheme accuracy were carried out by investigating the computational results based on the theoretical values. Therefore, in this paper we will center our effort on the demonstration of the numerical simulation results.

The characteristic-based method was employed for the solutions of time-domain Maxwell's equations through the application of explicit central-difference scheme by Shang [7] in the early of 90s. The implicit formulation collaborating with the flux-vector-splitting technique and the lower-upper approximate factorization scheme was soon developed for the same purpose. The latter was shown to yield results in good agreement with data generated by the FDTD technique [8], and to be able to

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solve the electromagnetic scattering problems involved with moving and/or vibrating perfect boundary in one dimension [9,10].

Three key features associated with the characteristic-based method are given as follows. The governing equations are recast into the curvilinear coordinate system prior to the application of the flux vector splitting technique. All field quantities are defined in the center of the grid cell meaning that every field quantity is the averaged value over the entire computational cell. It approximates the Maxwell equations by evaluating flux across every cell face and then balancing them for the changes of field quantities within each computational cell. This allows us to easily model problems with varying cell size. These features are in contrast to the traditional methods. The method of moment and the FDTD method define field quantities at the grid nodes and the Maxwell equations are approximated by directly applying explicit finite difference scheme. Since the change of cell size does not alter the definition of field component, the characteristic-based method is considered to be a more suitable numerical approach for solving electromagnetic problems involved with time-varying grid cells such as moving dielectric slab.

2. Governing Equations

The propagation of electromagnetic fields in source-free region is governed by the well-known Maxwell's equations:

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0 \quad (1)$$

$$\frac{\partial \vec{D}}{\partial t} - \nabla \times \vec{H} = 0 \quad (2)$$

$$\nabla \cdot \vec{D} = 0 \quad (3)$$

$$\nabla \cdot \vec{B} = 0 \quad (4)$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic field intensities, $\vec{D}$ and $\vec{B}$ are the electric and magnetic flux densities. They are related by $\vec{D} = \epsilon_0 \vec{E}$ and $\vec{B} = \mu_0 \vec{H}$ where ϵ_0 and μ_0 are the permittivity (dielectric constant) and permeability of vacuum. These two expressions are known

as the constitutive relations. Inside medium, they become $\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$ and $\vec{B} = \mu \vec{H} = \mu_r \mu_0 \vec{H}$ with ϵ_r and μ_r are the relative permittivity and permeability of medium, respectively. The need to modify the constitutive relations arises when one tries to solve electromagnetic fields inside a moving medium. Prior to the formulation of the governing equations, we first reduce the complexity of problem by assuming the incident Gaussian pulse is propagating in the positive-x direction and polarized such that $\vec{E} = \hat{z} E_z$ and $\vec{B} = -\hat{y} B_y$. The dielectric slab is assumed to have a finite thickness and move along the x-axis with a constant velocity $\vec{v} = v \hat{x}$, either approaching ($v < 0$) or receding from ($v > 0$) the incident pulse. Under such arrangement and according to Tai's work [11], the electric and magnetic flux densities are rewritten as

$$\vec{D} = \epsilon \alpha \cdot \vec{E} + \Omega \times \vec{H} \quad (5)$$

$$\vec{B} = \mu \alpha \cdot \vec{H} - \Omega \times \vec{E} \quad (6)$$

$$\text{where } \Omega = \frac{(n^2 - 1)\beta}{(1 - n^2\beta^2)c} \hat{x}, \alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}, a = \frac{1 - \beta^2}{1 - n^2\beta^2},$$

$\beta = \frac{v}{c}$, the index of refraction $n = \sqrt{\mu_r \epsilon_r}$, and c is the speed of light.

Inserting (5) and (6) into (1) and (2) with the assumption that $\vec{E} = \hat{z} E_z$ and $\vec{B} = -\hat{y} B_y$, we have

$$\frac{\partial B_y}{\partial t} - \frac{\partial E_z}{\partial y} = 0 \quad (7)$$

$$a \frac{\partial B_y}{\partial t} - \frac{\partial E_z}{\partial x} = -\Omega \frac{\partial E_z}{\partial t} \quad (8)$$

$$a \frac{\partial D_z}{\partial t} - \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = +\Omega \frac{\partial H_y}{\partial t} \quad (9)$$

Rearranging (7) through (9) in conservative form and then transforming them from the Cartesian system (t, x, y) into curvilinear system (τ, ξ, η), we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & \Omega/\epsilon \\ 0 & -\Omega/\mu & a \end{bmatrix} \frac{\partial Q}{\partial \tau} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} = 0 \quad (10)$$

where $Q = Jq$, $F = J(\xi_x f + \xi_y g)$, $G = J(\eta_x f + \eta_y g)$, $J = \left| \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \xi} \right|$, and three variable vectors are given as $q = [0, B_y, D_z]^T$, $f = [0, -E_z, -H_y]^T$, and $g = [E_z, 0, 0]^T$. An implicit characteristic-based approach for numerical solution of Eq. (10) is formulated by first applying the central difference operator to the governing equation. Equation (10) becomes

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & \Omega/\varepsilon \\ 0 & -\Omega/\mu & a \end{bmatrix} \left(\frac{Q^{n+1} - Q^n}{\Delta\tau} \right) + \frac{\delta_i F}{\Delta\xi} + \frac{\delta_j G}{\Delta\eta} = 0 \quad (11)$$

where the central difference operator is defined as

$$\delta_k(\psi) = (\psi)_{k+1/2} - (\psi)_{k-1/2} \quad (12)$$

In Eq. (11) the superscripts (n) and $(n+1)$ on variable vector Q stand for two successive time levels, and $\Delta\xi$ and $\Delta\eta$ represent the discretization of the computational domain along two coordinate directions. In Eq. (12) the half-integer index in the definition of central difference operator indicates that the flux vector ψ , F or G , is evaluated at cell face. Accordingly, Eq. (12) results in the flux difference for the k^{th} cell.

3. The Problem

The problem of interest is defined in Figure 1. As shown both regions A and C are free space and region B is the dielectric slab. The excitation source is a plane electromagnetic Gaussian pulse having only components of E_z and B_y , and propagates in the positive- x direction toward the dielectric slab. The Gaussian pulse has a peak value of 1 V/m, a truncated level of 100 dB, a width of about 0.695 ns measured from the peak to the level of $e^{-0.5}$, a span of 2 meters measured from cut-off to cut-off, and a highest frequency content of about one GHz. The dielectric slab is set to be precisely 50 cm thick for easy calculation and is assumed to be made of nonmagnetic ($\mu_r = 1$), linear, homogeneous, isotropic, and lossless material with $\varepsilon_r = 4$. The dielectric slab, as indicated by two dotted lines, may be stationary or move with a constant velocity of 10 percent of the light speed either approaching or receding from the incident pulse. Note that depicted in Figure 1 is the electric field component of the

incident pulse. The reflected and transmitted electric fields are recorded at $x = 1.25$ m and 4.25 m, respectively. The field inside medium is recorded at the center of the dielectric slab. Note also that the sampling point located inside medium moves as the dielectric slab moves while the other two are fixed in the reference frame. For other numerical settings, a grid density of 250 points per meter is used and the numerical time step is such that it takes 20 steps for the numerical pulse to march one grid cell.

As the grid system is concerned, the total cell number is constant and the cell size is uniform when the dielectric slab is at rest as illustrated in Figure 2(a). When the dielectric slab travels, both cell number and cell size become time dependent. Figure 2(b) shows the grid system changed due to the fact that the dielectric slab moves toward the incidence. The most far right cell in region A (cell A_N) is truncated by the dielectric slab while the most far left cell of region C (cell C_0) is added into the grid system. On the other hand, Figure 2(c) illustrates the changes in the grid system when the dielectric slab moves in the opposite direction, away from the incidence. Portion of the cell C_1 in region C is eliminated by the dielectric slab whereas an extra cell A_{N+1} in region A is introduced into the grid system. It is pointed out that during the simulation these variations must be carefully taken into account by updating the effective cell areas and the corresponding numerical time steps in order to retain the numerical accuracy.

Up to this point, we expect to observe the following phenomena due to movement of the dielectric slab. The

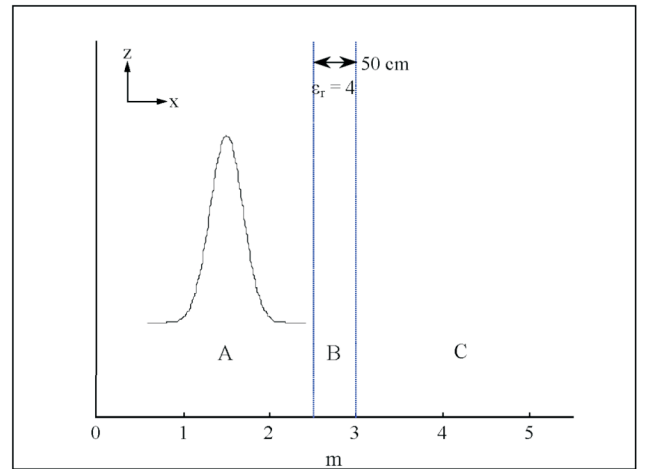


Figure 1. Definition of the problem. Dotted lines represent the two air-medium interfaces.

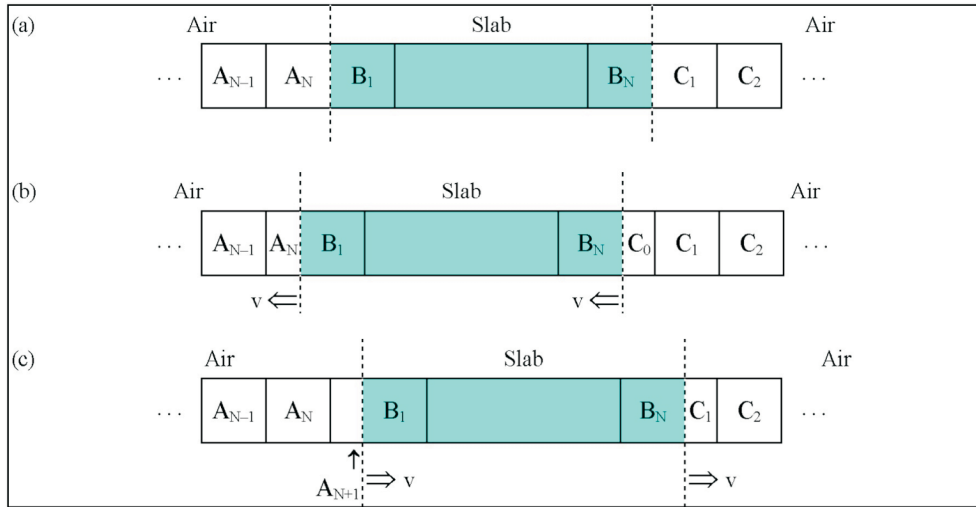


Figure 2. Grid system when slab is (a) stationary, (b) approaching, (c) Receding from the incidence.

Doppler effects can be observed on the reflected pulse width. Since the electromagnetic fields inside medium move along with the dielectric slab as propagating on, the transmitted pulses after emerging from slab bear notable time differences with respect to the sampling location.

The boundary conditions in the present simulation are quite straightforward. At the interface between two media numerical method assures that the tangential components of both electric field intensity and magnetic field

intensity are continuous. At the outer computational boundary, the numerical out-going fields are set not to bounce back into the computational domain.

4. Results

For clear examination on the computational results, three plots of the electric field intensity demonstrating the interaction between the electromagnetic pulse and the moving dielectric slab are given in Figure 3 respec-

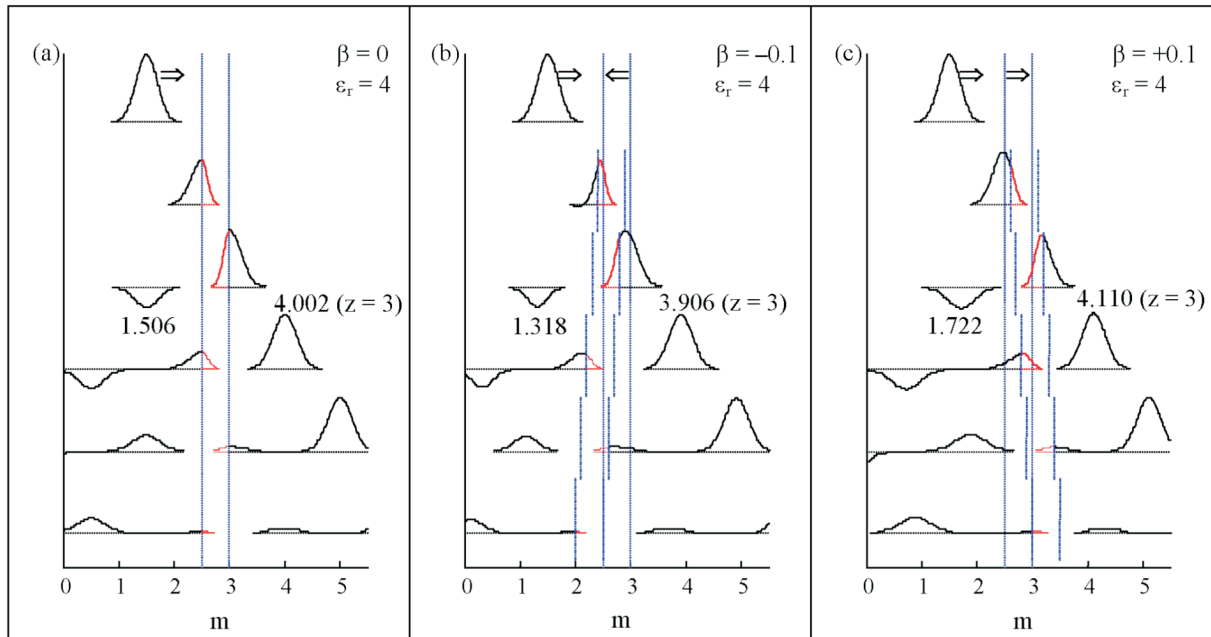


Figure 3. Electromagnetic pulses propagate into dielectric slab: (a) stationary, (b) approaching, (c) receding. Dash-dotted lines = moving interfaces; dotted lines = stationary interfaces. The dielectric constant of the slab is 4. The time interval is equal to 1 m/c.

tively for three different cases. And for easy investigation we define a time constant (Δt) between two consecutive snap shots in Figure 3 such that during this period of time the electromagnetic pulse propagate exactly one meter in distance in free space, i.e., $\Delta t = 1 \text{ m/c}$. Symbol c stands for the speed of light and is set to be $3 \times 10^8 \text{ m/s}$ in the present work. Since the index of refraction of the medium is 2, it is easily noticed from Figure 3(a) that the speed of electromagnetic pulse inside the motionless dielectric slab is one half of that in free space, and therefore that the transmitted pulse has a 0.5 m/c lag when compared with that in free space. Figures 3(b) and 3(c) respectively illustrate two opposite situations when the dielectric slab moves toward and away from the incident pulse. Based on the case where the dielectric slab is stationary, further examinations are carried out by comparing the locations of the peak value of the reflected pulses. Assuming that the starting position of the incident pulse peak is $x = 1.5 \text{ m}$, the total distance (D) traveled can be calculated by equation (13) with $\beta = v/c$, and letter z represents any positive integer number corresponding to the discretized time instance. Upon examining the reflected pulse in the third row ($z = 2$) of Figure 3, the electromagnetic pulse propagates exactly 2 meters forward and backward when the dielectric slab is stationary ($\beta = 0$) during the time interval of $2 \cdot \Delta t$. The location of the reflected pulse peak (R) then can be computed by equation (14) for the present set up. The calculated results of the reflected pulse along with the theoretical values are summarized in Table 1. Also listed in this table are the locations of the transmitted pulse peak. Since electromagnetic fields are slowed down by one half of the light speed, for a 0.5 cm thick dielectric slab the transmitted fields would take exactly one Δt to propagate through it. Keep it in mind that if the dielectric slab is moving at a constant velocity, the transmitted electromagnetic fields move along with medium as they are propagating on. Taking the stationary slab as a reference and focusing on

the fourth row ($z = 3$) of Figure 3, we can predict the locations of the transmitted pulse peak (T) simply by adding the difference ($\beta \cdot c \cdot \Delta t$) to that of the stationary case. As can be seen from Table I they are in good agreement.

$$D = \frac{z \cdot c \cdot \Delta t}{1 - \beta} \quad \text{for reflected pulse} \quad (13)$$

$$R = \frac{2\beta}{1 - \beta} + 1.5 \quad \text{with } z = 2 \quad (14)$$

The Doppler effects on the reflected pulse due to the motion of the dielectric slab are observed in Figure 4 where changes in the pulse width and differences in the pulse location are obvious as given in the legend. The reflected electric field from perfect electric conductor is also included to serve as a reference. The pulse width is measured from the pulse peak to its $e^{-0.5}$ in magnitude. On the basis of the stationary slab, the reflected pulse becomes narrower when slab moves toward the incidence; and it is broader when the slab moves in the opposite direction. Also shown in both Figures 3 and 4 are the multi-reflection/transmission phenomena of electromag-

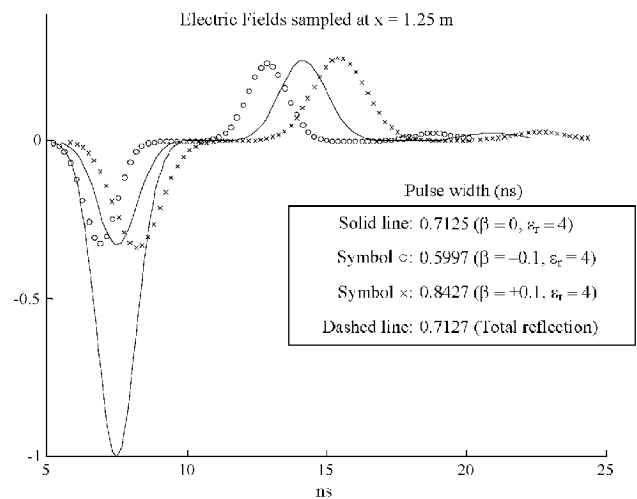


Figure 4. Electric field intensities sampled at $x = 1.25 \text{ m}$.

Table 1. Locations of the reflected and transmitted pulse: calculated vs. theoretical

Slab Velocity	Location of reflected pulse peak (R)		Location of transmitted pulse peak (T)	
β	Calculated (m)	Theoretical (m)	Calculated (m)	Theoretical (m)
0	1.506	1.500	4.002	4.000
-0.1	1.318	1.318	3.906	3.900
+0.1	1.722	1.722	4.106	4.100

netic fields inside the dielectric slab. It is noted from Figure 4 that the up-right electric fields are reflected from the right interface inside the medium and then emerge from the left interface. The spectrum corresponding to the electric fields given in Figure 4 are plotted in Figure 5. When compared to the $\beta = 0$ case, it is obvious that the maximums and minima shift to the lower frequency end if the slab recedes from the incident pulse, and shift to the higher frequency end if the slab approaches the incident pulse due to the Doppler effects.

The transmitted fields recorded at $x = 4.25$ m are illustrated in Figure 6, and their spectra are in Figure 7.

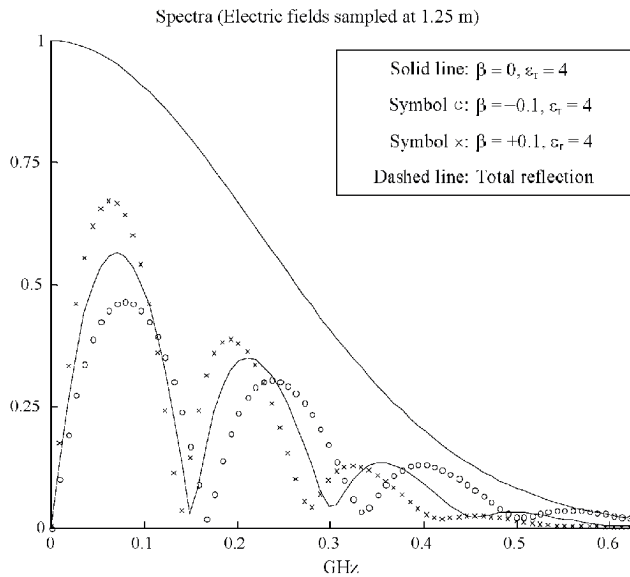


Figure 5. Spectra of the electric fields sampled at $x = 1.25$ m.

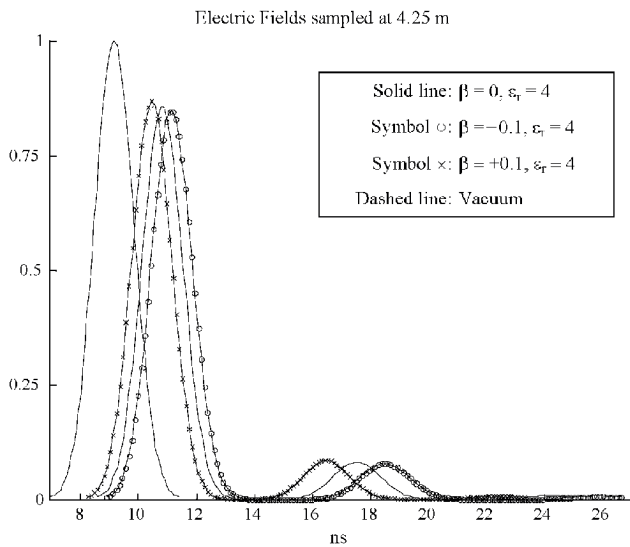


Figure 6. Electric field intensities sample at $x = 4.25$ m.

Similarly, as a reference, the dashed line represents the electromagnetic fields propagating in free space, in the absence of the dielectric slab. It is observed that all electric fields are positive- z polarized for the fact that the secondary are reflected from the left interface. The widths of the primary transmitted pulses are: 0.7130 ns ($\beta = -0.1$), 0.7048 ns ($\beta = 0$), and 0.6925 ns ($\beta = +0.1$). This is because that in the $\beta = -0.1$ case the dielectric slab is moving away from the sampling point, while in the $\beta = +0.1$ case the dielectric slab is approaching the sampling point. It is therefore evident that the maximums in the spectrum bear reverse trend as that of the reflected fields (Figure 5).

The electric fields inside the dielectric slab are recorded at the middle point and plotted in Figure 8. It is interesting to compute the time needed for the electromagnetic pulse to reach the sampling point, since this sampling point is moving along with the dielectric slab. The arrival times of the pulse peak for various situations are 5.0080 ns ($\beta = 0$), 4.6943 ns ($\beta = -0.1$), 5.3633 ns ($\beta = +0.1$), and 4.1703 ns (free space). The corresponding distance that the electromagnetic pulses travel in free space during the above time intervals are 1.5024 m ($\beta = 0$), 1.4083 m ($\beta = -0.1$), 1.6090 m ($\beta = +0.1$), and 1.2511 m (free space). These numbers are reasonable based on the following facts that the electromagnetic pulse is slowed down by one half inside the dielectric slab, that the dielectric slab is moving at a constant speed of ± 0.1 c , and that the sampling point is located at 0.25 m from

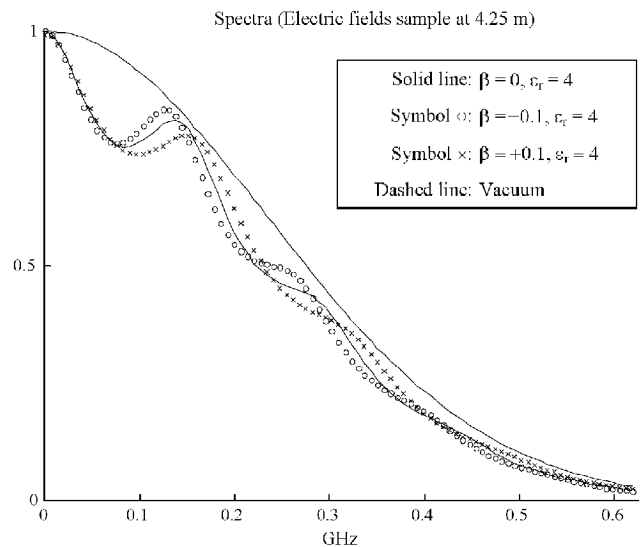


Figure 7. Spectra of the transmitted electric field intensities sample at $x = 4.25$ m.

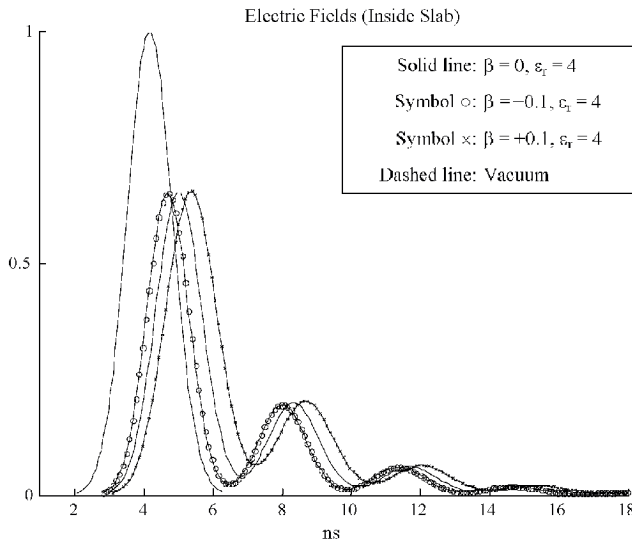


Figure 8. Electric field intensities inside slab.

the interface. Finally, the pulse widths of the primary transmitted pulses are 0.6690 ns ($\beta = 0$), 0.6098 ns ($\beta = -0.1$), 0.7368 ns ($\beta = +0.1$), and 0.6560 ns (free space). Their spectra are given in Figure 9 where the maximum shifts toward higher frequency end when slab approaches the incident pulse while toward lower frequency end when slab recedes from the incident pulse.

5. Conclusion

This study has shown that the characteristic-based method numerically solves the Maxwell-Minkowski equations for the propagation of electromagnetic pulse onto a moving dielectric slab in one dimension. The effects of the moving slab on the electromagnetic fields are also shown in both time domain and frequency domain. Some of the computational results are compared with the theoretical values and found to be in good agreement. It is direct evidence that the characteristic-based method has successfully approximated the Maxwell-Minkowski equations. To extend the existing code for three-dimensional problems featured with moving or vibrating media of finite dimensions is our future work.

Acknowledgement

The author is grateful to Mr. Michael F. Chen of Cartell Chemical Co., Ltd., Chia-Yi, Taiwan, for making this work possible through the industry-academy coo-

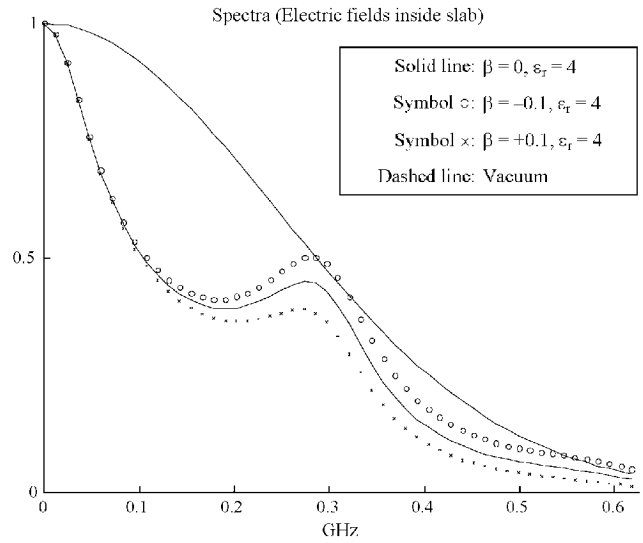


Figure 9. Spectra of the electric field intensities inside the slab.

peration project WFC-E-A1-9511-09.

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Manuscript Received: Aug. 8, 2007

Accepted: Mar. 19, 2009