

Free Vibration Analysis of Waves in a Microstretch Elastic Plate

Rajneesh Kumar^{1*} and Geeta Partap²

¹*Department of Mathematics, Kurukshetra University,
Kurukshetra, Haryana, India - 136 119*

²*Department of Mathematics, Dr. B. R. Ambedkar National Institute of Technology,
Jalandhar, Punjab, India - 144011*

Abstract

The free vibration analysis of waves in a homogeneous isotropic microstretch elastic plate subjected to stress free conditions is investigated. The secular equations for homogeneous isotropic microstretch elastic plate for symmetric and skew-symmetric wave modes are derived. The mathematical model has been simplified by using the Helmholtz decomposition technique and the frequency equations for different mechanical situations are obtained and discussed. The special cases such as short wavelength and regions of secular equations are deduced and discussed. The dispersion curves for symmetric and skew-symmetric modes are computed numerically and presented graphically. Results of some earlier workers have been deduced as particular cases.

Key Words: Microstretch elastic plate, Secular equations, Phase velocity, Micropolar elastic plate

1. Introduction

A micropolar continuum is a collection of inter-connected particles in the form of small rigid bodies undergoing both translational and rotational motions. Typical examples of such materials are granular media and multi-molecular bodies whose microstructures act as an evident part in their macroscopic responses. Rigid chopped fibres, elastic solids with rigid granular inclusions and other industrial materials such as liquid crystals are examples of such materials.

Eringen [1] extended his work to include the effect of axial stretch during the rotation of molecules and developed the theory of micropolar elastic solid with stretch. The material points in this continuum possess not only classical translational degree of freedom represented by the deformation vector field but also intrinsic rotations and an intrinsic axial stretch. The difference between these solids and micropolar elastic solids stems from the presence of scalar microstretch and a vector first moment.

Eringen [2] developed the theory of thermo-microstretch elastic solids. Eringen [3] also derived the equations of motions, constitutive equations and boundary conditions for thermo-microstretch fluids and obtained the solution of the problem for acoustical waves in bubbly liquids. Microstretch continuum is a model for Bravais lattice with basis on the atomic level and two phase dipolar solids with a core on the macroscopic level. Composite materials reinforced with chopped elastic fibres, porous media whose pores are filled with gas or inviscid liquid, asphalt or other elastic inclusions and solid-liquid crystals etc. are examples of microstretch solids.

Liu and Hu [4] investigated the inclusion problem of microstretch. Svanadze [5] constructed fundamental solution of the system of equations of steady oscillations in the theory of microstretch elastic solids. De Cicco [6] investigated the stress concentration effects in microstretch elastic bodies. Kumar, Singh and Chadha [7] discussed the axisymmetric problem in microstretch. Kumar, Singh and Chadha [8] investigated plane strain problem in microstretch elastic solid. Kumar and Partap [9] investigated reflection of plane waves in a heat flux dependent

*Corresponding author. E-mail: rajneesh_kuk@rediffmail.com

microstretch thermoelastic solid half spaces. Eringen [10] developed the equations of motion and the constitutive relations in a microstretch elastic solid. Kumar and Partap [11] studied propagation of Rayleigh-Lamb waves in micropolar isotropic elastic plate. Graff [12] investigated wave motion in elastic solids. Lamb [13] did investigations on waves in an elastic plate.

The present investigation is aimed to study the free vibration analysis of waves in an infinite homogeneous, isotropic microstretch elastic plate of thickness $2d$.

2. Basic Equations

The equations of motion and the constitutive relations in a microstretch elastic solid without body forces, body couples and stretch force given by Eringen [10] are

$$\lambda_0 \nabla \phi^* + (\lambda + 2\mu + K) \nabla (\nabla \cdot \vec{u}) - (\mu + K) \nabla \times \nabla \times \vec{u} + K \nabla \times \vec{\phi} = \rho \frac{\partial^2 \vec{u}}{\partial t^2} \quad (2.1)$$

$$(\alpha + \beta + \gamma) \nabla (\nabla \cdot \vec{\phi}) - \gamma \nabla \times (\nabla \times \vec{\phi}) + K \nabla \times \vec{u} - 2K \vec{\phi} = \rho j \frac{\partial^2 \vec{\phi}}{\partial t^2} \quad (2.2)$$

$$\alpha_0 \nabla^2 \phi^* - \lambda_1 \phi^* - \lambda_0 \nabla \cdot \vec{u} = \frac{1}{2} \rho j_0 \frac{\partial^2 \phi^*}{\partial t^2} \quad (2.3)$$

$$t_{ij} = (\lambda_0 \phi^* + \lambda u_{r,r}) \delta_{ij} + \mu (u_{i,j} + u_{j,i}) + K (u_{j,i} - \epsilon_{ijr} \phi_r) \quad (2.4)$$

$$m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i} + b_0 \epsilon_{mij} \phi_m^* \quad (2.5)$$

$$\lambda_i^* = \alpha_0 \phi_{i,i}^* + b_0 \epsilon_{ijm} \phi_{j,m} \quad (2.6)$$

where λ , μ , α , β , γ , K , α_0 , λ_0 , λ_1 , b_0 are material constants, ρ is the density, j is the microinertia, j_0 is the microinertia of microelement, t_{ij} and m_{ij} are components of stress and couple stress tensors respectively, $\vec{u} = (u_1, u_2, u_3)$ is the displacement vector, $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$ is the microrotation vector, λ_i^* is the component of microstretch, ϕ^* is the scalar point microstretch function, δ_{ij} is the Kronecker delta.

3. Formulation of the Problem

We consider a homogeneous isotropic microstretch

elastic plate of thickness $2d$. The origin of the coordinate system (x, y, z) is taken on middle surface of the plate and z -axis normal to it along the thickness as illustrated in Figure 1.

For two dimensional problem, we take $\vec{u}(u_1, 0, u_3)$ and $\vec{\phi} = (0, \phi_2, 0)$.

Introducing the dimensionless quantities defined by the expressions

$$\begin{aligned} x' &= \frac{\omega^*}{c_1} x, \quad z' = \frac{\omega^*}{c_1} z, \quad u'_1 = \frac{\omega^*}{c_1} u_1, \quad u'_3 = \frac{\omega^*}{c_1} u_3, \quad t' = \omega^* t, \\ \phi'_2 &= \frac{\omega^{*2}}{c_1^2} j \phi_2, \quad \phi'^* = \frac{\omega^{*2} j \phi^*}{c_1^2}, \quad \omega'^2 = \frac{K}{\rho j}, \quad \omega'^2 = \frac{\omega^2}{\omega^{*2}}, \\ t'_{ij} &= \frac{1}{\lambda} t_{ij} m'_{ij} = \frac{\omega^*}{\lambda c_1} m_{ij}, \quad p = \frac{K}{\rho c_1^2}, \quad \delta^2 = \frac{c_2^2}{c_1^2}, \quad \delta_2^2 = \frac{c_1^2}{c_4^2}, \\ \delta_3^2 &= \frac{\lambda_0}{K}, \quad \delta^{*2} = \frac{K}{\rho c_4^2}, \quad \delta_4^2 = \frac{\lambda_1 c_1^2}{\alpha_0 \omega^{*2}}, \quad \delta_5^2 = \frac{\lambda_0 j}{\alpha_0}, \quad \delta_6^2 = \frac{\rho c_1^2 j_0}{2 \alpha_0} \end{aligned} \quad (3.1)$$

where $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$, $c_2^2 = \frac{\mu + K}{\rho}$, $c_4^2 = \frac{\gamma}{\rho j}$, ω^* is the

characteristic frequency of the medium, c_1 and c_2 are respectively longitudinal and shear wave velocity in the medium.

Introducing the potential functions q and ψ through the relations

$$u_1 = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \quad (3.2)$$

and using equation (3.1) in equations (2.1)–(2.3) and after suppressing the primes for convenience, we obtain

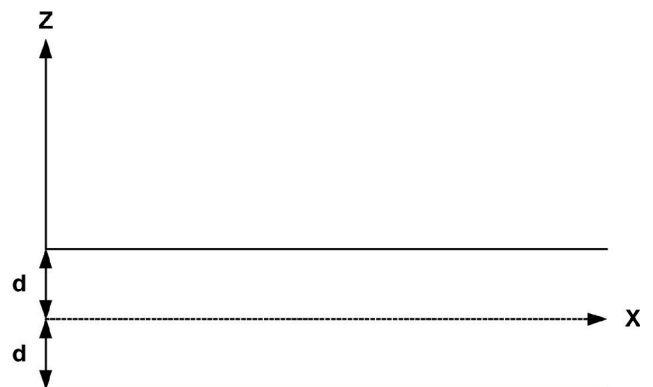


Figure 1. Geometry of the problem.

$$(\nabla^2 - \frac{\partial^2}{\partial t^2})q + \delta_3^2 \phi^* = 0 \quad (3.3)$$

$$\nabla^2 \psi - \frac{\phi_2}{\delta^2} - \frac{1}{\delta^2} \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (3.4)$$

$$\nabla^2 \phi_2 + \delta^{*2} \nabla^2 \psi - 2\delta_2^2 \phi_2 - \delta_2^2 \frac{\partial^2 \phi_2}{\partial t^2} = 0 \quad (3.5)$$

$$\nabla^2 \phi^* - \delta_3^2 \nabla^2 q - \delta_4^2 \phi^* - \delta_6^2 \frac{\partial^2 \phi^*}{\partial t^2} = 0 \quad (3.6)$$

Boundary conditions

The non-dimensional mechanical boundary conditions at $z = \pm d$ are given by

$$t_{33} = 0, t_{31} = 0, m_{32} = 0, \lambda_3^* = 0 \quad (3.7)$$

4. Formal Solution of the Problem

We assume the solutions of equations (3.3)–(3.6) of the form

$$(q, \psi, \phi_2, \phi^*) = [f(z), g(z), w(z), h(z)] e^{i\xi(x-ct)} \quad (4.1)$$

where $c = \omega / \xi$ is the non-dimensional phase velocity, ω and ξ are respectively the circular frequency and wave number.

Using equation (4.1) in equations (3.3)–(3.6) and solving the resulting differential equations, the expressions for q , ψ , ϕ_2 and ϕ^* are obtained as

$$q = (ACosm_1 z + BSinm_1 z + CCosm_2 z + DSinm_2 z) e^{i\xi(x-ct)} \quad (4.2)$$

$$\psi = (A'Cosm_3 z + B'Sinm_3 z + C'Cosm_4 z + D'Sinm_4 z) e^{i\xi(x-ct)} \quad (4.3)$$

$$\phi_2 = \delta^2 [(b^2 - m_3^2)(A'Cosm_3 z + B'Sinm_3 z) + (b^2 - m_4^2)(C'Cosm_4 z + D'Sinm_4 z)] e^{i\xi(x-ct)} \quad (4.4)$$

$$\phi^* = -\frac{1}{\delta_3^2} [(a^2 - m_1^2)(ACosm_1 z + BSinm_1 z) + (a^2 - m_2^2)(CCosm_2 z + DSinm_2 z)] e^{i\xi(x-ct)} \quad (4.5)$$

where

$$m_i^2 = \xi^2 (c^2 a_i^2 - 1), i = 1, 2, 3, 4; a^2 = \xi^2 (c^2 - 1),$$

$$b^2 = \xi^2 (\frac{c^2}{\delta^2} - 1),$$

$$(a_1^2, a_2^2) = \frac{1}{2} \left\{ \left[1 + \delta_6^2 - \frac{1}{\omega^2} (\delta_4^2 - \delta_3^2 \delta_5^2) \right] \pm \left[\left\{ 1 - \delta_6^2 - \frac{1}{\omega^2} (\delta_4^2 - \delta_3^2 \delta_5^2) \right\}^2 + \frac{4}{\omega^2} \{ \delta_4^2 + \delta_6^2 (\delta_4^2 - \delta_3^2 \delta_5^2) \} \right]^{1/2} \right\},$$

$$(a_3^2, a_4^2) = \frac{1}{2} \left\{ \left[\delta_2^2 + \frac{1}{\delta^2} + \frac{\delta^{*2}}{\omega^2 \delta^2} (1 - \frac{2\delta_2^2 \delta^2}{\delta^{*2}}) \right] \pm \left[\left\{ \frac{1}{\delta^2} - \delta_2^2 + \frac{\delta^{*2}}{\omega^2 \delta^2} (1 - \frac{2\delta_2^2 \delta^2}{\delta^{*2}}) \right\}^2 + \frac{4\delta_2^2}{\omega^2 \delta^2} \{ \delta^{*2} - 2(\delta_2^2 \delta^2 - 1) \} \right]^{1/2} \right\}$$

5. Derivation of the Secular Equations

Using boundary conditions (3.7) on the surfaces $z = \pm d$ of the plate and using equations (3.2), (4.2)–(4.5), and solving a system of eight simultaneous equations, we obtain the following secular equations after applying lengthy algebraic reductions and manipulations

$$\left\{ 1 + \frac{QR(m_1^2 - a^2)}{PS(m_4^2 - b^2)} + \frac{QV(m_3^2 - b^2)}{PU(m_2^2 - a^2)} + \frac{Q^2 RV(m_1^2 - a^2)(m_3^2 - b^2)}{P^2 SU(m_2^2 - a^2)(m_4^2 - b^2)} \right\} \left[\frac{\tan m_1 d}{\tan m_3 d} \right]^{\pm 1}$$

$$- \left\{ \frac{m_1(m_1^2 - a^2)}{m_2(m_2^2 - a^2)} + \frac{QRm_1(m_1^2 - a^2)}{PSm_2(m_4^2 - b^2)} + \frac{QVm_1(m_3^2 - b^2)}{PUm_2(m_2^2 - a^2)} + \frac{Q^2 RVm_1(m_3^2 - b^2)}{P^2 SUM_2(m_4^2 - b^2)} \right\} \left[\frac{\tan m_2 d}{\tan m_3 d} \right]^{\pm 1}$$

$$- \left\{ \frac{m_3(m_3^2 - b^2)}{m_4(m_4^2 - b^2)} + \frac{QR(m_1^2 - a^2)m_3}{PS(m_4^2 - b^2)m_4} + \frac{QVm_3(m_3^2 - b^2)}{PUm_4(m_2^2 - a^2)} + \frac{Q^2 RVm_3(m_1^2 - a^2)}{P^2 SUM_4(m_2^2 - a^2)} \right\} \left[\frac{\tan m_1 d}{\tan m_4 d} \right]^{\pm 1}$$

$$+ \left\{ \frac{m_1 m_3 (m_1^2 - a^2)(m_3^2 - b^2)}{m_2 m_4 (m_2^2 - a^2)(m_4^2 - b^2)} + \frac{QRm_1 m_3 (m_1^2 - a^2)}{PSm_2 m_4 (m_4^2 - b^2)} + \frac{QVm_1 m_3 (m_3^2 - b^2)}{PUm_2 m_4 (m_2^2 - a^2)} + \frac{Q^2 RVm_1 m_3}{P^2 SUM_2 m_4} \right\} \left[\frac{\tan m_2 d}{\tan m_4 d} \right]^{\pm 1}$$

$$\begin{aligned}
& -\frac{RV(m_2^2 - m_1^2)(m_4^2 - m_3^2)}{SUM_2m_4(m_4^2 - b^2)(m_2^2 - a^2)} \left[\frac{\tan m_1 d \tan m_2 d}{\tan m_3 d \tan m_4 d} \right]^{\pm 1} \\
& = \frac{-4\xi^2(1 - \frac{P}{2\delta^2})^2 m_1 m_3 (m_2^2 - m_1^2)(m_4^2 - m_3^2)}{(b^2 - \xi^2 + \frac{P\xi^2}{\delta^2})^2 (m_4^2 - b^2)(m_2^2 - a^2)} \quad (5.1)
\end{aligned}$$

where $P = b^2 - \xi^2 + \frac{P\xi^2}{\delta^2}$, $Q = -2i\xi(1 - \frac{P}{2\delta^2})$, $R = \frac{i\xi b_0}{\delta_3^2}$,

$$S = \gamma\delta^2, U = \frac{\alpha_0}{\delta_3^2}, V = i\xi b_0 \delta^2.$$

Here the exponent +1 refers to skew-symmetric and -1 refers to symmetric modes.

Particular Cases

(i) Micropolar elastic plate

In the absence of microstretch effect ($\alpha_0 = \lambda_0 = \lambda_1 = b_0 = 0$), the secular equation (5.1) reduces to

$$\begin{aligned}
& \left[\frac{\tan m_1 d}{\tan m_3 d} \right]^{\pm 1} - \frac{m_3(m_3^2 - b^2)}{m_4(m_4^2 - b^2)} \left[\frac{\tan m_1 d}{\tan m_4 d} \right]^{\pm 1} \\
& = -\frac{4\xi^2(1 - \frac{P}{2\delta^2})^2 a m_3 (m_4^2 - m_3^2)}{(b^2 - \xi^2 + \frac{P\xi^2}{\delta^2})^2 (m_4^2 - b^2)} \quad (5.2)
\end{aligned}$$

The equation (5.2) agrees with equation given by Kumar and Partap [11].

(ii) Elastic plate

In the absence of micropolarity effect ($p = 0, m_3 = b$), the secular equations (5.2) reduces to

$$\left[\frac{\tan m_1 d}{\tan m_3 d} \right]^{\pm 1} = -\frac{4\xi^2 ab}{(b^2 - \xi^2)^2} \quad (5.3)$$

The equation (5.3) agrees with equation given by Graff [12].

6. Regions of the Secular Equation

Here depending on whether $c < \delta, 1, 1/|a_i|$, $i = 1, 2, 3, 4$, we may have a, b, m_i ($i = 1, 2, 3, 4$) being purely imaginary, zero or real. Then the frequency equation (5.1)

is correspondingly altered as follows:

Region I

This region is characterized by $c < \delta, 1, \frac{1}{|a_i|}$, $i = 1, 2,$

3, 4. In this case, we replace a, b, m_i with $i\alpha', ib'$ and $i\alpha_i$, $i = 1, 2, 3, 4$ respectively. The secular equation (5.1) becomes

$$\begin{aligned}
& \left\{ 1 + \frac{QR(\alpha_1^2 - \alpha'^2)}{PS(\alpha_4^2 - b'^2)} + \frac{QV(\alpha_3^2 - b'^2)}{PU(\alpha_2^2 - \alpha'^2)} \right. \\
& \quad \left. + \frac{Q^2RV(\alpha_1^2 - \alpha'^2)(\alpha_3^2 - b'^2)}{P^2SU(\alpha_2^2 - \alpha'^2)(\alpha_4^2 - b'^2)} \right\} \left[\frac{\tanh \alpha_1 d}{\tanh \alpha_3 d} \right]^{\pm 1} \\
& - \left\{ \frac{\alpha_1(\alpha_1^2 - \alpha'^2)}{\alpha_2(\alpha_2^2 - \alpha'^2)} + \frac{QR\alpha_1(\alpha_1^2 - \alpha'^2)}{PS\alpha_2(\alpha_4^2 - b'^2)} + \frac{QV\alpha_1(\alpha_3^2 - b'^2)}{PU\alpha_2(\alpha_2^2 - \alpha'^2)} \right. \\
& \quad \left. + \frac{Q^2RV\alpha_1(\alpha_3^2 - b'^2)}{P^2SU\alpha_2(\alpha_4^2 - b'^2)} \right\} \left[\frac{\tanh \alpha_2 d}{\tanh \alpha_3 d} \right]^{\pm 1} \\
& - \left\{ \frac{\alpha_3(\alpha_3^2 - b'^2)}{\alpha_4(\alpha_4^2 - b'^2)} + \frac{QR(\alpha_1^2 - \alpha'^2)\alpha_3}{PS(\alpha_4^2 - b'^2)\alpha_4} + \frac{QV\alpha_3(\alpha_3^2 - b'^2)}{PU\alpha_4(\alpha_2^2 - \alpha'^2)} \right. \\
& \quad \left. + \frac{Q^2RV\alpha_3(\alpha_1^2 - \alpha'^2)}{P^2SU\alpha_4(\alpha_2^2 - \alpha'^2)} \right\} \left[\frac{\tanh \alpha_1 d}{\tanh \alpha_4 d} \right]^{\pm 1} \\
& + \left\{ \frac{\alpha_1\alpha_3(\alpha_1^2 - \alpha'^2)(\alpha_3^2 - b'^2)}{\alpha_2\alpha_4(\alpha_2^2 - \alpha'^2)(\alpha_4^2 - b'^2)} + \frac{QR\alpha_1\alpha_3(\alpha_1^2 - \alpha'^2)}{PS\alpha_2\alpha_4(\alpha_4^2 - b'^2)} \right. \\
& \quad \left. + \frac{QV\alpha_1\alpha_3(\alpha_3^2 - b'^2)}{PU\alpha_2\alpha_4(\alpha_2^2 - \alpha'^2)} + \frac{Q^2RV\alpha_1\alpha_3}{P^2SU\alpha_2\alpha_4} \right\} \left[\frac{\tanh \alpha_2 d}{\tanh \alpha_4 d} \right]^{\pm 1} \\
& - \frac{RV(\alpha_2^2 - \alpha'^2)(\alpha_4^2 - \alpha'^2)}{SU\alpha_2\alpha_4(\alpha_4^2 - b'^2)(\alpha_2^2 - \alpha'^2)} \left[\frac{\tanh \alpha_1 d \tanh \alpha_2 d}{\tanh \alpha_3 d \tanh \alpha_4 d} \right]^{\pm 1} \\
& = \frac{4\xi^2(1 - \frac{P}{2\delta^2})^2 \alpha_1\alpha_3(\alpha_2^2 - \alpha'^2)(\alpha_4^2 - \alpha'^2)}{(\frac{P\xi^2}{\delta^2} - b'^2 - \xi^2)^2 (\alpha_4^2 - b'^2)(\alpha_2^2 - \alpha'^2)} \quad (6.1)
\end{aligned}$$

Region II

This region is characterized by $\delta < c < 1$. In this case, we have $b = b, m_3 = m_3, m_4 = m_4, m_i = i\alpha_i$, ($i = 1, 2$) and the secular equation becomes

$$\begin{aligned}
& \left\{ 1 + \frac{QR(\alpha_1^2 - \alpha'^2)}{PS(m_4^2 - b^2)} + \frac{QV(m_3^2 - b^2)}{PU(\alpha_2^2 - \alpha'^2)} \right. \\
& \quad \left. + \frac{Q^2RV(\alpha_1^2 - \alpha'^2)(m_3^2 - b^2)}{P^2SU(\alpha_2^2 - \alpha'^2)(m_4^2 - b^2)} \right\} \left[\frac{\tanh \alpha_1 d}{\tanh m_3 d} \right]^{\pm 1}
\end{aligned}$$

$$\begin{aligned}
& - \left\{ \frac{\alpha_1(\alpha_1^2 - \alpha'^2)}{\alpha_2(\alpha_2^2 - \alpha'^2)} + \frac{QR\alpha_1(\alpha_1^2 - \alpha'^2)}{PS\alpha_2(m_4^2 - b^2)} + \frac{QV\alpha_1(m_3^2 - b^2)}{PU\alpha_2(\alpha_2^2 - \alpha'^2)} \right. \\
& \quad \left. + \frac{Q^2RV\alpha_1(m_3^2 - b^2)}{P^2SU\alpha_2(m_4^2 - b^2)} \right\} \left[\frac{\tanh \alpha_2 d}{\tan m_3 d} \right]^{\pm 1} \\
& - \left\{ \frac{m_3(m_3^2 - b^2)}{m_4(m_4^2 - b^2)} + \frac{QR(\alpha_1^2 - \alpha'^2)m_3}{PS(m_4^2 - b^2)m_4} + \frac{QVm_3(m_3^2 - b^2)}{PUm_4(\alpha_2^2 - \alpha'^2)} \right. \\
& \quad \left. + \frac{Q^2RVm_3(\alpha_1^2 - \alpha'^2)}{P^2SUM_4(\alpha_2^2 - \alpha'^2)} \right\} \left[\frac{\tanh \alpha_1 d}{\tan m_4 d} \right]^{\pm 1} \\
& + \left\{ \frac{\alpha_1 m_3(\alpha_1^2 - \alpha'^2)(m_3^2 - b^2)}{\alpha_2 m_4(\alpha_2^2 - \alpha'^2)(m_4^2 - b^2)} + \frac{QR\alpha_1 m_3(\alpha_1^2 - \alpha'^2)}{PS\alpha_2 m_4(m_4^2 - b^2)} \right. \\
& \quad \left. + \frac{QV\alpha_1 m_3(m_3^2 - b^2)}{PU\alpha_2 m_4(\alpha_2^2 - \alpha'^2)} + \frac{Q^2RV\alpha_1 m_3}{P^2SU\alpha_2 m_4} \right\} \left[\frac{\tanh \alpha_2 d}{\tan m_4 d} \right]^{\pm 1} \\
& - \frac{RV(\alpha_2^2 - \alpha_1^2)(m_4^2 - m_3^2)}{SU\alpha_2 m_4(m_4^2 - b^2)(\alpha_2^2 - \alpha'^2)} \left[\frac{\tanh \alpha_1 d}{\tan m_3 d} \frac{\tanh \alpha_2 d}{\tan m_4 d} \right]^{\pm 1} \\
& = \frac{-4\xi^2(1 - \frac{P}{2\delta^2})^2 \alpha_1 m_3(\alpha_2^2 - \alpha_1^2)(m_4^2 - m_3^2)}{(b^2 - \xi^2 + \frac{P\xi^2}{\delta^2})^2 (m_4^2 - b^2)(\alpha_2^2 - \alpha'^2)} \quad (6.2)
\end{aligned}$$

Region III

This region is characterized by $\xi < \omega$, $c > 1$ and the secular equation is given by equation (5.1).

7. Waves of Short Wavelength

Some information on the asymptotic behavior is obtained by letting $\xi \rightarrow \infty$, $\frac{\tanh \alpha_i d}{\tanh \alpha_j d} \rightarrow 1$, $i = 1, 2$; $j = 3, 4$. If

we take $\xi > \frac{\omega}{\delta}$, it follows that $c < \delta$, 1. Then we replace a , b , m_i with $i\alpha'$, ib' , $i\alpha_i$ and secular equation (6.1) reduces to

$$\begin{aligned}
& 4\xi^2(1 - \frac{P}{2\delta^2})^2 \alpha_1 \alpha_2 \alpha_3 \alpha_4 (\alpha_1 + \alpha_2)(\alpha_3 + \alpha_4) \\
& = (\frac{P\xi^2}{\delta^2} - b'^2 - \xi^2)^2 \times \\
& \left[(\alpha_1^2 + \alpha_2^2 + \alpha_1 \alpha_2 - \alpha'^2)(\alpha_4^2 + \alpha_3^2 + \alpha_3 \alpha_4 - b'^2) \right. \\
& \quad + \frac{QR}{PS}(\alpha_1^2 - \alpha'^2)(\alpha_2^2 - \alpha'^2) + \frac{QV}{PU}(\alpha_3^2 - b'^2)(\alpha_4^2 - b'^2) \\
& \quad + \frac{Q^2RV}{P^2SU}(\alpha_3 \alpha_4 + b'^2)(\alpha_1 \alpha_2 + \alpha'^2) \\
& \quad \left. - \frac{RV}{SU}(\alpha_1 + \alpha_2)(\alpha_3 + \alpha_4) \right] \quad (7.1)
\end{aligned}$$

These are merely Rayleigh surface wave equations. The Rayleigh results enter here since, for such small wavelengths, the finite thickness plate appears as a half-space. Hence vibrational energy is transmitted mainly along the surface of the plate.

8. Lamé Modes

A special class of exact solutions, called the Lamé modes, but evidently first identified by Lamb [13] can be obtained by considering the special case $b^2 = \xi^2(1 - \frac{P}{\delta^2})$, the roots for this case are in Region II and the frequency equation (6.2) reduces to

Symmetric modes: $\tan m_3 d = \infty \Rightarrow m_3 = n\pi / 2d$, $n = 1, 3, 5, \dots$

Anti symmetric modes: $\tan m_3 d = 0 \Rightarrow m_3 = n\pi / 2d$, $n = 0, 2, 4, \dots$

Here, the frequency is given by

$$\omega = \frac{\sqrt{4b^2 d^2 + n^2 \pi^2 (1 - \frac{P}{\delta^2})}}{2da_3 \sqrt{1 - \frac{P}{\delta^2}}} \quad (8.1)$$

9. Example Results

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results. The material chosen for this purpose is Magnesium crystal (microstretch elastic solid), the physical data for which is given below

$$\begin{aligned}
\rho &= 1.74 \times 10^3 \text{ Kg} / m^3, \quad \lambda = 9.4 \times 10^{10} \text{ N} / m^2, \\
\mu &= 4.0 \times 10^{10} \text{ N} / m^2, \quad K = 1.0 \times 10^{10} \text{ N} / m^2, \\
\gamma &= 0.779 \times 10^{-9} \text{ N}, \quad b_0 = 0.5 \times 10^{-9} \text{ N}, \\
j &= 0.2 \times 10^{-19} m^2, \quad j_0 = 0.185 \times 10^{-19} m^2, \\
\lambda_0 &= 0.6 \times 10^{10} \text{ N} / m^2, \quad \lambda_1 = 0.5 \times 10^{10} \text{ N} / m^2, \\
\alpha_0 &= 0.779 \times 10^{-9} \text{ N}, \quad d = 0.01 \text{ m}.
\end{aligned}$$

The phase velocity of symmetric and skew-symmetric modes of wave propagation has been computed for various values of wave number from secular equation (5.1). The corresponding numerically computed values of phase velocity have been represented graphically in Figure 2

and Figure 3 for different modes ($n = 0$ to $n = 6$). The solid curves correspond to microstretch elastic plate (MES) and dashed curves refer to micropolar elastic plate (ME).

The phase velocity of lowest symmetric mode ($n = 0$) remains constant with the variation in wave number, whereas the phase velocity of lowest skew-symmetric mode varies for wave number $\xi \leq 2$ and becomes constant for wave number $\xi \geq 2$. The phase velocity of higher modes of wave propagation, symmetric and skew symmetric attains quite large values at vanishing wave number, which sharply slashes down to become steady with increasing wave number.

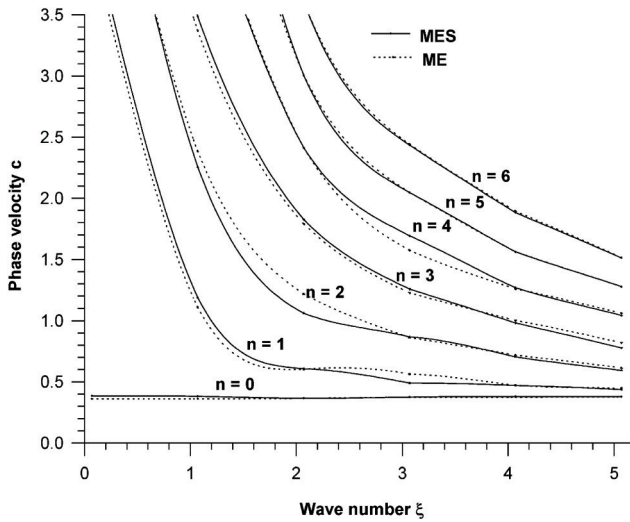


Figure 2. Phase velocity profiles for symmetric modes of wave propagation.

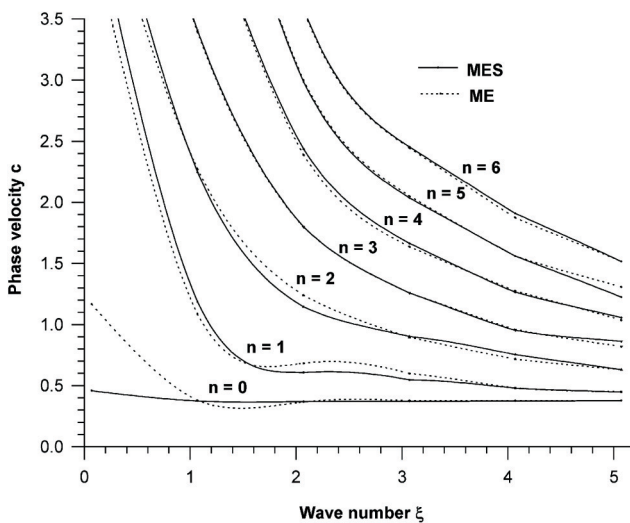


Figure 3. Phase velocity profiles for skew symmetric modes of wave propagation.

For symmetric modes of wave propagation, we observe the following (i) for $n = 1$, phase velocity for ME is smaller than in MES for wave number $\xi \leq 2$ and for wave number $\xi \geq 2$ and $\xi \leq 4$, the values of phase velocity for ME is more in comparison with MES and for wave number $\xi \geq 4$, phase velocity profiles coincide in respect of MES and ME (ii) for mode $n = 2$, phase velocity for ME is more than in case of MES for wave number lying between 1.0 and 3.0 and phase velocity profiles coincide in respect of ME and MES for wave number $\xi \leq 1.0$ and $\xi \geq 3.0$ (iii) at $n = 4$, the phase velocity profiles in respect of MES and ME coincide for wave number $\xi \leq 2.0$ and $\xi \geq 4.0$, whereas for wave number lying between 2.0 and 4.0 the values of phase velocity for ME are less in comparison with MES (iv) For modes $n = 3, 5, 6$, phase velocity profiles for MES and ME coincide.

For skew-symmetric modes of wave propagation, we observe the following (a) for lowest mode $n = 0$, phase velocity in ME is more than in MES for wave number $\xi \leq 1.0$ and for wave number lying between 1.0 and 2.0, phase velocity for MES is more than in ME and for wave number $\xi \geq 2.0$, phase velocity in MES and ME is nearly same (b) for $n = 1$, the values of phase velocity for ME are lesser than in MES for wave number $\xi \leq 1.5$ and for wave number $\xi \geq 1.5$ and $\xi \leq 4$, the values of phase velocity for ME are more than in MES and for wave number $\xi \geq 4$, phase velocity profiles coincide in respect of MES and ME (c) for $n = 2$, for wave number lying between 1.0 and 3.0, phase velocity for ME is more than in case of MES and phase velocity for ME is slightly less than in case of MES for wave number $\xi \leq 1.0$ and $\xi \geq 3.0$ (d) for higher modes $n = 3, 4, 5, 6$ phase velocity for MES and ME is nearly same.

10. Conclusion

(i) The analysis of free vibrations in infinite homogeneous, isotropic microstretch elastic plate is investigated after deriving the secular equations (ii) It is noticed that the motion of free vibrations is governed by the Rayleigh – Lamb type secular equations (iii) At short wave limit, the secular equations in case of symmetric and skew-symmetric modes of wave propagation in a stress free plate reduces to the Rayleigh surface frequency equations (iv) Significant stretch effect is observed on the phase velocity.

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