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Equilibrium Stabilization Method to Control Chaotic Behavior of Continuous Systems

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Abstract

Chaos is acomplicated phenomenon in nonlinear dynamical systems. A dynamic controller, based on the stability transformation method (STM), has been used to stabilize both multiple fixed points and unknown unstable periodic orbits (UPOs) in dynamical systems. However, in these methods, there is not any unified algorithm in order to stabilize the fixed points. Here, we present a universal and simple chaos control algorithm called method of selecting an unstable fixed point (SUFP), which is able to stabilize unstable selected fixed points, by generating a stability matrix. Although this algorithm modifies a system by applying an input feedback, the designed feedback does not relocate the positions of fixed points but changes their stabilities. Results show that all tested chaotic trajectories are attracted to selected points, independent of initial values.

Key Words: Stability Matrix, Chaotic System, Fixed Point, Selecting Unstable Fixed Point, SUFP, Eigenvalue, Algorithm, Lorenz Model

1. Introduction

Chaos is a naturally complex phenomenon in deterministic nonlinear dynamical systems. Chaotic behavior has been found in most model systems in biology, physics, chemistry, ecology, etc. Moreover, engineering applications of chaos are rapidly growing in several areas, such as laser technologies, mechanical and chemical engineering, neural networks and telecommunications, etc. Recently, the field of controlling chaotic systems has been under elaborate investigation. The E. Ott, C. Grebogi and J. A. Yorke (OGY) method [1–4] is a good example of a great feedback control strategy. This model is implemented in order to control the chaotic behavior of practical systems with almost no analytic knowledge about a system; however, its sensitivity with respect to noise [5] is high. Actually, the disadvantages of this method are the weak isolation of the Poincaré section and calculation of the precise perturbations [6,7]. Another useful method is the Pyragas's one [8], which is called the time-delay (DFC) method. This optional feedback control method improves noise issues, but time delay evaluation is required for implementing the method.

Kittel et al. [5] proposed feeding the varying time delay to the feedback controller for an autonomous system, where the calculation of discretized delayed time is difficult. Later, Rezaie et al. [9] proposed a simple adaptive algorithm with a continuously varying time delay. In these algorithms, it is not necessary to know how UPOs are controlled. Although the main target of chaos control is to locate and stabilize the UPOs in a chaotic attractor, in some cases, the stabilizations of unstable fixed points are given higher priority. Classical control theory, including state vector feedback control [10,11] and speed feedback control [10,12,13], is commonly used to control unstable equilibrium points of autonomous chaotic systems. However, for a complicated chaotic system, it is difficult to select a single feedback variable to control several unstable fixed points. In order to solve this problem, dislocated feedback control and raised feedback control are introduced [14–16].

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Here, we propose a SUFP method, which is effective and applicable in controlling the chaotic behavior of unstable systems. This method stabilizes the equilibrium points of a chaotic system by introducing a stability matrix, which is derived according to the characteristics of the unstable equilibrium point. Then, in simulation and result section we implement the method to stabilize the Lorenz model and a new 3D autonomous model.

2. Method of Chaos Control

The trajectory of continuous chaotic systems crossing between collections of saddle fixed points and the collection of these fixed points are unstable [17]. Provided that applying feedback stables one of the points, the trajectory of the continuous chaotic system will be attracted to that point and the chaotic behavior of system will be controlled. In order to control the chaotic behavior of a system using our proposed algorithm, five steps are introduced:

1. Finding fixed points.
2. Analyzing stability of selected fixed points by linearization.
3. Selecting one unstable fixed point.
4. Generating a stability matrix (C) according to the selected unstable fixed point.
5. Generating input feedback according to the stability matrix. Consequently, $\dot{X} = Cf(X)$ will be a controlled version of $\dot{X} = f(X)$, and both systems have the same fixed points. Notice that input feedback is $U = (C - I)f(X)$ where I is the unit matrix.

Each step is explained as follows:

2.1 Finding Fixed Points

Suppose the following system of equations:

$$\begin{aligned} \dot{X}_1 &= f_1(X_1, X_2, \ldots, X_n), \\ \dot{X}_2 &= f_2(X_1, X_2, \ldots, X_n), \\ &\vdots \\ \dot{X}_n &= f_n(X_1, X_2, \ldots, X_n) \end{aligned} \tag{1}$$

where $X_1, X_2, \ldots, X_n \in R_n$. The fixed points of systems are acquired by solving:

$$\begin{aligned} &f_1(X_1^*, X_2^*, \ldots, X_n^*) = 0, \\ &f_2(X_1^*, X_2^*, \ldots, X_n^*) = 0, \\ &\qquad\vdots \\ &f_n(X_1^*, X_2^*, \ldots, X_n^*) = 0 \end{aligned} \tag{2}$$

2.2 Linearization

By linearizing the stable points, the Jacobian matrix is calculated as:

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial X_1} & \dfrac{\partial f_1}{\partial X_2} & \cdots & \dfrac{\partial f_1}{\partial X_n} \\ \dfrac{\partial f_2}{\partial X_1} & \dfrac{\partial f_2}{\partial X_2} & \cdots & \dfrac{\partial f_n}{\partial X_n} \\ \vdots & \vdots & & \vdots \\ \dfrac{\partial f_2}{\partial X_1} & \dfrac{\partial f_2}{\partial X_2} & \cdots & \dfrac{\partial f_n}{\partial X_n} \end{bmatrix} (X_1^*, X_2^*, \ldots, X_n^*) \tag{3}$$

where the sign of the eigenvalues determines the stability of the points:

$$|\lambda \cdot I - A| = 0 \tag{4}$$

2.3 Selecting One Unstable Fixed Point

When at least one eigenvalue of the selected fixed point is positive, then that point is unstable [17]. In this step it is better to select an unstable fixed point that is closer to the initial point.

2.4 Generating Stability Matrix

After finding fixed points, linearization and selecting an unstable fixed point, the Jacobian matrix related to the selected fixed point (matrix A) can be formed. According to the structure of matrix A, matrix C is generated.

In this section, we prove the existence of the stability matrix (matrix C), then we explain how this matrix is obtained.

Theorem 1. Consider an arbitrary and non-singular $n \times n$ matrix with negative or positive eigenvalues, called A. Thus, at least there is a $n \times n$ matrix, C, whose eigenvalues of matrix product CA, have the same magnitudes as the eigenvalues of matrix A, while the signs of eigenvalues of matrix product CA are negative.

Proof. Assuming an arbitrary and non-singular matrix A, where $\lambda_{11}, \lambda_{22}, \ldots, \lambda_{nn}$ are its eigenvalues and, V_1,

V_2, ..., V_n are its eigenvectors. The Jordan form of A is:

$$\Lambda = V^{-1}AV = \begin{bmatrix} \pm\lambda_{11} & 0 \text{ or } 1 & 0 & \dots & 0 \\ 0 & \pm\lambda_{22} & 0 \text{ or } 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \text{ or } 1 \\ 0 & 0 & \dots & 0 & \pm\lambda_{nn} \end{bmatrix} \quad (5)$$

where $\lambda_{11}, \lambda_{22}, \ldots, \lambda_{nn}$ can be equal or not.

First a matrix is designed (matrix K) that the signs of eigenvalues of matrix $K\Lambda$ are negative.

$$K\Lambda = \begin{bmatrix} \pm 1 & k_{12} & k_{13} & \dots & k_{1n} \\ 0 & \pm 1 & k_{23} & \dots & k_{2n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & k_{(n-1)n} \\ 0 & 0 & \dots & 0 & \pm 1 \end{bmatrix} \begin{bmatrix} \mp\lambda_{11} & 0 \text{ or } 1 & 0 & \dots & 0 \\ 0 & \mp\lambda_{22} & 0 \text{ or } 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \text{ or } 1 \\ 0 & 0 & \dots & 0 & \mp\lambda_{nn} \end{bmatrix} \quad (6)$$

$$= \begin{bmatrix} -\lambda_{11} & 0 \text{ or } 1 & 0 & \dots & 0 \\ 0 & -\lambda_{22} & 0 \text{ or } 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \text{ or } 1 \\ 0 & 0 & \dots & 0 & -\lambda_{nn} \end{bmatrix}$$

Obviously, it is always possible to determine the $k_{12}, k_{13}, \ldots, k_{(n-1)n}$ so that eigenvalues of $K\Lambda$ are all negative. By naming the matrix VKV^{-1} as matrix C and considering $A = V\Lambda V^{-1}$:

$$\begin{aligned} CA &= (VKV^{-1})(V\Lambda V^{-1}) = VK(V^{-1}V)\Lambda V^{-1} = VK(I)\Lambda V^{-1} \\ &= V(K\Lambda)V^{-1} \end{aligned} \quad (7)$$

Due to the similarity between the product of CA and $K\Lambda$($CA^{-} \rightarrow K\Lambda$), there is a matrix such as C where CA has negative eigenvalues. Thus, by knowing A and K matrices, C matrix is generated.

The matrix structure K is given as below:

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & \dots & k_{1n} \\ 0 & k_{22} & k_{23} & k_{24} & \dots & k_{2n} \\ 0 & 0 & k_{33} & k_{34} & \dots & k_{3n} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & k_{(n-1)n} \\ 0 & 0 & \dots & 0 & 0 & k_{nn} \end{bmatrix} \quad (8)$$

where the values of elements of matrix K are obtained according to the sign and repeatability of eigenvalues of matrix Λ.

Eqs. 9 and 10 indicate how the elements of matrix K are defined.

$$\text{if } \lambda_{ij} \le 0 \Rightarrow \begin{cases} k_{ij} = 0 & (\text{for } i > j) \\ k_{ij} = 1 & (\text{for } i = j) \\ k_{ij} = 0 & (\text{for } i < j) \end{cases} \quad (9)$$

and

$$\text{If } \lambda_{ij} > 0 \Rightarrow \begin{cases} k_{ij} = 0 \ (\text{for } i > j) \\ k_{ij} = -1 \ (\text{for } i = j) \\ k_{ij} = 0 \ (\text{for } i < j \text{ and } L > i) \\ k_{ij} = 0 \ (\text{for } i < j \text{ and } L \le i \text{ and } j > R + L - 1) \\ k_{ij} = (-1)^{i+j+1}\left(\dfrac{2}{\lambda^{j-i}}\right) \\ (\text{for } i < j \text{ and } L \le i \text{ and } j \le R + L - 1 \end{cases} \quad (10)$$

where R is a constant showing the number of repetitive eigenvalues and L is a constant showing the row where the first repetitive eigenvalue begins in matrix Λ. In order to clarify the method, an example is considered.

Example:

Assume:

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 8 & 20 & 18 & 7 \end{bmatrix} \quad (11)$$

The Jordan form of Eq. 11 is:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad (12)$$

and the eigenvectors of matrix A are:

$$V_1 = \begin{bmatrix} 8 \\ -8 \\ 8 \\ -8 \end{bmatrix}, \ V_2 = \begin{bmatrix} -4 \\ 8 \\ -16 \\ 32 \end{bmatrix}, \ V_3 = \begin{bmatrix} 6 \\ -8 \\ 8 \\ 0 \end{bmatrix}, \ V_4 = \begin{bmatrix} -7 \\ 8 \\ -8 \\ 8 \end{bmatrix} \quad (13)$$

Now, our goal is to find the matrix C whereas matrix product CA has negative eigenvalues. K matrix is generated by using Eq. 8

$$K = \begin{bmatrix} -1 & k_{12} & k_{13} & k_{14} \\ 0 & -1 & k_{23} & k_{24} \\ 0 & 0 & -1 & k_{34} \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{14}$$

Since λ_{11} is positive, distinct element ($R = 1$) and there is in the first row ($L = 1$), the fourth condition of 10 is satisfied and $k_{12} = k_{13} = k_{14} = 0$. Also, since λ_{22} is a repetitive element ($R = 3$) and the first repetitive eigenvalue begins in the second row ($L = 2$), the fifth condition of 10 is established and $k_{23} = k_{34} = 1$ and $k_{24} = -1$. Therefore, matrix $K\Lambda$ is obtained as follows:

$$\begin{aligned} K\Lambda &= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -\frac{1}{2} \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \end{aligned} \tag{15}$$

Finally, according to Eq. 7, matrix C is generated:

$$C = \begin{bmatrix} 7 & 15 & 8.5 & 1.5 \\ -12 & -23 & -12 & -2 \\ 16 & 28 & 13 & 2 \\ -16 & -24 & -8 & -1 \end{bmatrix} \tag{16}$$

where the eigenvalues of matrix CA have the identical magnitude to the eigenvalues of matrix A, but the signs of matrix CA are all negative ($\lambda_{11} = -1$, $\lambda_{22} = -2$, $\lambda_{33} = -2$, $\lambda_{44} = -2$).

2.5 Generating Input Feedback

The input feedback is simply calculated as:

$$\begin{aligned} U(X_1, X_2, ..., X_n) &= (C) f(X_1, X_2, ..., X_n) - f(X_1, X_2, ..., X_n) \\ &= (C - I) f(X_1, X_2, ..., X_n) \end{aligned} \tag{17}$$

It should be reminded that $(C) f(X_1, X_2, ..., X_n)$ is the stabilized version of $f(X_1, X_2, ..., X_n)$.

3. Simulations and Discussion

In order to assess the efficiency of the proposed algorithm and further understanding of how it works, the SUFP algorithm was applied to two chaos models. Then, results of stabilization algorithm on these two models are presented and interpreted separately.

3.1 The Lorenz Model

The Lorenz model which a nonlinear, non-periodic, three-dimensional and deterministic [18], is defined in the form of a system of differential equations (18). The model suggests that atmosphere, as an example of a nonlinear system, exhibits regime-like structures that are, although quite deterministic, subject to sudden and apparently erratic change. Describing details of the Lorenz model is beyond than the scope of this study, thus we only try to introduce the model mathematically and leave it further meteorology interpretation to the readers. This famous model indicates the unpredictable behavior of the weather [19] as:

$$f(X_1, X_2, X_3) = \begin{cases} \dot{X}_1 = \sigma(X_2 - X_1) \\ \dot{X}_2 = X_1(\rho - X_3) - X_2 \\ \dot{X}_3 = X_1 X_2 - \beta X_3 \end{cases} \tag{18}$$

where $\dot{X}_1$ and $\dot{X}_2$ and $\dot{X}_3$ are derivatives of system states, X_1, X_2, and $X_3 \in R$, with respect to time. Behavior of the original system (Eq. 18) is chaotic for system parameters $\sigma = 10$, $\beta = 8/3$, and $\rho = 28$ as depicted in Figure 1. The fixed points of the Lorenz system have been shown in the left column of Table 1. According to the proposed the algorithm, three stability matrices are generated for the three unstable fixed points of the system. The third and forth columns of Table 1 respectively show the eigenvalues of the system before and after applying the stability algorithm. To clarify the algorithm, let us consider that the point (0, 0, 0) is selected to be stabilized, then:

$$C = \begin{bmatrix} 0.2597 & -0.5771 & 0 \\ -1.6159 & -0.2597 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

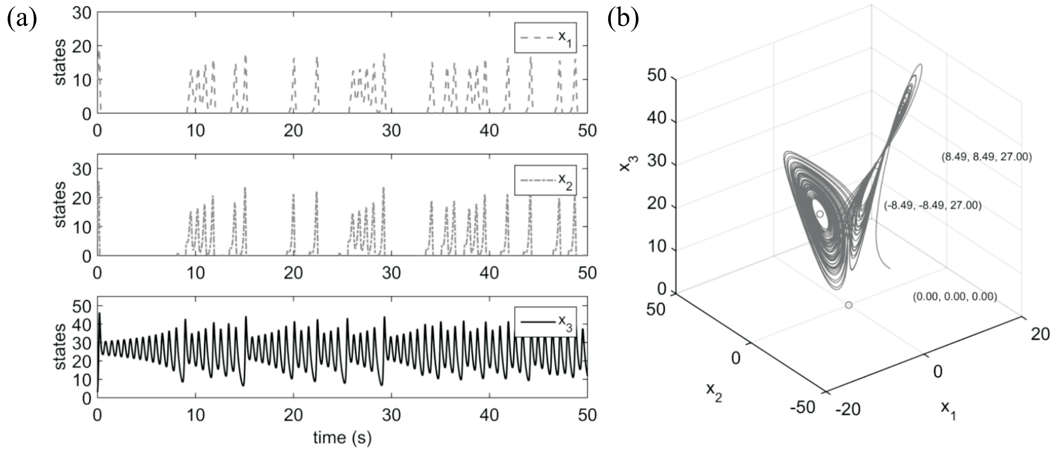

Figure 1. The chaotic behavior of the Lorenz system with initial point (10, 5, 3). (a) x_1, x_2 and x_3 are unstable vs. time. (b) The trajectory attract to none of fixed points. This system has three fixed points as listed in Table 1.

Table 1. Fixed points, C matrices, and eigenvalues of the Lorenz system before and after applying feedback. The feedback changes signs of all positive eigenvalues to negative and consequently stabilizes the system for each selected fixed point

Fixed point	Matrix C	Eigenvalues of $f(X_1, X_2, X_3)$	Eigenvalues of $Cf(X_1, X_2, X_3)$
(0, 0, 0)	$\begin{bmatrix} 0.2597 & -0.5771 & 0 \\ -1.6159 & -0.2597 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	λ_1 = -22.83 λ_2 = 11.83 λ_3 = -2.67	λ_1 = -22.83 λ_2 = -11.83 λ_3 = -2.67
(8.49, 8.49, 27)	$\begin{bmatrix} 0.4461 & -0.7496 & -0.5685 \\ -0.5574 & -0.711 & 0.2192 \\ -0.674 & 0.3494 & -0.735 \end{bmatrix}$	λ_1 = -17.65 λ_2 = -11.31 λ_3 = 7.22	λ_1 = -17.65 λ_2 = -11.31 λ_3 = -7.22
(-8.49, -8.49, 27)	$\begin{bmatrix} 0.4461 & -0.7496 & 0.5685 \\ -0.5574 & -0.711 & -0.2192 \\ 0.674 & -0.3494 & -0.735 \end{bmatrix}$	λ_1 = -13.85 λ_2 = 0.09 λ_3 = 0.09	λ_1 = -13.85 λ_2 = -0.09 λ_3 = -0.09

By assuming controllable variables, the input feedback is obtained:

$$U(X_1, X_2, X_3) = (C - I)f(X_1, X_2, X_3)$$
$$= \begin{cases} U_1 = -8.75X_1 - 6.83X_2 + 0.57X_1X_3 \\ U_2 = -14.9X_2 - 19.12X_1 + 1.26X_1X_3 \\ U_3 = 0 \end{cases} \quad (20)$$

Thus, eigenvalues of the new system will be λ_{11} = -22.83, λ_{22} = -11.83 and λ_{33} = -2.67. A 2D representation of X_1, X_2 and X_3 states exhibiting chaotic behavior vs. time is shown in Figure 1a and 1b, which respectively represent 2D and 3D portraits of X_1, X_2 and X_3 when the initial point is (10, 5, 3). In the Lorenz chaotic model, the trajectory is a function of time and it can be imagined as a roller coaster tracing an invisible object [20]. Figure 2 shows the behavior of the system after stabilization, in 2D and 3D portraits. To validate the algorithm with respect to different initial conditions, two more simulations were run (Figure 3). The results illustrated that the chaos control algorithm is independent of initial conditions.

Lyapunov exponents of the Lorenz system before and after applying feedback have been shown in Figure 4. According to [21,22], the chaotic system has at least one positive Lyapunov exponent (Figure 4a). In Figure 4b the chaotic behavior of the system is controlled and all Lyapunov exponents are negative.

4.2 A 3D Autonomous Chaotic System

In 2010, Dadras et al. [23] introduced a 3D smooth quadratic autonomous system having five real equilibrium

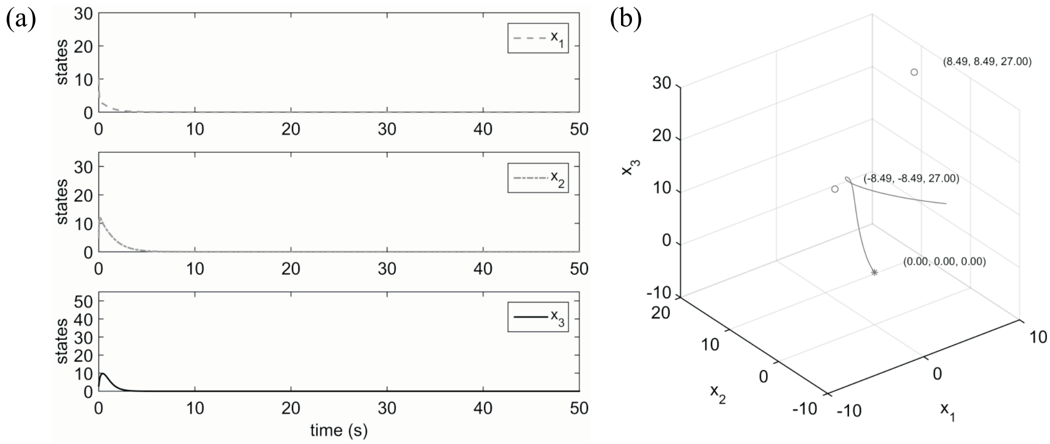

Figure 2. The trajectory behavior of the Lorenz system with initial point (10, 5, 3) after applying feedback. (a) x_1, x_2 and x_3 are stable vs. time. (b) The trajectory is attracted to fixed points (0, 0, 0) when the initial point is (10, 5, 3).

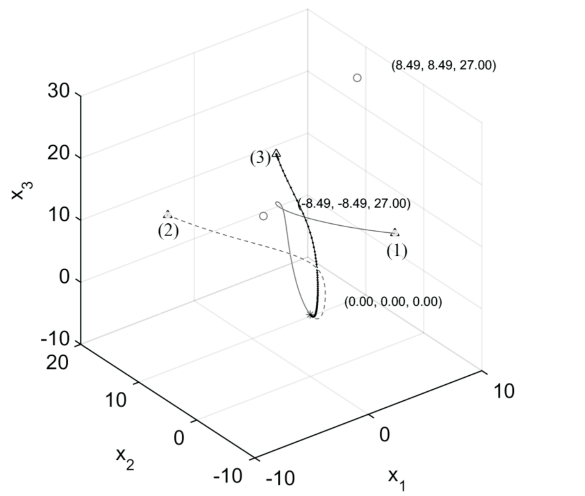

Figure 3. All trajectories of the Lorenz system attract to the fixed point (0, 0, 0), when the initial pointis (1): (10, 5, 3), (2): (-10, 5, 20) or (3): (-3, 0, 28).

points. This system is able to generate one-to four-wing chaotic attractors and, periodic motion with the variation of only one parameter. The following differential equations define the 3D autonomous chaotic system:

$$\begin{cases} \dot{X}_1 = X_2 - aX_1 + X_2X_3 \\ \dot{X}_2 = bX_2 - X_1X_3 \\ \dot{X}_3 = cX_1X_2 - dX_3 - hX_1^2 \end{cases} \tag{21}$$

where $\dot{X}_1$ and $\dot{X}_2$ and $\dot{X}_3$ are derivatives of system states X_1, X_2 and $X_3 \in R$, $a, b, c, d \in R^+$ and $h \in R$. By setting $a = 2.9$, $b = 3$, $c = 8$, $d = 11$ and $h = 0.5$ the system makes a chaotic attractor with two wings (Figure 5). For each unstable fixed point, a matrix C, and its corresponding eigenvalues were estimated and listed in Table 2. The behavior of the system before applying the feedback is shown in Figure 5.

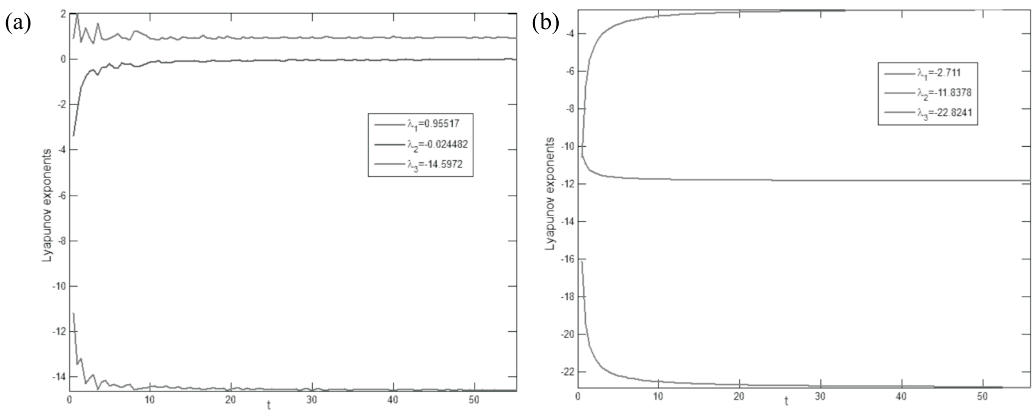

Figure 4. The behavior of the Lorenz system with (-3, 2, 5) as the initial condition. (a) Lyapunov exponents of the Lorenz system before applying feedback. λ_1 is positive and the system has chaotic behavior. (b) Lyapunov exponents of the Lorenz system after applying feedback. λ_1, λ_2, λ_3 are negative and the system has not chaotic behavior.

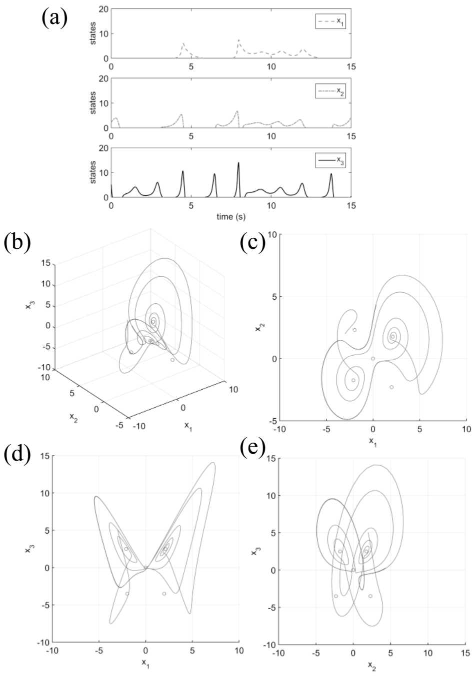

Figure 5. The chaotic behavior of the 3D autonomous system with (-3, 2, 5) as the initial condition. (a) The states of x_1, x_2 and x_3 are unstable vs. time. (b) The trajectory behaves like a roller coaster around the five fixed points. To clearly show the 3D portrait of the system with five fixed points, above (c), side (d) and front (e) views were added. This system has five fixed points as shown in Table 2.

Figure 5a illustrates the chaotic states of three variables for the original 3D autonomous system vs. time. Since representing the five fixed points of the system and behavior of the trajectory was messy and vague, the 3D portraits are shown from the top (Figure 5c), side (Figure 5d) and front (Figure 5e) views.

Table 2 represents characteristics and parameters of the stability matrices for each fixed point. Signs of positive eigenvalues were converted to negative by applying the proposed algorithm which stabilized the selected fixed point.

Figure 6a-e depicted 2D and 3D portraits of the stabilized system when the initial point was (10, 5, 3) and the stabilized point was (0, 0, 0). In this case, according to Table 2, the matrix C is as follow:

$$C = \begin{bmatrix} 1 & -0.34 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{22}$$

and the input feedback is obtained as follow:

$$U(X_1, X_2, X_3) = (C - I) f(X_1, X_2, X_3) = \begin{cases} U_1 = 0.339 X_1 X_3 - X_2 \\ U_2 = 2 X_1 X_3 - 6 X_2 \\ U_3 = 0 \end{cases} \tag{23}$$

Table 2. Fixed points, C matrices, and eigenvalues of the 3D autonomous system before and after applying feedback. The feedback changes signs of all positive eigenvalues to negative and consequently stabilizes the system for each selected fixed point

Fixed point	Matrix C	Eigenvalues of $f(X_1, X_2, X_3)$	Eigenvalues of $Cf(X_1, X_2, X_3)$
(0, 0, 0)	$\begin{bmatrix} 1 & -0.34 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	λ_1 = -11.00 λ_2 = 3.00 λ_3 = -2.90	λ_1 = -2.90 λ_2 = -3.00 λ_3 = -11.00
(1.98, -2.30, -3.49)	$\begin{bmatrix} -0.38 & -0.20 & 0.31 \\ 0.20 & -1.06 & 0.10 \\ 2.8 & -0.90 & 0.44 \end{bmatrix}$	λ_1 = -14.31 λ_2 = 1.71 λ_3 = 1.71	λ_1 = -14.31 λ_2 = -1.71 λ_3 = -1.71
(2.11, 1.75, 2.49)	$\begin{bmatrix} -0.45 & 0.28 & -0.36 \\ -0.24 & -1.12 & 0.16 \\ -2.38 & -1.21 & 0.57 \end{bmatrix}$	λ_1 = -12.03 λ_2 = 0.56 λ_3 = 0.56	λ_1 = -12.03 λ_2 = -0.56 λ_3 = -0.56
(-1.98, 2.30, -3.49)	$\begin{bmatrix} -0.38 & -0.20 & -0.31 \\ 0.20 & -1.06 & -0.10 \\ -2.8 & 0.90 & 0.44 \end{bmatrix}$	λ_1 = -14.31 λ_2 = 1.71 λ_3 = 1.71	λ_1 = -14.31 λ_2 = -1.71 λ_3 = -1.71
(-2.11, -1.75, 2.49)	$\begin{bmatrix} -0.45 & 0.28 & 0.36 \\ -0.24 & -1.12 & -0.16 \\ 2.38 & 1.21 & 0.57 \end{bmatrix}$	λ_1 = -12.03 λ_2 = 0.56 λ_3 = 0.56	λ_1 = -12.03 λ_2 = -0.56 λ_3 = -0.56

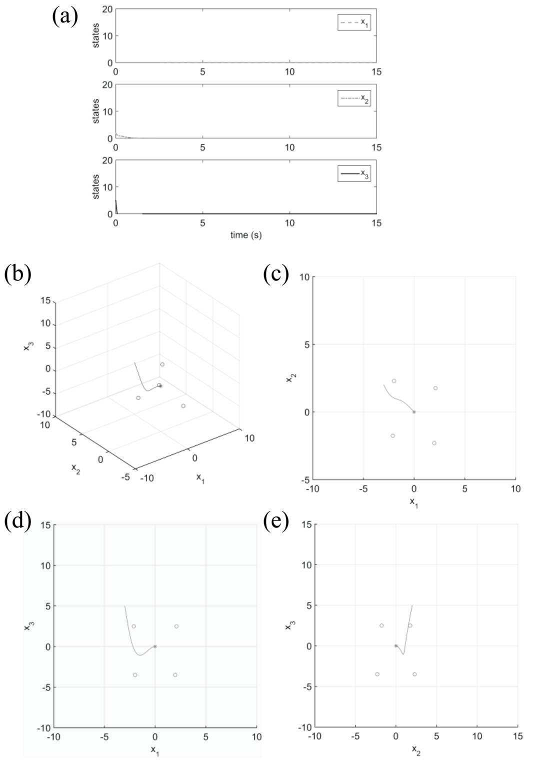

Figure 6. The behavior of the 3D autonomous system with (10, 5, 3) as the initial condition, after applying feedback control. (a) The states of x_1, x_2 and x_3 are stable vs. time. (b) The trajectory smoothly is attracted to (0, 0, 0). Above (c), side (d) and front (e) view.

Likewise, Figure 7a and b indicate that the stabilization algorithm is independent of the selected fixed point. The chaotic system and the controlled system are shown in Figure 8a and Figure 8b.

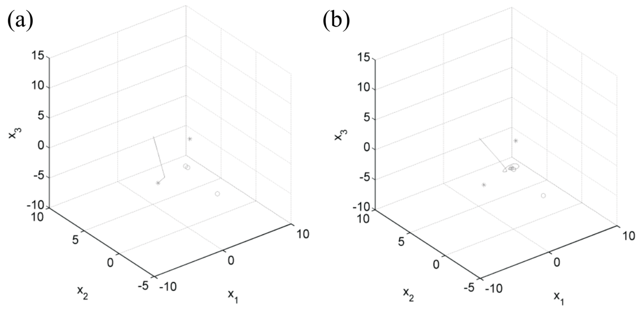

Figure 7. The behavior of the stabilized 3D autonomous system when initial condition are (a) (-10, 5, 20) and (b) (-3, 0, 28).

4. Conclusion

This paper proposes a simple controlling design method for the continuous chaotic nonlinear systems, by introducing a new stability matrix and corresponding feedback. Although this algorithm modifies a system by applying an input feedback, the designed feedback does not relocate the positions of the fixed points, but it changes their stabilities. We indicated that by applying the proposed feedback for a selected unstable fixed point, the trajectory is attracted to this point, independent of its initial conditions. The algorithm was successfully tested on several chaotic systems; however, only the results of the tests on the Lorenz and 3D autonomous model were presented here. Since by considering the model limitations, the algorithm is independent of the model structures, it can be postulated that this algorithm works properly for almost all chaotic models which all states are controllable.

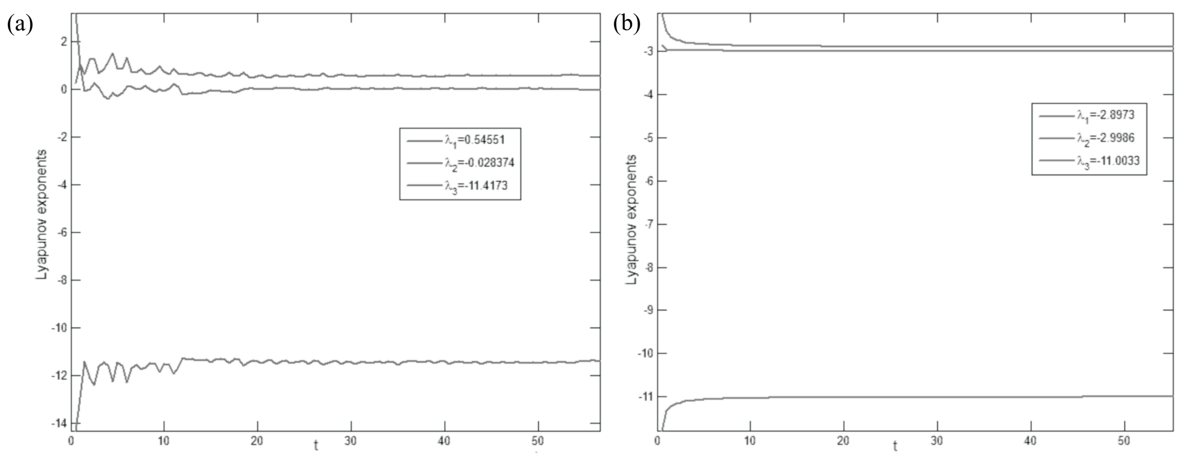

Figure 8. The behavior of the 3D autonomous system with (10, 5, 3) as the initial condition. (a) Lyapunov exponents of the 3D autonomous system before applying feedback. λ_1 is positive and the system has chaotic behavior. (b) Lyapunov exponents of the 3D autonomous system after applying feedback. λ_1, λ_2, λ_3 are negative and the system has not chaotic behavior.

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