The Modified Carlsson's Model for Setting the Optimum Process Mean

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Abstract

In 1984, Carlsson addressed the method for setting the optimum process mean when the net income takes production cost and different selling prices into account. He adopted the piecewise linear profit function to measure the net income for a product quality characteristic is normally distributed. However, the non-normal quality characteristic of product maybe occur in the industrial application. In this paper, the author proposes the modified Carlsson's models with beta and Weibull quality characteristics for determining the optimum process mean. A numerical example and sensitivity analysis of parameters will be provided for illustration.

***Key Words*:** Piecewise Linear Profit Function, Beta Distribution, Weibull Distribution, Process Mean

1. Introduction

For modern industrial statistics, one addresses product inspection, statistical process control, quality design, and product reliability. To determine the optimum process parameters is an important topic for the statistical process control. The optimal selection of process mean (manufacturing target) will directly affect the process defective rate of product, scrap cost, rework cost, inspection cost, and the loss to the customer. There are considerable attentions paid to this work. The piecewise linear profit function of quality characteristic is usually applied in the filling/canning problem for determining the optimum manufacturing target, e.g., Carlsson [1], Golhar and Pollock [2], Misiorek and Barnett [3], and Lee, et al. [4,5], and so forth.

Carlsson [1] developed a method to determine the optimal process mean of normal quality characteristic of larger-the-better type. The net income takes production cost and different selling prices into account. He adopted the piecewise linear profit function to measure the net income. The manufacturer's income per item depends upon whether the product is accepted or rejected, reprocessed, or sold at a reduced price. If the product is an accepted item, the customer is willing to pay an additional price for good quality. If the product is a rejected item, the manufacturer may have to compensate the customer for bad quality, not only by reducing the price, but also by a price reduction proportional to the deficit in quality. The objective is to obtain the optimal process mean and maximize the expected net income per item.

Golhar and Pollock [2] considered a canning problem of normal quality characteristic which needs to determine the optimal mean and the upper limit of ingredient. The under-filled cans and overfilled cans are emptied and the ingredients are reprocessed. The objective in Golhar and Pollock's [2] model is to find the optimum value of the average weight and the maximum weight of the ingredients in a can that maximizes the expected profit per can. Misiorek and Barnett [3] proposed the modified Golhar and Pollock's [2] model with extra cost parameters. The overflowing material of filling is either recaptured or lost and the cost of the containers is considered. The under-filled container is top-

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ped-up or is emptied-out and material reused. They also considered the normal quality characteristic, found the optimum process mean, and maximized the expected profit per fill.

Lee et al. [4,5] presented the problem of jointing determination of optimum process mean and screening limits. The quality characteristic of performance variable or surrogate variable is considered as the screening variable. Their models involved selling and discounted prices as well as production, inspection, rework and penalty costs. The normal and bivariate normal distribution are assumed and used in Lee et al.'s [4,5] models. The objective of model is to maximize the expected profit per item.

For modern statistical quality control, one addresses the process control, quality improvement, and quality design for promoting the product and service quality. Recently, many researchers have contributed their papers in an integrated production, inventory, and quality model with economic manufacturing quality and imperfect product quality for obtaining the maximum expected total profit of product. In 1997, Sana et al. [6,7] proposed a volume flexible inventory model considering the demand rate of defective items sold at a reduced price as a function of the reduction rate of the selling price. They further addressed the imperfect production system including the unit production cost taken to be a function of the finite production rate. In 2010, Sana [8,9] considered the production-inventory model with imperfect production system for defective items restored to their original quality by rework. His model also discusses and provides an optimal solution for product reliability parameter and dynamic production rate. Sana [10] discussed that the enterprises' initiatives affect the time varying demand for a multi-item economic order quantity problem. Sana [11,12] further contributed his papers for the problem about price sensitive demand with random sales price. In 2011, Sana [13] further presented a economic order quantity model when demand is price demand and partial backorder is permitted. Sana [14,15] and Rov et al. [16] also proposed the imperfect quality consideration for the production, inventory, and supply chain system.

The distribution of the product quality characteristic is usually assumed to be normal in the selection of optimum process mean. Chandra [17, p. 138] pointed out that "The beta distribution could be more flexible when compared with the normal probability distribution because it can accommodate different finite ranges and different shapes from left skewed to symmetrical to right skewed. This makes the beta distribution a much better candidate for fitting real-life distributions of quality characteristics".

In the real world, however, many quality characteristics often show various types of skewed distribution. Hence, one needs a practical distribution fitting real situations very well. A Weibull distribution are often adopted in the reliability analysis model addressing the ages with time. It has been successfully applied in describing the lifetime of certain electronic systems. This makes the Weibull distribution a much better candidate for fitting the larger-the-better quality characteristics.

In this paper, the author proposes the modified Carlsson's [1] model with beta and Weibull distributions for determining the optimum process mean. A numerical example with sensitivity analysis of parameters is provided for comparing Carlsson's [1] solution and our results.

2. Carlsson's Model with Normal Qaulity Characteristic

In Carlsson's [1] model, the objective is to maximize the expected net income per item and obtain the optimal process mean. The net income per item depends upon whether the quality characteristic x is less than or greater than or equal to the lower specification limit, L. From Chandra [17, pp. 141], there are five assumptions in Carlsson's [1] model as follows:

1. The quality characteristic X has a lower specification limit, L.
2. Products with value of the quality characteristic $X \geq L$ are accepted and sold at full price, f_2. Customers will pay an additional price of c_2 $(x - L)$, where x is the value of X.
3. Products with values of $X < L$ are reprocessed and sold at a reduced price, f_1.
4. The quality characteristic X is normally distributed

with a known variance σ^2 and an unknown mean μ, which is the decision variable.

5. The producer's manufacturing cost per item (UC) is separated into a fixed cost b and a variable cost c, which depends on the value of the quality characteristic X, which is x. Hence, the cost per item is $UC = b + cx = b_0 + c_0 (x - L)$, where $b_0 = b + cL$, $c_0 = c$, $b_0 > 0$, and $c_0 > 0$.

From Chandra [17, pp. 140–145], we have the following Eqs. (1)–(4).

The producer may have to compensate the customer for bad quality, not only by reducing the price to f_1, but also by a price reduction proportional to the deficit in quality which is equal to c_1 $(x - L)$. Hence, the net income per item to the manufacturer for $x < L$ is

$$\begin{aligned} p_1(x) &= f_1 + c_1(x-L) - [b_0 + c_0(x-L)] \\ &= (f_1 - b_0) - (c_0 - c_1)(x-L) \end{aligned} \tag{1}$$

The net income per item to the producer for $x \geq L$ is

$$\begin{aligned} p_2(x) &= f_2 + c_2(x-L) - [b_0 + c_0(x-L)] \\ &= (f_2 - b_0) - (c_0 - c_2)(x-L) \end{aligned} \tag{2}$$

Let $s_1 = f_2 - b_0$, $r_1 = f_1 - b_0$, $s_1 > r_1$, $g_1 = c_0 - c_2$, and $1 - \rho_1 = (c_0 - c_1)/(c_0 - c_2)$.
Hence, Eqs. (1)–(2) can be merged into

$$P(X) = \begin{cases} r_1 - g_1(1-\rho_1)(x-L), & x < L \\ s_1 - g_1(x-L), & x \geq L \end{cases} \tag{3}$$

The expected net income per item is

$$\begin{aligned} &E[P(X)] \\ &= \int_{-\infty}^{L} [r_1 - g_1(1-\rho_1)(x-L)] f(x)dx + \int_{L}^{\infty} [s_1 - g_1(x-L)] f(x)dx \\ &= s_1 - \Phi(-\frac{\mu - L}{\sigma})[s_1 - r_1 - g_1\rho_1(\mu - L)] \\ &-g_1\rho_1\sigma\phi(\frac{\mu - L}{\sigma}) - g_1(\mu - L) \end{aligned} \tag{4}$$

where $\Phi(\cdot)$ is the cumulative probability of a standard normal random variable with probability density function $\phi(\cdot)$

By setting the partial derivative of the above Eq. (4) with respect to 0, Carlsson [1] adopted the numerical methods for finding μ that maximizes the expected net income per item.

3. Modified Carlsson's Model with Beta Quality Characteristic

Consider the beta distribution can accommodate different finite ranges and different shapes from left skewed to symmetrical to right skewed for a product quality characteristic. Assume that the random variable X is a beta distribution. Hence, the expected net income per item of modified Carlsson's [1] model is to

Maximize

$$\begin{aligned} E[P(X)] &= \int_{a+\delta}^{L} [r_1 - g_1(1-\rho_1)(x-L)] f(x)dx \\ &+ \int_{L}^{b+\delta} [s_1 - g_1(x-L)] f(x)dx \\ &= \int_{a+\delta}^{L} [r_1 + g_1L(1-\rho_1)] f(x)dx - \int_{a+\delta}^{L} g_1(1-\rho_1)xf(x)dx \\ &+ \int_{L}^{b+\delta} (s_1 + g_1L) f(x)dx - \int_{L}^{b+\delta} g_1xf(x)dx \\ &= [r_1 + g_1L(1-\rho_1)]\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} B(\frac{L-\delta-a}{b-a};\ \alpha,\ \beta) \\ &- g_1(1-\rho_1)\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \{(\delta+a)B(\frac{L-\delta-a}{b-a};\ \alpha,\ \beta) \\ &+ (b-a)B(\frac{L-\delta-a}{b-a};\ \alpha+1,\ \beta)\} - g_1\{(\delta+a)[1 \\ &- B(\frac{L-\delta-a}{b-a};\ \alpha,\ \beta)\ \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}] \\ &+ (b-a)\ [1 - B(\frac{L-\delta-a}{b-a};\ \alpha+1,\ \beta)\ \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}]\} \\ &+ (s_1 + g_1L)[1 - B(\frac{L-\delta-a}{b-a};\ \alpha,\ \beta)\ \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}] \end{aligned} \tag{5}$$

subject to

$$0 \leq a + \delta \leq L \leq b + \delta \tag{6}$$

where a is the minimum value of beta quality characteristic X; b is the maximum value of X; α is the shape parameter of X, $\alpha \geq 0$; β is the shape parameter of X, $\beta \geq 0$; δ is the location parameter of X;

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{1}{b-a} [\frac{(x-\delta)-a}{b-a}]^{\alpha-1} [\frac{b-(x-\delta)}{b-a}]^{\beta-1}, \quad a \leq x-\delta \leq b \tag{7}$$

$$B(z;\ \alpha,\ \beta) = \int_0^z t^{\alpha-1}(1-t)^{\beta-1} dt \tag{8}$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \tag{9}$$

Let $y = \frac{L-\delta-a}{b-a}$ and $I_y(\alpha,\ \beta) = \int_0^y \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} t^{\alpha-1}(1-t)^{\beta-1} dt$. Hence, the problem of Eqs. (5)–(6) can be rewritten as

Maximize

$$\begin{aligned} E[P(X)] &= [r_1 + g_1 L(1-\rho_1)] I_y(\alpha,\ \beta) - g_1(1-\rho_1)\{(\delta+a) I_y(\alpha,\ \beta) \\ &\quad + (b-a) I_y(\alpha+1,\ \beta) \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+\beta+1)}\} \\ &\quad - g_1\{(\delta+a)[1-I_y(\alpha,\ \beta)] + (b-a)[1-I_y(\alpha+1,\ \beta) \\ &\quad \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+\beta+1)}]\} + (s_1 + g_1 L)[1 - I_y(\alpha,\ \beta)] \end{aligned} \tag{10}$$

subject to

$$0 \leq a+\delta \leq L \leq b+\delta \tag{11}$$

If α and β are both positive integers, then for $0 \leq y \leq 1$,

$$I_y(\alpha,\ \beta) = \sum_{i=\alpha}^{\alpha+\beta-1} C_i^{\alpha+\beta-1} y^i (1-y)^{\alpha+\beta-1-i} \tag{12}$$

That is, $F_Y(y) = P(\alpha \leq U \leq \alpha+\beta-1) = 1 - F_U(\alpha-1)$, where U is a binomial random variable with parameters $n = \alpha+\beta-1$ and $p = y$. Hence, if α and β are both positive integers, then the problem of Eqs. (10)–(11) can be rewritten as

Maximize

$$\begin{aligned} E[P(X)] &= [r_1 + g_1(1-\rho_1)][1-F_U(\alpha-1)] \\ &\quad - g_1(1-\rho_1)\{(\delta+a)[1-F_U(\alpha-1)] + (b-a) \\ &\quad \cdot [1-F_V(\alpha)]\frac{\alpha}{\alpha+\beta}\} - g_1\{(\delta+a)F_U(\alpha-1) \\ &\quad + (b-a)[1-[1-F_V(\alpha)]\frac{\alpha}{\alpha+\beta}]\} + (s_1 + g_1 L) F_U(\alpha-1) \end{aligned} \tag{13}$$

subject to

$$0 \leq a+\delta \leq L \leq b+\delta \tag{14}$$

where U is the binomial random variable with parameters $n = \alpha + \beta - 1$ and $p = y$; V is the binomial rnadomvariable with parameters $n = \alpha + \beta$ and $p = y$.

The above problem of determining the optimum process mean μ $(= \delta + a + (b-a)\frac{\alpha}{\alpha+\beta})$ is to determine the optimal value of δ such that the expected net income per item is maximized. Since, the problem of Eqs. (13)–(14) is one dimensional, we can use the simple interval search method for obtaining the optimum location parameter δ^*.

4. Modified Carlsson's Model with Weibull Quality Characteristic

Consider the Weibull distribution can describe the lifetime of certain electronic product quality characteristic. Assume that the random variable X is a Weibull distribution. Hence, the expected net income per item of modified Carlsson's [1] model is to

Maximize

$$\begin{aligned} E[P(X)] &= \int_0^L [r_1 - g_1(1-\rho_1)(x-L)] f(x) dx \\ &\quad + \int_L^\infty [s_1 - g_1(x-L)] f(x) dx \\ &= \int_0^L [r_1 + g_1 L(1-\rho_1)] f(x) dx \\ &\quad - \int_0^L g_1(1-\rho_1) x f(x) dx + \int_L^\infty (s_1 + g_1 L) f(x) dx \end{aligned}$$

$$-\int_L^\infty g_1 x f(x)dx = [r_1 + g_1 L(1-\rho_1)][1-e^{-(\frac{L-v}{\theta})^\Psi}]$$
$$+(s_1 + g_1 L)e^{-(\frac{L-v}{\theta})^\Psi} - g_1(1-\rho_1)\{-Le^{-(\frac{L-v}{\theta})^\Psi}$$
$$+\frac{\theta}{\Psi}\Gamma(\frac{1}{\Psi})I[\frac{1}{\Psi}, (\frac{L-v}{\theta})^\Psi]\} - g_1\{Le^{-(\frac{L-v}{\theta})^\Psi}$$
$$+\frac{\theta}{\Psi}\Gamma(\frac{1}{\Psi})[1-I[\frac{1}{\Psi}, (\frac{L-v}{\theta})^\Psi]]\} = [r_1 + g_1 L(1-\rho)]$$
$$+(s_1 - r_1)e^{-(\frac{L-v}{\theta})^\Psi} - g_1\frac{\theta}{\Psi}\Gamma(\frac{1}{\Psi})$$
$$+g_1\rho_1\frac{\theta}{\Psi}\Gamma(\frac{1}{\Psi})I[\frac{1}{\Psi}, (\frac{L-v}{\theta})^\Psi] \quad (15)$$

subject to

$$0 \le v \le L \quad (16)$$

where Ψ is the scale parameter of Weibull quality characteristic X, $\Psi > 0$; θ the scale parameter of X, $\theta > 0$; v is the location parameter of X;

$$f(x) = \frac{\Psi}{\theta}(\frac{x-v}{\theta})^{\Psi-1}e^{-(\frac{x-v}{\theta})^\Psi}, \; x-v \ge 0 \quad (17)$$

$$\Gamma(z) = \int_0^\infty u^{z-1}e^{-u}du \quad (18)$$

$$I[z,p] = \frac{1}{\Gamma(z)}\int_0^p u^{z-1}e^{-u}du \quad (19)$$

The above problem of determining the optimum process mean μ ($=\theta\Gamma(1+\frac{1}{\Psi}) + v$) is to determine the optimal value of v such that the expected net income per item is maximized. Since, the problem of Eqs. (15)– (16) is one dimensional, we can use the simple interval search method for obtaining the optimum location parameter v^*. For Eq. (15), $I[z, p]$ is the incomplete gamma function. If z is a positive integer, then $I[z, p] = 1 - \sum_{i=0}^{z-1}\frac{p^i}{i!}e^{-p}$. Since $\lim_{p\to 0} I[z,p] = 1$ with $z > 0$ and $p > 0$, the values of the incomplete gamma function are always assumed to be between zero and one. As the location parameter v increases, the function $I[\frac{1}{\Psi}, (\frac{L-v}{\theta})^\Psi]$ and $e^{-(\frac{L-v}{\theta})^\Psi}$ increase. Hence, the optimum location parameter v^* should be the lower specification limit, L, for modified Carlsson's [1] model with Weibull distribution.

5. Numerical Example

Consider a numerical example given in Chandra [17, pp. 140–145]. Let $L = 10$, $b_0 = 5$, $f_2 = 8$, $f_1 = 6$, $c_0 = 1$, $c_2 = 0.8$, and $c_1 = 0.6$. Hence, the corresponding values are $s_1 = 3$, $r_1 = 1$, $g_1 = 0.2$, and $\rho_1 = -1$. The quality characteristic is normally distributed with the unknown mean μ and the known standard deviation $\sigma = 0.05$. By solving Eq. (4), the optimal solution is $\mu^* = 10.148$ with $E[P(X)] = 2.967$.

Assume that the quality characteristic is a beta distribution with the unknown δ and the known parameters $a = 8$, $b = 14$, $\alpha = 2$ and $\beta = 4$. By solving the problem of Eqs. (13)–(14), the optimal solution is $\delta^* = 1.946$ ($\mu^* = 11.946$) with $E[P(X)] = 1.805$.

Assume that the quality characteristic is a Weibull distribution with the unknown v and the known parameters $\theta = 1$ and $\Psi = 0.5$. By solving the problem of Eqs. (15)–(16), the optimal solution is $v^* = 10$ ($\mu^* = 12$) with $E[P(X)] = 8.152$.

Tables 1–3 list ±20% change for parameter value and present the effect on the process mean and the expected net income per item. If the change percentage of the expected net income per item is larger than 10%, then the parameter has a significant effect on the expected net income per item. From Tables 1–3, we have the following conclusions:

1. All profit and cost parameters (b_0, c_0, c_1, c_2, f_1 and f_2) have a slight effect on the optimum process mean. The parameters of beta and Weibull distributions, (a, b, α, β, θ, Ψ) have a major effect on the optimum process mean.
2. The parameters have a major effect on the expected net income per item for Carlsson's [1] model with normal quality characteristic are b_0 and f_2. The parameters have a major effect on the expected net income per item for model with beta quality characteristic are b_0, c_0, c_2, f_2, a, b and α. The parameters have a major effect on the expected net income per item for model

Table 1. The effect of parameters for optimal solution under modified model with normal quality characteristic

b_0	μ	$E[P(X)]$
4	10.148	3.967
6	10.148	1.967
f_1	μ	$E[P(X)]$
4.8	10.156	2.966
7.2	10.131	2.970
f_2	μ	$E[P(X)]$
6.4	10.117	1.373
9.6	10.158	4.566
c_0	μ	$E[P(X)]$
0.8	10.262	3.000
1.2	10.136	2.939
c_1	μ	$E[P(X)]$
0.48	10.148	2.967
0.72	10.148	2.967
c_2	μ	$E[P(X)]$
0.64	10.138	2.945
0.96	10.173	2.993
σ	μ	$E[P(X)]$
0.04	10.121	2.973
0.06	10.174	2.961

Table 2. The effect of parameters for optimal solution under modified model with beta quality characteristic

b_0	(δ^*, μ^*)	$E[P(X)]$
4	(1.946, 11.946)	2.805
6	(1.946, 11.946)	0.805
f_1	(δ^*, μ^*)	$E[P(X)]$
4.8	(1.956, 11.956)	1.804
7.2	(1.930, 11.930)	1.807
f_2	(δ^*, μ^*)	$E[P(X)]$
6.4	(1.923, 11.923)	0.208
9.6	(1.958, 11.958)	3.404
c_0	(δ^*, μ^*)	$E[P(X)]$
0.8	(2.000, 12.000)	3.000
1.2	(1.917, 11.917)	0.616
c_1	(δ^*, μ^*)	$E[P(X)]$
0.48	(1.955, 11.955)	1.804
0.72	(1.933, 11.933)	1.807
c_2	(δ^*, μ^*)	$E[P(X)]$
0.64	(1.899, 11.899)	0.858
0.96	(1.990, 11.990)	2.760
a	(δ^*, μ^*)	$E[P(X)]$
6.4	(3.513, 12.446)	1.489
9.6	(0.371, 11.438)	2.123
b	(δ^*, μ^*)	$E[P(X)]$
11.2	(1.985, 11.052)	2.361
16.8	(1.881, 12.814)	1.252
α	(δ^*, μ^*)	$E[P(X)]$
1	(5.000, 14.200)	36.181
3	(5.000, 15.571)	245.772
β	(δ^*, μ^*)	$E[P(X)]$
3	(1.915, 12.315)	1.808
5	(1.963, 11.677)	1.804

with Weibull quality characteristic are b_0, c_0, c_1, c_2, f_2, θ and Ψ. The modified model with Weibull quality characteristic has the largest expected net income per item than those of models with normal and beta quality characteristics. The modified model with beta quality characteristic has the smallest expected net income per item. Hence, one needs to have a correct estimate on these parameters for obtaining the exact expected net income per item.

6. Conclusion

For the larger-the-better quality characteristic, it is not possible to increase the process mean without any bound because of increasing the manufacturing cost. Carlsson [1] considered the piecewise linear profit function for product with normal quality characteristic. In this paper, the author has presented the modified Carlsson's [1] model with beta and Weibull distributions for maximizing the expected net income per item. From the above numerical results, we have the conclusions that (1) the scale and shape parameters of beta and Weibull distributions have a major effect on the optimum process mean and the expected net income per item; (2) the full price of the accepted item, f_2, and the manufacturing cost with $X = L$, b_0, have a major effect on the expected net income per item for three models. Further

Table 3. The effect of parameters for optimal solution under modified model with Weibull quality characteristic

b_0	(v^*, μ^*)	$E[P(X)]$
4	(10, 12)	9.152
6	(10, 12)	7.152
f_1	(v^*, μ^*)	$E[P(X)]$
4.8	(10, 12)	8.123
7.2	(10, 12)	8.181
f_2	(v^*, μ^*)	$E[P(X)]$
6.4	(10, 12)	6.590
9.6	(10, 12)	9.714
c_0	(v^*, μ^*)	$E[P(X)]$
0.8	(10, 12)	2.952
1.2	(10, 12)	8.152
c_1	(v^*, μ^*)	$E[P(X)]$
0.48	(10, 12)	7.752
0.72	(10, 12)	5.352
c_2	(v^*, μ^*)	$E[P(X)]$
0.64	(10, 12)	6.232
0.96	(10, 12)	6.872
θ	(v^*, μ^*)	$E[P(X)]$
2	(10, 14)	6.166
3	(10, 16)	5.772
Ψ	(v^*, μ^*)	$E[P(X)]$
1/3	(10, 16)	5.639
1/4	(10, 34)	1.911

study will extend this method to an integrated quality and inventory model for obtaining the optimal inventory and quality control.

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