

The Joint Determination of Optimum Process Mean and Economic Order Quantity

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Abstract

In Chen and Liu's [1] model with traditional production system, they neglected the effect of product quality on the retailer's order quantity. Their model only considered the order quantity obeying the uniform distribution. In fact, the retailer's order quantity is concerned with product quality. Chen and Liu's [1] model with simple manufacturing cost did not consider the used cost of customers. Hence, the modified Chen and Liu's [1] model should be addressed for determining the optimal process parameter. In this study, the author proposes a modified Chen and Liu's [1] model with quality loss and single sampling inspection plan. Assume that the retailer's order quantity is concerned with the manufacturer's product quality and the quality characteristic of product is normally distributed. Taguchi's symmetric quadratic quality loss function is applied in evaluating the product quality. The optimal retailer's order quantity and the manufacturer's process mean will be simultaneously determined by maximizing the expected total profit of society including the manufacturer and the retailer.

Key Words: Economic Order Quantity, Process Mean, Taguchi's Quadratic Quality Loss Function

1. Introduction

The supply chain system is a major topic for the manufacturing industries in order to obtain the maximum expected total profit of society including the manufacturer and the retailer. The manufacturer's objective needs to consider the sale revenue, the manufacturing cost, the inspection cost, and the inventory cost for having the maximum expected profit. The retailer's objective needs to consider the order quantity, the holding cost, the goodwill loss of cost, and the used cost of customer for having the maximum expected profit. Hence, the market needs to solve the problem of "how to get a trade-off between them". Goyal [2] first proposed an integrated inventory model with the total costs from the producers and the purchasers and obtained the optimum production quantity and order cycle. Lu [3] considered

the problem of an integrated inventory model with a single producer and multi-purchasers and presented a heuristic method for obtaining the model's optimal solution. Hill [4] modified the Goyal's [2] and Lu's [3] models for determining the optimum production quantity and schedule under the minimum expected total cost. Goyal and Nebebe [5] proposed the optimum production and transportation policy with a minimum cost between the producers and the purchasers. Goyal [6] further formulated the modified Hill's [4] model for obtaining the optimum transportation quantity under the minimum time cost per unit. Seifert et al. [7] explained the resulting procurement change and proposed the benefits of using spot markets from a supply chain perspective. They developed the mathematical model that determines the optimal order quantity to purchase via forward contracts and spot markets. Haksoz and Seshadri [8] discussed about supply chain system with spot market and presented a literature review about recent work. Chen

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and Liu [9] presented the optimum profit model between the producers and the purchasers for the supply chain system with pure procurement policy from the regular supplier and mixed procurement policy from the regular supplier and the spot market. Chen and Liu [1] further proposed an optimal consignment policy considering a fixed fee and a per-unit commission. Their model determines a higher manufacturer's profit than the traditional production system and coordinates the retailer to obtain a large supply chain profit. Li and Liu [10] considered the problem about the retailer determining his optimal order quantity and the manufacturer determining his optimal reserve capacity. Their model can make two sides of the supply chain increase profit. Darwish [11] proposed the optimum process mean setting for a single purchaser and a single purchaser in the filling industry. The optimum process mean, shipment quantity of product, and numbers of shipment need to be determined in his model. Chen and Huang [12] addressed that the retailers purchase their products from options and online spot markets for hedging the risk of demand uncertainty. Arshinder et al. [13] proposed a review of supply chain coordination and presented its research directions for further study.

In 1997, Sana et al. [14,15] proposed a volume flexible inventory model considering the demand rate of defective items sold at a reduced price as a function of the reduction rate of the selling price. They further addressed the imperfect production system including the unit production cost taken to be a function of the finite production rate. In 2010, Sana [16,17] considered the production-inventory model with imperfect production system for defective items restored to their original quality by rework. His model also discusses and provides an optimal solution for product reliability parameter and dynamic production rate. Sana [18] discussed that the enterprises' initiatives affect the time varying demand for a multi-item economic order quantity problem. Sana [19] further presented a economic order quantity model when demand is price demand and partial backorder is permitted.

In Chen and Liu's [1] model with traditional production system, they neglected the effect of product quality on the retailer's order quantity. They only considered the order quantity obeying the uniform distribution. In fact, the retailer's order quantity is concerned

with the product quality. Chen and Liu's [1] model with simple manufacturing cost did not consider the used cost of customers in the traditional production system. Hence, the modified Chen and Liu's [1] model needs to be addressed for determining the optimum process parameters.

Economic selection of process mean is an important problem for modern statistical process control. It will affect the expected profit/cost per item. Recently, many researchers have addressed this work. Both 100% inspection and sampling inspection are considered for different models. Taguchi [20] presented the quadratic quality loss function for redefining the product quality. Hence, the optimum product quality should be the quality characteristic with minimum bias and variance. Recently, his quality loss function has been successfully applied in the problem of optimum process mean setting.

In this paper, the author proposes a modified Chen and Liu's [1] model with quality loss and adopting the single sampling inspection plan for determining the lot quality of manufacturer. Assume that the retailer's order quantity is concerned with the manufacturer's product quality and the quality characteristic of product is normally distributed. Taguchi's [20] symmetric quadratic quality loss function will be applied in evaluating the product quality. The optimal retailer's order quantity and the manufacturer's process mean will be simultaneously determined by maximizing the expected total profit of society including the manufacturer and the retailer. The motivation behind this work stems from the fact that the neglect of the quality loss within the specification limits should have the overestimated expected total profit of society.

2. Literature Review --- Chen and Liu's [1] Traditional System Model

Chen and Liu's [1] traditional system model is actually based on the standard news-vendor model without spot markets. They consider a single period manufacturer-retailer relationship in which a regular manufacturer produces short-life cycle products and a retailer orders products from the regular manufacturer and then sells to the end customer. Some assumptions are as follows:

1. A retailer purchases a finished product from a regular supplier and resells it at a price, R , to the end customer.

2. The regular manufacturer produces each unit at a cost, C .
3. The regular manufacturer and the retailer enter into a contract at a wholesale price, W .
4. The regular manufacturer sets the wholesale price to maximize his expected profit while offering the buyer a specific order quantity, Q .
5. When realized demand exceeds procurement quantity, unmet demand occurs a goodwill lost, S , for the retailer; therefore, demand uncertainty exposes the retailer to risks associated with mismatches between the procurement quantity and demand. When realized demand is less than procurement quantity, the retailer occurs a carrying cost, H , for the retailer.
6. The procurement lead time is long relative to the selling season, so that the buyer cannot observe demand before placing the order.
7. The consumer demand, X , is an uniform distribution, i.e.,

$$X \sim U[\mu_x - (\sigma_x/2), \mu_x + (\sigma_x/2)]$$
8. where μ_x is the mean of X and σ_x is the variability of X .

The retailer's profit is given by

$$\pi_{PS}^R = \begin{cases} RX - WQ - H(Q - X), & X < Q \\ RQ - WQ - S(X - Q), & X \geq Q \end{cases} \quad (1)$$

The retailer's expected profit can be expressed as

$$E(\pi_{PS}^R) = \int_{\mu_x - (\sigma_x/2)}^Q [Rx - WQ - H(Q - x)]f(x)dx + \int_Q^{\mu_x + (\sigma_x/2)} [(R - W)Q - S(x - Q)]f(x)dx \quad (2)$$

where $f(x)$ is the density function of X .

Let the partial derivative of the retailer's expected profit function with respect to Q be zero, i.e., $\frac{dE(\pi_{PS}^R)}{dQ} =$

0. From Appendix of Chen and Liu (2008), the optimal order quantity

$$Q^* = \left(\mu_x - \frac{\sigma_x}{2} \right) + \frac{(R - W + S)\sigma_x}{R + H + S} \quad (3)$$

The regular manufacturer maximizes his expected profit and determines the wholesale price per unit based

on the retailer's order quantity Q^* . Hence, the regular manufacturer's expected profit is expressed as

$$E(\pi_{PS}^S) = (W - C)Q^* \quad (4)$$

where C is the regular manufacturer's production cost per unit.

Let the partial derivative of the regular manufacturer's expected profit function with respect to W be zero, i.e., $\frac{dE(\pi_{PS}^S)}{dW} = 0$. The optimal W value of equation (4) yields

$$W^* = \frac{R + 2C - H + S}{4} + \frac{(R + H + S)\mu_x}{2\sigma_x} \quad (5)$$

Substituting equation (5) into equation (3), the optimal Q value can be rewritten as

$$Q^* = \frac{1}{4} \left[(2\mu_x + \sigma_x) - \frac{2(C + H)\sigma_x}{R + H + S} \right] \quad (6)$$

From equations (2), (4), (5), and (6), the retailer's and manufacturer's expected profit can be expressed as

$$E(\pi_{PS}^R) = \frac{[2R\mu_x - 2C\sigma_x + R\sigma_x + (2\mu_x - \sigma_x)H + (2\mu_x + \sigma_x)S]^2}{16\sigma_x(R + H + S)} \quad (7)$$

$$E(\pi_{PS}^S) = \mu_x(R + H) - \frac{1}{2\sigma_x} \left\{ (R + H + S) \left[\left(\mu_x + \frac{\sigma_x}{2} \right)^2 - \frac{1}{16} \left(2\mu_x + \sigma_x - \frac{2\sigma_x(C + H)}{R + H + S} \right)^2 \right] \right\} \quad (8)$$

3. Modified Chen and Liu's [1] Traditional System Model

Chen and Liu's [1] traditional system model with constant manufacturing cost per unit for the manufacturer is too simple. In fact, the manufacturing cost per unit should include the constant and variable production costs. The variable production cost is proportional to the value of quality characteristic. Chen and Liu's [1] model also did not consider the used cost of customers. The neglect of the quality loss within the specification limits

should have the overestimated expected profit per item for the retailer. Assume that the quality characteristic $Y \sim N(\mu_y, \sigma_y^2)$ and $X|Y \sim N(\lambda_1 + \lambda_2 Y, \sigma^2)$, where λ_1, λ_2 , and σ^2 are constants. Hence, we have $X \sim N(\lambda_1 + \lambda_2 \mu_y, \lambda_2^2 \sigma_y^2 + \sigma^2)$ and $Y|X \sim N\left(\frac{\lambda_2 \sigma_y^2 (x - \lambda_1) + \mu_y \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}, \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}\right)$. Hence, the author proposes the following

modified Chen and Liu's [1] model.

The retailer's profit is given by

$$\pi_{PS}^R = \begin{cases} RX - WQ - H(Q - X) - X \cdot \text{Loss}(Y), & X < Q, -\infty < Y < \infty \\ RQ - WQ - S(X - Q) - Q \cdot \text{Loss}(Y), & X \geq Q, -\infty < Y < \infty \end{cases} \quad (9)$$

where Y is the normal quality characteristic of product, $Y \sim N(\mu_y, \sigma_y^2)$; μ_y is the unknown mean of Y ; σ_y is the known standard deviation of Y ; $\text{Loss}(Y)$ is the quality loss per unit, $\text{Loss}(Y) = k(Y - y_0)^2$; k is the quality loss coefficient; y_0 is the target value of product.

From Appendix, the retailer's expected profit is

$$E(\pi_{PS}^R) = E(\pi_1) + E(\pi_2) + E(\pi_3) + E(\pi_4) \quad (10)$$

where

$$E(\pi_1) = (R + H) \left\{ (\lambda_1 + \lambda_2 \mu_y) \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) - \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} - (W + H) Q \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \quad (11)$$

$$E(\pi_2) = (R - W + S) Q \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] - S \left\{ (\lambda_1 + \lambda_2 \mu_y) \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] + \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \quad (12)$$

$$\begin{aligned} E(\pi_3) = & kA^2 \left\{ \mu_k^3 \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) + 3\mu_k^2 \sigma_k \left[-\phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] + 3\mu_k \sigma_k^2 \left[-\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right. \right. \\ & \cdot \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) + \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \left. \right] \\ & + \sigma_k^3 \left[-\left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right)^2 \cdot \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) - 2\phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \\ & + 2kAB \left\{ (\lambda_1 + \lambda_2 \mu_y)^2 \left[\Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] - 2 \cdot (\lambda_1 + \lambda_2 \mu_y) \cdot \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right. \\ & \left. + (\lambda_2^2 \sigma_y^2 + \sigma^2) \left[-\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \cdot \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) + \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right\} \\ & + k(B^2 + C_0) \left\{ (\lambda_1 + \lambda_2 \mu_y) \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) - \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \end{aligned} \quad (13)$$

$$\begin{aligned} E(\pi_4) = & kA^2 Q \left\{ (\lambda_1 + \lambda_2 \mu_y)^2 \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] + 2(\lambda_1 + \lambda_2 \mu_y) \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right. \\ & + (\lambda_2^2 \sigma_y^2 + \sigma^2) \left[\left[\left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] + \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right] \left. \right\} \end{aligned}$$

$$\begin{aligned}
& +2kQAB \left\{ (\lambda_1 + \lambda_2 \mu_y) \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \\
& \left. + \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \\
& + kQ(B^2 + C_0) \left\{ 1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \quad (14)
\end{aligned}$$

where $\mu_k = \lambda_1 + \lambda_2 \cdot \mu_y$, $\sigma_k = \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}$, $A = \frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}$; $B = \frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}$; $C_0 = \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}$; $\Phi(\cdot)$ is the cumulative distribution function of standard normal random variable; $\phi(\cdot)$ is the probability density function of standard normal random variable.

Assume that the retailer's order quantity is equal to the lot size of single sampling inspection plan. A single sampling inspection plan is adopted for determining the lot quality of manufacturer. If the lot is accepted, then the selling price of product per unit is W . If the lot is rejected, then a product is scrapped and sold at a price S_p . The manufacturer's profit under the normal quality characteristic is given by

$$\pi_{PS}^S = \begin{cases} WQ - ni - Qc\mu_y, & D \leq d_0 \\ S_p Q - ni - Qc\mu_y, & D > d_0 \end{cases} \quad (15)$$

where n is the sample size; c is the variable production cost per unit; i is the inspection cost per unit; d_0 is the acceptance number; D is the number of non-conformance in the sample.

The manufacturer's expected profit is

$$E(\pi_{PS}^S) = (WQ - ni - Qc\mu_y)P_a + (S_p Q - ni - Qc\mu_y)(1 - P_a) \quad (16)$$

where

$$P_a = \sum_{d=0}^{d_0} \frac{e^{-np} \cdot (np)^d}{d!} \quad (17)$$

$$p = P(Y < L) + P(Y > U) = 1 - \left[\Phi \left(\frac{U - \mu_y}{\sigma_y} \right) - \Phi \left(\frac{L - \mu_y}{\sigma_y} \right) \right] \quad (18)$$

L is the lower specification limit of product; U is the upper specification limit of product.

The expected total profit of society including the retailer and the manufacturer is

$$ETP(Q, \mu_y) = E(\pi_{PS}^R) + E(\pi_{PS}^S) \quad (19)$$

The trade-off model between Eqs. (10) and (16) needs to obtain the optimal retailer's order quantity (Q^*) and the optimal manufacturer's process mean (μ_y^*) with the maximum expected profits for the retailer and the manufacturer. Let $L < \mu_y < U$. One can adopt direct search method for obtaining the optimal μ_y^* with the maximum expected total profit of society for Eq. (19) with the given order quantity. The combination (Q^*, μ_y^*) with maximum expected total profit of society is the optimal solution.

4. Numerical Example and Sensitivity Analysis

Assume that some parameters are as follows: $R = 70$, $W = 10$, $S = 3$, $H = 2$, $\lambda_1 = 1$, $\lambda_2 = 2$, $n = 16$, $d_0 = 0$, $\sigma = 1$, $y_0 = 10$, $\sigma_y = 0.5$, $i = 0.05$, $k = 1$, $c = 0.5$, $L = 8$, $U = 12$, and $S_p = 2$. By setting $8 < \mu_y < 12$ and solving Eq. (19) by direct search method, one obtains the optimal process mean $\mu_y^* = 9.15$. The corresponding optimal order quantity $Q^* = 21$ with retailer's expected profit $E(\pi_{PS}^R) = 864.90$, manufacturer's expected profit $E(\pi_{PS}^S) = 86.74$, and expected total profit of society $ETP(Q, \mu_y) = 951.64$.

Table 1 lists $\pm 20\%$ change for parameter value and presents the effect on the process mean, the order quantity, the retailer's expected profit, the manufacturer's expected profit, and the expected total profit of society. If the change percentage of expected profit is larger than 10%, then the parameter has a major effect on the expected profit. From Table 1, one has the following conclusions:

1. The order quantity is constant and the process mean increases as the sale price per unit, R , increases. The sale price per unit has a major effect on the retailer's and society's expected profits.
2. The order quantity and the process mean are constants as the goodwill loss per unit of stockout, S , increases. The goodwill loss per unit of stockout has a slight effect on the retailer's, manufacturer's and society's expected profits.

Table 1. The effect of parameters for optimal solution

R	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
56	21	9.11	601.38	81.67	683.05	-28.22
84	21	9.19	1130.16	91.06	1221.21	28.33
S	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
2.4	21	9.15	864.95	86.74	951.69	0.005
3.6	21	9.15	864.86	86.74	951.59	-0.005
W	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
8	21	9.09	913.66	45.61	959.27	0.80
12	21	9.19	818.08	127.62	945.69	-0.62
k	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
0.8	21	9.23	911.93	94.67	1006.60	5.78
1.2	21	9.08	823.10	77.35	900.45	-5.38
σ_y	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
0.4	21	9.15	990.72	107.79	1098.51	15.43
0.6	21	9.09	743.32	42.61	785.93	-17.41
σ	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
0.9	21	9.09	803.30	78.84	882.14	-7.30
1.2	21	9.30	952.98	104.29	1057.27	11.10
λ_1	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
0.8	21	9.16	853.73	87.89	941.62	-1.05
1.2	21	9.14	875.81	85.55	961.35	1.02
λ_2	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
1.6	19	9.89	899.19	95.02	994.22	4.47
2.2	22	9.05	809.15	76.08	885.23	-6.98
c	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
0.40	21	9.15	864.90	105.96	970.86	2.02
0.60	21	9.15	864.90	67.53	932.43	-2.02
i	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
0.040	21	9.15	864.90	86.88	951.78	1.43
0.060	21	9.15	864.90	86.61	951.51	-1.40
S_p	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
1.6	21	9.16	863.72	86.63	950.34	-0.14
2.4	21	9.14	866.07	86.94	953.00	0.14
n	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
8	21	9.03	877.85	90.34	968.19	1.74
12	21	9.10	870.57	88.11	958.68	0.74
20	21	9.19	860.07	85.93	946.00	-0.59
24	21	9.22	856.28	85.01	941.29	-1.09
d_0	Q	μ_y	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	$ETP(Q, \mu_y)$	Per
1	21	8.96	884.38	103.03	987.41	3.76
2	21	8.82	895.26	108.47	1003.73	5.47
3	21	8.72	901.32	112.01	1013.33	6.48
4	20	8.62	912.34	108.08	1020.42	7.23

Note: $\text{Per} = \frac{ETP(Q, \mu_y) - 951.64}{951.64} \cdot 100\%$

3. The order quantity is constant and the process mean increases as the wholesale price, W , increases. The wholesale price has a major effect on the manufacturer's expected profit.
4. The order quantity is constant and the process mean decreases as the quality loss coefficient, k , increases. The quality loss coefficient has a slight effect on the retailer's, manufacturer's and society's expected profits.
5. The order quantity is constant and the process mean decreases as the standard deviation of quality characteristic of product, σ_y , increases. The standard deviation of quality characteristic of product has a major effect on the retailer's, manufacturer's, and society's expected profits.
6. The order quantity is constant and the process mean increases as the standard deviation of conditional random variable $X|Y$, σ , increases. The standard deviation of conditional random variable $X|Y$ has a major effect on the retailer's, manufacturer's, and society's expected profits.
7. The order quantity is constant and the process mean decreases as the intercept of mean for conditional random variable $X|Y$, λ_1 , increases. The intercept of mean for conditional random variable $X|Y$ has a slight effect on the retailer's, manufacturer's, and society's expected profits.
8. The order quantity increases and the process mean decreases as the slope of mean for conditional random variable $X|Y$, λ_2 , increases. The slope of mean for conditional random variable $X|Y$ has a slight effect on the retailer's, manufacturer's, and society's expected profits.
9. The order quantity and the process mean are constants as the variable production cost per unit, c , increases. The variable production cost per unit has a major effect on the manufacturer's expected profit.
10. The order quantity and the process mean are constants as the inspection cost per unit, i , increases. The inspection cost per unit has a slight effect on the retailer's, manufacturer's and society's expected profits.
11. The order quantity is constant and the process mean increases as the discounted price per unit for the non-conformance product scrapped, S_p , increases. The discounted price per unit for the non-conformance product scrapped has a slight effect on the retailer's, manufacturer's, and society's expected profits.
12. The order quantity is constant and the process mean increases as the sample size, n , increases. The sample size has a slight effect on the retailer's, manufacturer's, and society's expected profits.
13. The order quantity and the process mean decrease as the acceptance number, d_0 , increases. The acceptance number has a slight effect on the retailer's, manufacturer's, and society's expected profits.

5. Conclusion

In this paper, the author has presented a modified Chen and Liu's [1] traditional system model with quality loss of product. Assume that the retailer's order quantity is concerned with the manufacturer's product quality and the quality characteristic of product is normally distributed. The quality of lot for manufacturer is decided by adopting a single sampling inspection plan. The process mean of quality characteristic and the order quantity of retailer are simultaneously determined in the modified model. From the above numerical results, one has the following conclusion: (1) The sale price per unit, the standard deviation of quality characteristic of product, and the standard deviation of conditional random variable $X|Y$ have a major effect on the expected profit of the retailer; (2) The wholesale price, the standard deviation of quality characteristic of product, the standard deviation of conditional random variable $X|Y$, and the variable production cost per unit have a major effect on the expected profit of the manufacturer; (3) The sale price per unit, the standard deviation of quality characteristic of product, and the standard deviation of conditional random variable $X|Y$ have a major effect on the expected total profit of the society. The extension to modified Chen and Liu's (2008) model with quality investment or mixed procurement model may be left for further study.

Appendix. Derivative of Expected Profit for Retailer

The retailer's profit is given by

$$\pi_{PS}^R = \begin{cases} RX - WQ - H(Q - X) - X \cdot \text{Loss}(Y), & X < Q, \quad -\infty < Y < \infty \\ RQ - WQ - S(X - Q) - Q \cdot \text{Loss}(Y), & X \geq Q, \quad -\infty < Y < \infty \end{cases} \quad (A1)$$

Hence, the expected profit of retailer can be divided by the following four parts.

$$\begin{aligned}
 1. & \int_{-\infty}^Q \int_{-\infty}^{\infty} [(R+H)x - (W+H)Q] f(x, y) dy dx \\
 &= \int_{-\infty}^Q \int_{-\infty}^{\infty} (R+H)xf(x, y) dy dx - \int_{-\infty}^Q \int_{-\infty}^{\infty} (W+H)Qf(x, y) dy dx \\
 &= \int_{-\infty}^Q (R+H)xm(x)dx - (W+H)Q \int_{-\infty}^Q m(x)dx \\
 &= (R+H) \left\{ (\lambda_1 + \lambda_2 \mu_y) \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right. \\
 &\quad \left. - \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \\
 &\quad - (W+H)Q \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \quad (A2)
 \end{aligned}$$

$$\begin{aligned}
 2. & \int_Q^{\infty} \int_{-\infty}^{\infty} (R-W+S)Qf(x, y) dy dx - S \int_Q^{\infty} \int_{-\infty}^{\infty} xf(x, y) dy dx \\
 &= (R-W+S)Q \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] - S \int_Q^{\infty} xm(x)dx \\
 &= (R-W+S)Q \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \\
 &\quad - S \left\{ (\lambda_1 + \lambda_2 \mu_y) \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \\
 &\quad \left. + \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \quad (A3)
 \end{aligned}$$

$$\begin{aligned}
 3. & \int_{-\infty}^Q \int_{-\infty}^{\infty} x \cdot \text{Loss}(y) f(x, y) dy dx \\
 &= \int_{-\infty}^Q x \int_{-\infty}^{\infty} \text{Loss}(y) f(x, y) dy dx \\
 &= \int_{-\infty}^Q x \cdot k \cdot m(x) p(y|x)(y - y_0)^2 dy dx \\
 &= \int_{-\infty}^Q xkm(x) \left\{ \left[\frac{\lambda_2^2 \sigma_y^2 (x - \lambda_1) + \mu_y \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0 \right]^2 + \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \right\} dx \\
 &= \int_{-\infty}^Q xkm(x) \left\{ \left(\frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \right)^2 \cdot x^2 \right. \\
 &\quad + 2 \cdot \frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \left[\frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0 \right] \cdot x \\
 &\quad \left. + \left[\frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0 + \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \right] \right\} dx \quad (A4)
 \end{aligned}$$

$$\text{Let } A = \frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}, B = \frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0, \text{ and } C_0 = \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}.$$

Eq. (A4) can be rewritten as

$$\begin{aligned}
 & \int_{-\infty}^Q xkm(x) [A^2 x^2 + 2ABx + B^2 + C_0] dx \\
 &= kA^2 \int_{-\infty}^Q x^3 m(x) dx + 2kAB \int_{-\infty}^Q x^2 m(x) dx \\
 &\quad + k(B^2 + C_0) \int_{-\infty}^Q xm(x) dx \\
 &= kA^2 \left\{ \mu_k^3 \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) + 3\mu_k^2 \sigma_k \left[-\phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \\
 &\quad + 3\mu_k \sigma_k^2 \left[-\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \cdot \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right. \\
 &\quad \left. + \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] + \sigma_k^3 \left[-\left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right)^2 \right. \\
 &\quad \left. \cdot \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) - 2\phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \\
 &\quad \left. + 2kAB \left\{ (\lambda_1 + \lambda_2 \mu_y)^2 \left[\Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \right. \\
 &\quad \left. - 2 \cdot (\lambda_1 + \lambda_2 \mu_y) \cdot \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right. \\
 &\quad \left. + (\lambda_2^2 \sigma_y^2 + \sigma^2) \left\{ -\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right. \right. \\
 &\quad \left. \cdot \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) + \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \right\} \\
 &\quad + k(B^2 + C_0) \left\{ (\lambda_1 + \lambda_2 \mu_y) \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right. \\
 &\quad \left. - \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \quad (A5)
 \end{aligned}$$

where $\mu_k = \lambda_1 + \lambda_2 \cdot \mu_y$ and $\sigma_k = \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}$.

$$\begin{aligned}
4. & \int_0^\infty \int_{-\infty}^\infty Q \text{Loss}(y) f(x, y) dy dx \\
&= Q \int_0^\infty km(x) p(y|x) (y - y_0)^2 dy dx \\
&= Q \int_0^\infty km(x) \left\{ \left[\frac{\lambda_2^2 \sigma_y^2 (x - \lambda_1) + \mu_y \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0 \right]^2 + \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \right\} dx \\
&= Q \int_0^\infty km(x) \left\{ \left[\frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \cdot x + \frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0 \right]^2 \right. \\
&\quad \left. + \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} \right\} dx \tag{A6}
\end{aligned}$$

$$\text{Let } A = \frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}, B = \frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0, \text{ and } C_0 = \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}.$$

Hence, Eq. (A6) can be rewritten as

$$\begin{aligned}
& Q \int_0^\infty km(x) [A^2 x^2 + 2ABx + B^2 + C_0] dx \\
&= kA^2 Q \int_0^\infty x^2 m(x) dx + 2kQAB \int_0^\infty xm(x) dx \\
&\quad + kQ(B^2 + C_0) \int_0^\infty m(x) dx \\
&= kA^2 Q \left\{ (\lambda_1 + \lambda_2 \mu_y)^2 \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \\
&\quad + 2(\lambda_1 + \lambda_2 \mu_y) \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \\
&\quad + (\lambda_2^2 \sigma_y^2 + \sigma^2) \left\{ \left[\left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \\
&\quad \left. + \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right\} \left. \right\} \\
&\quad + 2kQAB \left\{ (\lambda_1 + \lambda_2 \mu_y) \left[1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right] \right. \\
&\quad + \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2} \phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \left. \right\} \\
&\quad + kQ(B^2 + C_0) \left\{ 1 - \Phi \left(\frac{Q - \lambda_1 - \lambda_2 \mu_y}{\sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}} \right) \right\} \tag{A7}
\end{aligned}$$

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