

Optimal Design of Viscous Dampers for Vibration Control of Adjacent Building Structures

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Abstract

The objective of this paper is to investigate the dynamic characteristics of two adjacent building structures interconnected by viscous dampers under seismic excitations. The computational procedure for an analytical model including the system model formulation, complex modal analysis and seismic time history analysis is presented for this purpose. A numerical example is also provided to illustrate the analytical model. The complex modal analysis is conducted to determine the optimal damping ratio and the optimal damper coefficient of the linear viscous damper for each mode of the system. For the damper coefficient with optimal value, the responses can be categorized into underdamped and critically damped vibrations. The seismic time history analysis is conducted to assess the effectiveness of the linear viscous damper for vibration control. Based on the optimal damping ratio and the optimal damper coefficient of the linear viscous damper for a certain mode of the system, compared to the peak responses, the linear viscous damper can be used to more effectively suppress the root-mean-square responses of the two adjacent buildings.

Key Words: Adjacent Building Structure, Viscous Damper, Complex Modal Analysis, Seismic Time History Analysis

1. Introduction

The impact of closely neighbouring building structures during earthquakes has become a problem in modern cities. Viscous dampers interconnecting adjacent buildings have been used to prevent the pounding for engineering applications. Since the dynamic properties of viscous dampers between adjacent structures are complex, it is necessary to fully understand the mechanism of the system, which provides the essential information to accurately calculate the vibrations of the adjacent structures under seismic excitations as well as to reasonably assess the effectiveness of the viscous dampers for vibration control.

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tations. The computational procedure for an analytical model is presented for this purpose. A numerical example is also provided to illustrate the analytical model.

2. Analytical Model

The computational procedure for an analytical model including the system model formulation, complex modal analysis and seismic time history analysis is presented in Figure 1, which will be discussed in this section.

2.1 System Model Formulation

The system model formulation is the basis of both the complex modal analysis and seismic time history analysis. Figure 2 illustrates the analytical model in this study. Two adjacent structures with a total of N degrees of freedom are simulated by a L -story and $(N - L)$ -story shear building models in the left and right sides, respectively. $N = 2L$, $N < 2L$ and $N > 2L$ individually represent the two

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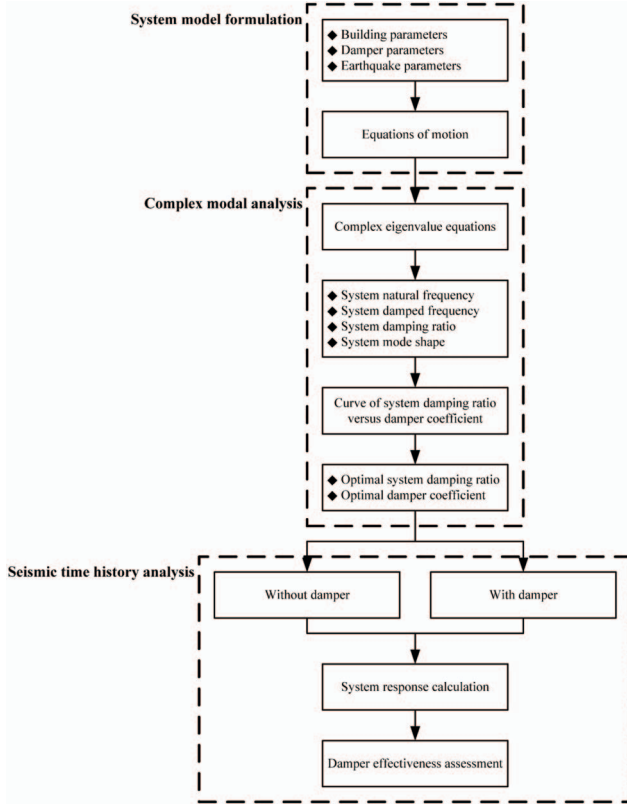


Figure 1. Computational procedure in this study.

buildings with equal height, the left building with taller height and the right building with taller height. M_i , K_i , C_i and $X_i(t)$ [$\dot{X}_i(t)$, $\ddot{X}_i(t)$] are the mass, stiffness, damping and horizontal displacement (velocity, acceleration) time history with respect to the ground motion of the floor with the i th degree of freedom ($i = 1, 2, \dots, N$) for the system, respectively, and t is the time. The elevation of each floor in the left building and that of the corresponding one in the right building are assumed to be equal, and the neighbouring floors at the same elevation is interconnected by a linear viscous damper. C_{di} is the damping coefficient of the linear viscous damper between the two floors with the i th and $(L + i)$ th degrees of freedom, where either $i = 1, 2, \dots, N - L$ for $N < 2L$ or $i = 1, 2, \dots, L$ for $N \geq 2L$. The seismic excitation of the left building is assumed to be identical to that of the right building, which can be simulated by the horizontal displacement (acceleration) time history of the ground motion $X_G(t)$ [$\dot{X}_G(t)$] [1].

Based on the above mentioned parameters, one can derive the equations of motion for the two adjacent buildings interconnected by viscous dampers under

seismic excitations [2–4], which can be written as

$$[m]_{N \times N} \{\ddot{x}(t)\}_{N \times 1} + [c]_{N \times N} \{\dot{x}(t)\}_{N \times 1} + [k]_{N \times N} \{x(t)\}_{N \times 1} = -[m]_{N \times N} \{1\}_{N \times 1} \ddot{X}_G(t) \quad (1)$$

where $[m]_{N \times N}$, $[k]_{N \times N}$ and $[c]_{N \times N}$ are the mass, stiffness and damping matrices of the system, respectively; $\{x(t)\}_{N \times 1}$, $\{\dot{x}(t)\}_{N \times 1}$ and $\{\ddot{x}(t)\}_{N \times 1}$ are the displacement, velocity and acceleration vectors of the system, respectively; $\{1\}_{N \times 1}$ is a column vector with all elements equal to unity. The mass matrix of the system can be expressed as

$$[m]_{N \times N} = \text{diag}[M_1, M_2, \dots, M_N] \quad (2)$$

The stiffness matrix of the system can be written as

$$[k]_{N \times N} = \begin{bmatrix} [k_L]_{L \times L} & [0]_{L \times (N-L)} \\ [0]_{(N-L) \times L} & [k_R]_{(N-L) \times (N-L)} \end{bmatrix} \quad (3)$$

$$[k_L]_{L \times L} = \begin{bmatrix} K_1 + K_2 & -K_2 & 0 & 0 & 0 & 0 \\ -K_2 & K_2 + K_3 & -K_3 & 0 & 0 & 0 \\ 0 & -K_3 & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & -K_{L-1} & 0 \\ 0 & 0 & 0 & -K_{L-1} & K_{L-1} + K_L & -K_L \\ 0 & 0 & 0 & 0 & -K_L & K_L \end{bmatrix} \quad (4)$$

$$[k_R]_{(N-L) \times (N-L)} = \begin{bmatrix} K_{L+1} + K_{L+2} & -K_{L+2} & 0 & 0 & 0 & 0 \\ -K_{L+2} & K_{L+2} + K_{L+3} & -K_{L+3} & 0 & 0 & 0 \\ 0 & -K_{L+3} & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & -K_{N-1} & 0 \\ 0 & 0 & 0 & -K_{N-1} & K_{N-1} + K_N & -K_N \\ 0 & 0 & 0 & 0 & -K_N & K_N \end{bmatrix} \quad (5)$$

where $[k_L]_{L \times L}$ and $[k_R]_{(N-L) \times (N-L)}$ are the stiffness matrices of the left and right buildings, respectively; $[0]_{(N-L) \times L}$ and $[0]_{L \times (N-L)}$ are both zero matrices. The damping matrix of the system can be expressed as

$$[c]_{N \times N} = [c_I]_{N \times N} + [c_D]_{N \times N} \quad (6)$$

$$[c_I]_{N \times N} = \begin{bmatrix} [c_L]_{L \times L} & [0]_{L \times (N-L)} \\ [0]_{(N-L) \times L} & [c_R]_{(N-L) \times (N-L)} \end{bmatrix} \quad (7)$$

$$[c_L]_{L \times L} = \begin{bmatrix} C_1 + C_2 & -C_2 & 0 & 0 & 0 & 0 \\ -C_2 & C_2 + C_3 & -C_3 & 0 & 0 & 0 \\ 0 & -C_3 & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & -C_{L-1} & 0 \\ 0 & 0 & 0 & -C_{L-1} & C_{L-1} + C_L & -C_L \\ 0 & 0 & 0 & 0 & -C_L & C_L \end{bmatrix} \quad (8)$$

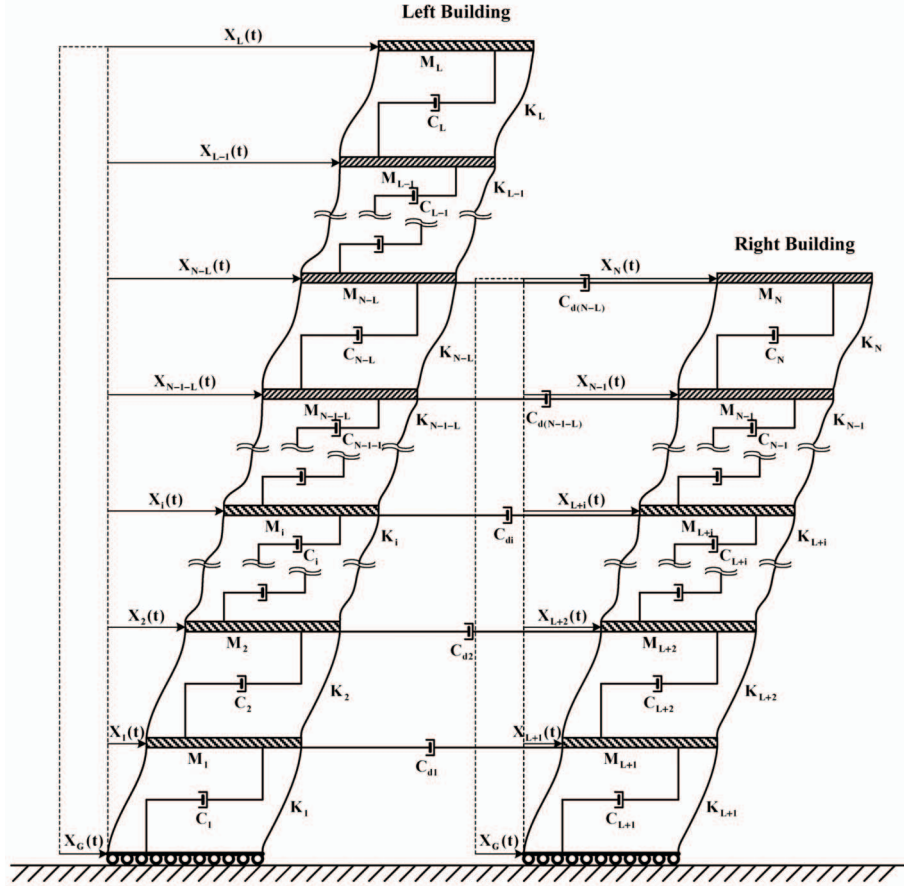


Figure 2. Analytical model in this study.

$$[c_R]_{(N-L) \times (N-L)} = \begin{bmatrix} C_{L+1} + C_{L+2} & -C_{L+2} & 0 & 0 & 0 & 0 \\ -C_{L+2} & C_{L+2} + C_{L+3} & -C_{L+3} & 0 & 0 & 0 \\ 0 & -C_{L+3} & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & -C_{N-1} & 0 \\ 0 & 0 & 0 & -C_{N-1} & C_{N-1} + C_N & -C_N \\ 0 & 0 & 0 & 0 & -C_N & C_N \end{bmatrix} \quad (9)$$

where $[c_L]_{N \times N}$ and $[c_D]_{N \times N}$ are the damping matrices of the adjacent building structures and viscous dampers, respectively; $[c_L]_{L \times L}$ and $[c_R]_{(N-L) \times (N-L)}$ are the damping matrices of the left and right buildings, respectively. For $N < 2L$,

$$[c_D]_{N \times N} = \begin{bmatrix} [c_V]_{(N-L) \times (N-L)} & [0]_{(N-L) \times (2L-N)} & -[c_V]_{(N-L) \times (N-L)} \\ [0]_{(2L-N) \times (N-L)} & [0]_{(2L-N) \times (2L-N)} & [0]_{(2L-N) \times (N-L)} \\ -[c_V]_{(N-L) \times (N-L)} & [0]_{(N-L) \times (2L-N)} & [c_V]_{(N-L) \times (N-L)} \end{bmatrix} \quad (10)$$

$$[c_V]_{(N-L) \times (N-L)} = \text{diag}[c_{d1}, c_{d2}, \dots, c_{d(N-L)}] \quad (11)$$

for $N \geq 2L$,

$$[c_D]_{N \times N} = \begin{bmatrix} [c_V]_{L \times L} & -[c_V]_{L \times L} & [0]_{L \times (N-2L)} \\ -[c_V]_{L \times L} & [c_V]_{L \times L} & [0]_{L \times (N-2L)} \\ [0]_{(N-2L) \times L} & [0]_{(N-2L) \times L} & [0]_{(N-2L) \times (N-2L)} \end{bmatrix} \quad (12)$$

$$[c_V]_{L \times L} = \text{diag}[c_{d1}, c_{d2}, \dots, c_{dL}] \quad (13)$$

where $[c_V]_{(N-L) \times (N-L)}$ and $[c_V]_{L \times L}$ are both the submatrices of $[c_D]_{N \times N}$; $[0]_{(2L-N) \times (N-L)}$, $[0]_{(N-L) \times (2L-N)}$, $[0]_{(2L-N) \times (2L-N)}$, $[0]_{(N-2L) \times L}$, $[0]_{L \times (N-2L)}$ and $[0]_{(N-2L) \times (N-2L)}$ are both zero matrices. The displacement, velocity and acceleration vectors of the system can be written, respectively, as

$$\{x(t)\}_{N \times 1} = \{X_1(t), X_2(t), \dots, X_n(t)\}^T \quad (14)$$

$$\{\dot{x}(t)\}_{N \times 1} = \{\dot{X}_1(t), \dot{X}_2(t), \dots, \dot{X}_n(t)\}^T \quad (15)$$

$$\{\ddot{x}(t)\}_{N \times 1} = \{\ddot{X}_1(t), \ddot{X}_2(t), \dots, \ddot{X}_n(t)\}^T \quad (16)$$

2.2 Complex Modal Analysis

The complex modal analysis is conducted for the

optimization of the coefficient, location and number of linear viscous dampers. Under the condition that both the damper location and number have been decided, the determination of the optimal damper coefficient is emphasized in this study. The optimal damper location and number can also be estimated by the same method. For this purpose, the analytical model including a linear viscous damper in a certain location of two adjacent buildings is taken as an example to illustrate the computational procedure.

The complex modal analysis based on the state-space method has been widely used to investigate the free vibrations of dynamic systems [5]. By using the state-space method, the N second-order equations of motion in Eq. (1) can be transformed into the $2N$ first-order state equations

$$[A]_{2N \times 2N} \{\dot{z}(t)\}_{2N \times 1} + [B]_{2N \times 2N} \{z(t)\}_{2N \times 1} = \{F(t)\}_{2N \times 1} \quad (17)$$

$$[A]_{2N \times 2N} = \begin{bmatrix} [c]_{N \times N} & [m]_{N \times N} \\ [m]_{N \times N} & [0]_{N \times N} \end{bmatrix} \quad (18)$$

$$[B]_{2N \times 2N} = \begin{bmatrix} [k]_{N \times N} & [0]_{N \times N} \\ [0]_{N \times N} & -[m]_{N \times N} \end{bmatrix} \quad (19)$$

$$\{F(t)\}_{2N \times 1} = \begin{Bmatrix} -[m]_{N \times N} \{\ddot{x}_G(t)\}_{N \times 1} \\ \{0\}_{N \times 1} \end{Bmatrix} \quad (20)$$

$$\{z(t)\}_{2N \times 1} = \begin{Bmatrix} \{x(t)\}_{N \times 1} \\ \{\dot{x}(t)\}_{N \times 1} \end{Bmatrix} \quad (21)$$

where $[A]_{2N \times 2N}$ and $[B]_{2N \times 2N}$ are both the coefficient matrices; $\{F(t)\}_{2N \times 1}$ is the state forcing vector; $\{z(t)\}_{2N \times 1}$ is the state vector; $[0]_{N \times N}$ is a zero matrix; $\{0\}_{N \times 1}$ is a zero vector.

With the consideration of the free vibrations for the two adjacent building structures interconnected by viscous dampers, Eq. (17) reduces to the homogeneous form

$$[A]_{2N \times 2N} \{\dot{z}(t)\}_{2N \times 1} + [B]_{2N \times 2N} \{z(t)\}_{2N \times 1} = \{0\}_{2N \times 1} \quad (22)$$

where $\{0\}_{2N \times 1}$ is a zero vector. The solutions to Eq. (22) have the exponential form

$$\{z(t)\}_{2N \times 1} = \{\theta\}_{2N \times 1} e^{\lambda t} \quad (23)$$

where λ is a scalar and $\{\theta\}_{2N \times 1}$ is a vector. Substituting

Eq. (23) into Eq. (22) and dividing by $e^{\lambda t}$ yields the complex eigenvalue equations

$$[\lambda[A]_{2N \times 2N} + [B]_{2N \times 2N}] \{\theta\}_{2N \times 1} = \{0\}_{2N \times 1} \quad (24)$$

The complex eigenvalues λ_i and the corresponding complex eigenvectors $\{\theta_i\}_{2N \times 1}$ ($i = 1, 2, \dots, 2N$) can be determined by solving Eq. (24). Based on λ_i and $\{\theta_i\}_{2N \times 1}$ ($i = 1, 2, \dots, 2N$), one can calculate the natural frequency f_{nj} , damped frequency f_{dj} , damping ratio ζ_j and mode shape ϕ_j for the j th mode ($j = 1, 2, \dots, N$) of the system [5].

By varying the damper coefficient with the other parameters of the analytical model constant, the curve of damping ratio versus damper coefficient for each mode of the system can be obtained. Under the objective function to maximize the modal damping ratio, the maximum ordinate and the corresponding abscissa of the mentioned curve can be individually determined to be the optimal damping ratio and the optimal damper coefficient of the linear viscous damper for each mode of the system.

2.3 Seismic Time History Analysis

The seismic time history analysis is conducted to assess the effectiveness of linear viscous dampers for vibration control. Based on the optimal parameters of the linear viscous damper obtained by the complex modal analysis, $X_i(t)$, $\dot{X}_i(t)$ and $\ddot{X}_i(t)$ ($i = 1, 2, \dots, N$), and the corresponding root-mean-square and peak values for two types of systems: with and without dampers, can be calculated according to the equations of motion for these systems under seismic excitations [1]. The effectiveness of the linear viscous damper can be assessed by comparing the response reduction rates between the mentioned two types of systems.

3. Numerical Example

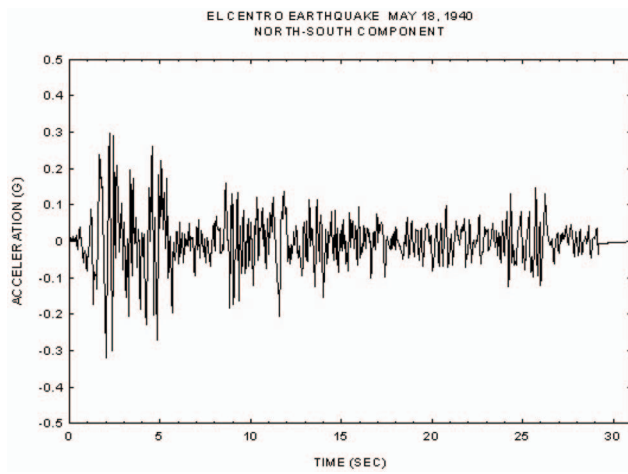
A numerical example is provided to illustrate the computational procedure for the analytical model presented in section 2. For this purpose, the system including a linear viscous damper between the third floor of each of the two three-story adjacent buildings with a total of six degrees of freedom ($N = 6, L = 3$) under seismic excitations is developed in this study. Table 1 lists the mass M_i , stiffness K_i and damping C_i ($i = 1, 2, \dots, 6$) for each degree of freedom of the two adjacent buildings. The damping coefficient of the linear viscous damper

Table 1. Parameters of the two adjacent buildings

Degree of freedom i	Floor in the left building	Floor in the right building	Mass M_i (kg)	Stiffness K_i (N/m)	Damping C_i (N-s/m)
1	1		980.71	1.04×10^6	200.0
2	2		980.71	9.82×10^5	140.0
3	3		980.71	8.50×10^5	120.0
4		1	784.60	1.04×10^6	200.0
5		2	784.60	9.82×10^5	140.0
6		3	784.60	8.50×10^5	120.0

C_{d3} varies from 0 to 3.0×10^4 N-s/m with an increment of 100.0 N-s/m. Figure 3 shows the north-south component of the El Centro earthquake accelerogram recorded in the Imperial Valley of Southern California on May 18, 1940 [1], which is selected as the horizontal acceleration time history of the ground motion $\ddot{X}_G(t)$.

The complex modal analysis is conducted to determine the optimal damping ratio and the optimal damper coefficient of the linear viscous damper for each mode of

**Figure 3.** North-south component of the El Centro earthquake accelerogram [1].

the system, and the results are shown in both Figure 4 and Table 2. Figures 4(a) to 4(f) depict the relationship between the damping ratio ζ_j ($j = 1, 2, \dots, 6$) and C_{d3} for each of the six modes of the system, respectively. For the curve of each mode, the natural frequency f_{nj} , damped frequency f_{dj} , ζ_j ($j = 1, 2, \dots, 6$) and C_{d3} for the case of origin (C_{d3} with zero value), and those for the case of maximum ordinate (C_{d3} with optimal value) are listed in Table 2, in which the maximum ordinate ζ_j ($j = 1, 2, \dots, 6$) is determined to be the optimal value. For C_{d3} with zero value, the underdamped vibration occurs for each of the six modes due to the fact that ζ_j ($j = 1, 2, \dots, 6$) contributed wholly from C_i ($i = 1, 2, \dots, 6$) is smaller than 0.005. For C_{d3} with optimal value, the responses can be categorized into underdamped and critically damped vibrations. The modes 1, 3 to 6 are dominated by the underdamped vibration, in which ζ_j ($j = 1, 2, \dots, 6$) contributed primarily from C_{d3} as well as secondarily from C_i ($i = 1, 2, \dots, 6$) is between 0.01 and 0.12. The mode 2 has the substantial contribution from the critically damped vibration with ζ_2 approaching unity. Under these conditions, ζ_2 is considered to be contributed entirely from C_{d3} and the contribution from C_i ($i = 1, 2, \dots, 6$) can be neglected.

Based on the optimal ζ_1 and C_{d3} , the seismic time

Table 2. Modal properties of the system

Mode j	Damper coefficient C_{d3} with zero value				Damper coefficient C_{d3} with optimal value			
	Natural frequency f_{nj} (Hz)	Damped frequency f_{dj} (Hz)	Damping ratio ζ_j	Damper coefficient C_{d3} (N-s/m)	Natural frequency f_{nj} (Hz)	Damped frequency f_{dj} (Hz)	Damping ratio ζ_j	Damper coefficient C_{d3} (N-s/m)
1	2.2560	2.2560	0.0012	0.0	2.2560	2.2539	0.0434	2400.0
2	2.5223	2.5223	0.0013	0.0	2.5223	0.0000	1.0000	19700.0
3	6.0842	6.0842	0.0030	0.0	6.0842	6.0454	0.1128	19000.0
4	6.8022	6.8022	0.0034	0.0	6.8022	6.7978	0.0358	8300.0
5	8.9102	8.9101	0.0042	0.0	8.9102	8.9097	0.0106	16500.0
6	9.9617	9.9616	0.0047	0.0	9.9617	9.9610	0.0118	16900.0

history analysis is conducted to assess the effectiveness of the linear viscous damper for vibration control, and the results are shown in both Figure 5 and Table 3. Figures 5(a) to 5(f) individually depict the horizontal displacement, velocity and acceleration time histories with respect to the ground motion of the third floor in the left building [$X_3(t)$, $\dot{X}_3(t)$, $\ddot{X}_3(t)$], and those in the right building [$X_6(t)$, $\dot{X}_6(t)$, $\ddot{X}_6(t)$] under seismic excitations for two types of systems: with and without dampers. The root-mean-square values, peak values and the corresponding response reduction rates between the men-

tioned two types of systems of $X_3(t)$, $\dot{X}_3(t)$ and $\ddot{X}_3(t)$ in the left building, and those of $X_6(t)$, $\dot{X}_6(t)$ and $\ddot{X}_6(t)$ in the right building are shown in Tables 3(a) and 3(b), respectively. It is revealed that the response reduction rate of the root-mean-square value and that of the peak value in the left building are individually greater than 67% and 48%, while the ones in the right building are greater than 63% and 32%, respectively. Consequently, compared to the peak responses, the linear viscous damper can be used to more effectively suppress the root-mean-square responses of the two adjacent buildings.

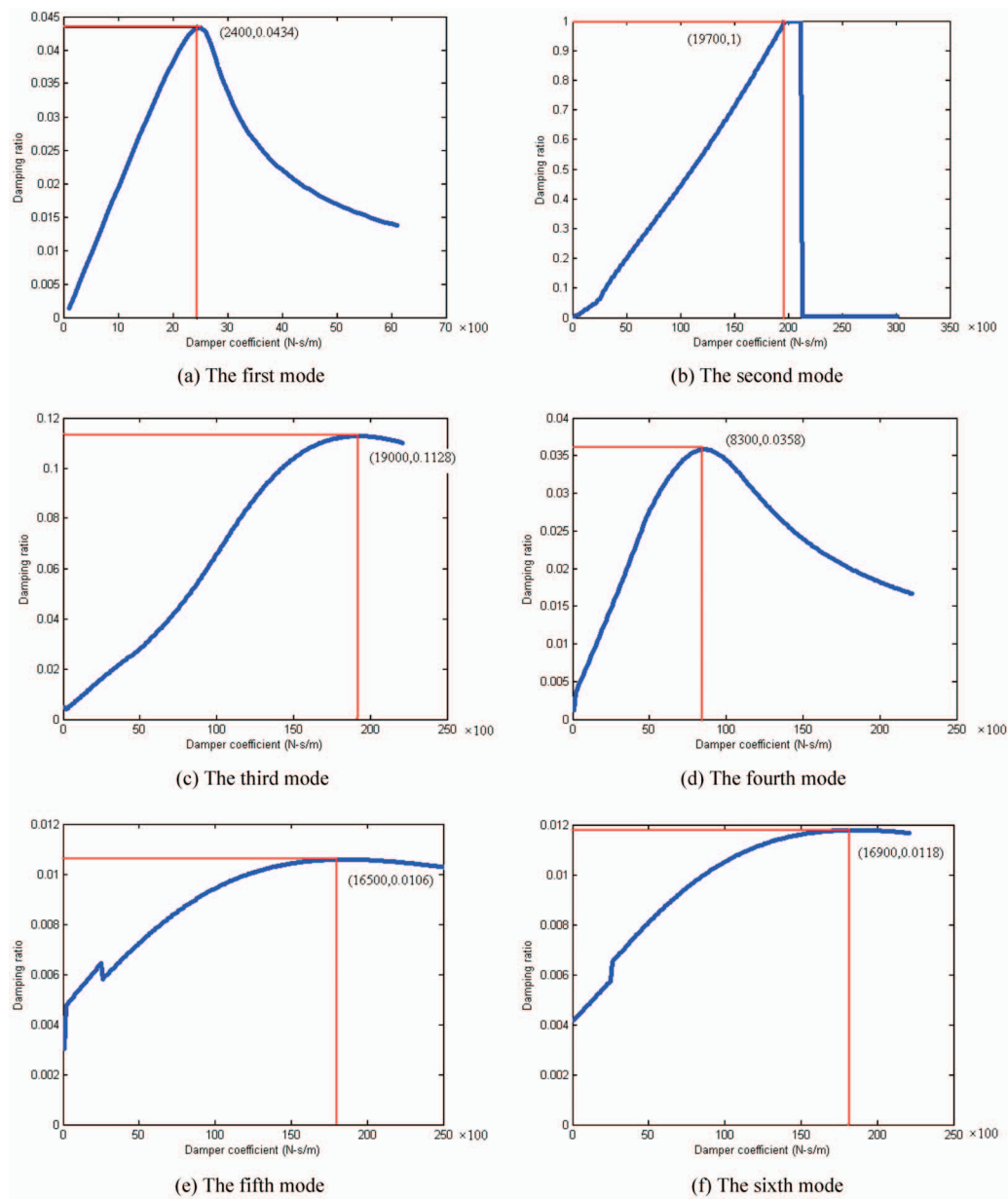


Figure 4. Curves of damping ratio versus damper coefficient of the system.

4. Conclusions

The objective of this paper is to investigate the dynamic characteristics of two adjacent building structures interconnected by viscous dampers under seismic excitations. The computational procedure for an analytical model including the system model formulation, complex modal analysis and seismic time history analysis is pre-

sented for this purpose. A numerical example is also provided to illustrate the analytical model.

The complex modal analysis is conducted to determine the optimal damping ratio and the optimal damper coefficient of the linear viscous damper for each mode of the system. For the damper coefficient with zero value, the underdamped vibration occurs for each mode due to the fact that the modal damping ratio contributed wholly

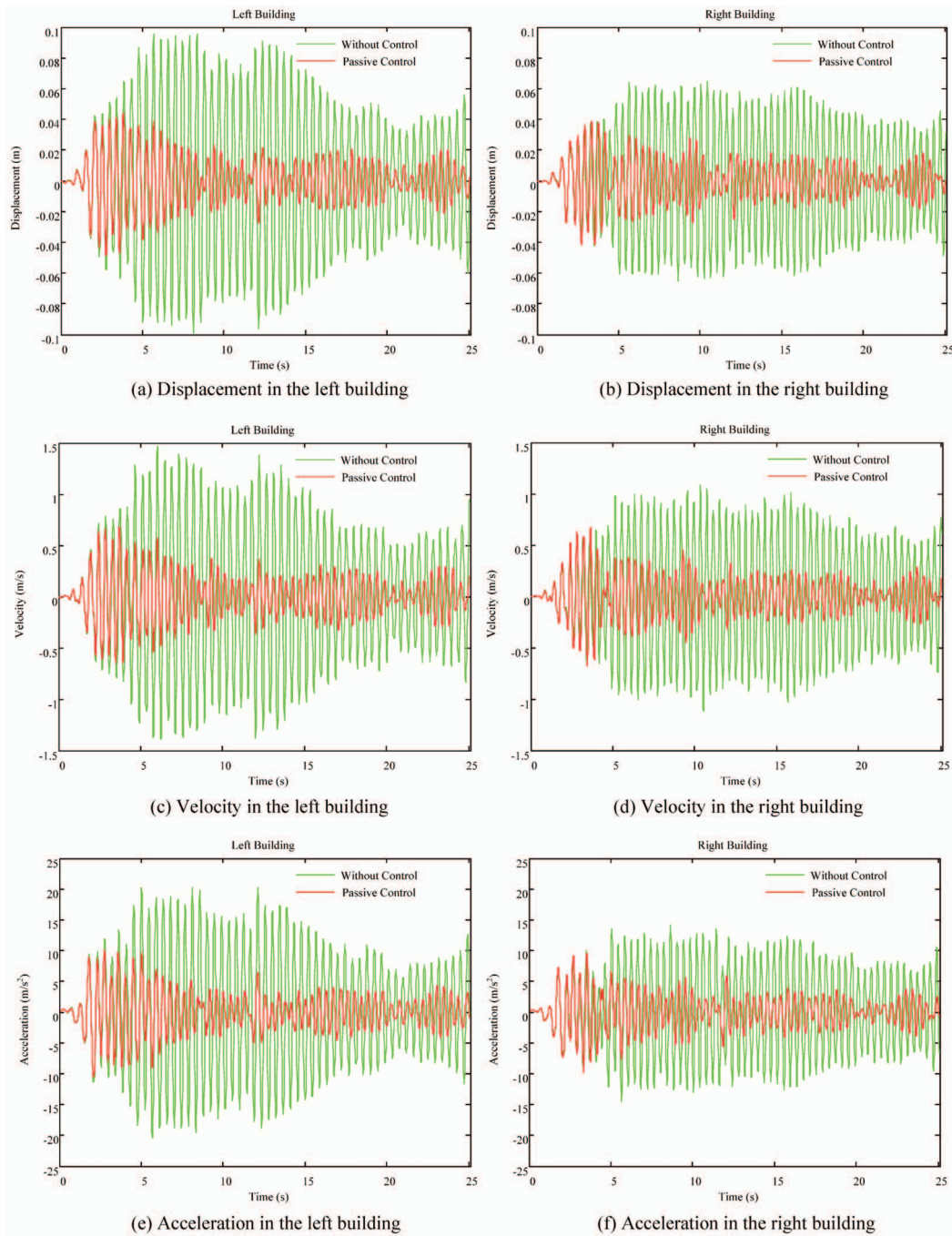


Figure 5. Response time histories of the third floor in the two adjacent buildings under seismic excitations.

Table 3. Dynamic responses of the third floor in the two adjacent buildings under seismic excitations

(a) Left building

System	Root-mean-square response			Peak response		
	Displacement (m)	Velocity (m/s)	Acceleration (m/s ²)	Displacement (m)	Velocity (m/s)	Acceleration (m/s ²)
Without damper	0.0387	0.5460	7.8605	0.0989	1.4669	20.4099
With damper	0.0117	0.1668	2.5251	0.0491	0.6776	10.4462
Response reduction rate (%)	69.7674	69.4505	67.8761	50.3539	53.8073	48.8180

(b) Right building

System	Root-mean-square response			Peak response		
	Displacement (m)	Velocity (m/s)	Acceleration (m/s ²)	Displacement (m)	Velocity (m/s)	Acceleration (m/s ²)
Without damper	0.0291	0.4604	5.9261	0.0650	1.1680	14.5859
With damper	0.0100	0.1481	2.1772	0.0419	0.6770	9.8352
Response reduction rate (%)	65.6357	67.8323	63.2608	35.5385	42.0377	32.5705

from the structural damping is smaller than unity. For the damper coefficient with optimal value, the responses can be categorized into underdamped and critically damped vibrations. For each of the modes dominated by the underdamped vibration, the modal damping ratio contributed primarily from the damper coefficient as well as secondarily from the structural damping is smaller than unity. While for each of the modes dominated by the critically damped vibration, the modal damping ratio approaching unity is considered to be contributed entirely from the damper coefficient and the contribution from the structural damping can be neglected.

The seismic time history analysis is conducted to assess the effectiveness of the linear viscous damper for vibration control. Based on the optimal damping ratio and the optimal damper coefficient of the linear viscous damper for a certain mode of the system, compared to the peak responses, the linear viscous damper can be used to more effectively suppress the root-mean-square responses of the two adjacent buildings.

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Manuscript Received: Feb. 22, 2016

Accepted: Jun. 27, 2016