

Simulation of subway tunnel construction process based on finite element analysis

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In the process of subway tunnel construction, the traditional design method of foundation pit support has less selection of calculation parameters, so it has relatively simple experience and cannot accurately express the complex characteristics of the soil. Based on the finite element analysis, this paper simulates the construction process of the subway tunnel, and uses the time-history analysis method to analyze the seismic response of the tunnel soil structure. Moreover, this paper uses the fast Fourier algorithm to perform Fourier transform on the bottom input of the finite element model and the surface acceleration time history curve, and studies the variation of peak acceleration with soil depth. In addition, this paper considers the mesh size of the finite element model, the selection of dynamic artificial boundaries and system damping ratio, and simulates various tunnel construction processes. Finally, this paper designs a controlled experiment to verify the performance of this method. The research results show that the predicted value obtained by the method proposed in this paper is very close to the actual value, so the method proposed in this paper is practical and effective.

Keywords: Finite element analysis, subway tunnel, construction simulation, simulation

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1. Introduction

With the continuous acceleration of urbanization and the increasing population density, many large cities have a series of problems such as population expansion and traffic congestion. The reasonable development and utilization of underground space is a very effective solution. The urban subway not only has the characteristics of safety, reliability, convenience and comfort on time, but also occupies a small area and basically does not damage the landscape. In addition, by transferring part of the street traffic to the underground, it can quickly transport a large number of passengers and greatly alleviate the problem of urban traffic congestion. Therefore, many large cities have turned their attention to the construction of subways. At present, many large and medium-sized cities are building or continuing to build subways, and underground development has become the future development trend of cities [1].

The excavation of underground rock and soil will cause surface settlement and deformation, and subway lines generally pass through the prosperous downtown area with densely populated, ground-based buildings, and dense underground pipe networks. This section has very high requirements for surface displacement and deformation caused by construction, and a slight mistake in the selection of construction methods will cause immeasurable losses. Severe cases will cause uneven settlement, cracking, and even collapse of the building causing casualties. For nearby underground railway tunnels, it will cause serious accidents such as excessive deformation and train derailment. Even for underground pipelines with small pipe diameters, excessive deformation will cause gas leakage and explosion, and cause water pipes to burst to form water disaster, and cable breaks to cause power outages or communication interruptions. These will seriously endanger production

and construction and the safety of people's lives and properties, and cause great direct and indirect economic losses [2]. It is very difficult to completely stop the settlement and deformation of the ground. Therefore, we can only take effective measures to reduce deformation according to the requirements of surface protection, so that buildings (structures) such as surface houses will not be damaged, and the ecological environment will not deteriorate [3].

In order to ensure the smooth progress of the tunnel project and the safety of the surrounding buildings, it is very important to carry out in-depth research on dynamic construction mechanics of tunnel engineering rock and soil, and to propose a more reliable prediction and control method of ground movement and deformation caused by tunnel excavation and the deformation of surrounding buildings. Therefore, how to reduce the disturbance of the tunnel engineering to the surrounding soil and minimize the impact of the tunnel engineering on the neighboring buildings and the ground are all the concerns of the actual engineering.

2. Related work

The literature [4] simulated and analyzed the dynamic loads in two directions generated during subway train operation, including the simulation of the lateral dynamic load of the subway train and the simulation of the longitudinal dynamic load of the subway train. By processing the measurement point data and performing spectrum analysis, the mathematical expression of the acceleration between the subway wheel and the rail is deduced, and the expression of the vibration load of the train is obtained. The literature [5] obtained the expression of vibration load generated during subway operation through spectrum analysis method. The literature [6] drew the subway acceleration waveform curve according to the collected data and obtained the mathematical expression of the vibration load during subway operation. Based on previous research results, the literature [7] established a model that can analyze the vertical vibration of vehicles separately, and used Hertian contact theory to establish a sub-way-track system model to simulate vibration, thereby determining the mathematical expression of vibration load generated during train operation. The literature [8] conducted experiments on the dynamic response of the tunnel soil under different pressure conditions and used the fatigue damage theory of the soil to study the test results, thereby depicting the dynamic characteristics of the disturbed soil under the long-term vibration load. The literature [9] obtained the expression of the vibration load generated during train operation by analyzing the frequency spectrum peak caused by train

operation and assuming the direction of the vibration load generated during train operation. In the case of comprehensive consideration of the unevenness of the track and the wheel transfer, as well as the train model and other factors, literature [10] derived the expression of the train vibration load based on the existing re-search results. The literature [11] used the seismic wave reflection method and the multi-degree-of-freedom railway vehicle operation mode to discuss the Biowave propagation model suitable for porous isotropic materials. The literature [12] conducted data collection by arranging test reference points and installing acceleration testers on the subway line. Through frequency spectrum analysis of the collected data, the mathematical expression of acceleration during subway driving is obtained. The literature [13] laid out test reference points and installs instruments and equipment for a certain subway line 1 to conduct on-site monitoring and discovery. At this monitoring site, during the operation of the subway, the vibration frequency of the underground structure and soil of the subway is most prominent in the frequency range of 60-80 Hz. Moreover, there is a vibration-enhanced area located 10-30m away from the centerline of the tunnel, and the frequency of the enhanced area is mainly concentrated below 10 Hz. The literature [14] proposed a numerical model to simulate the vibration generated by subway tunnels. The model fits the soil and train in the free field separately, and constrains their boundaries. Moreover, the numerical analysis results of the dynamic load response of two different types of tunnels under the harmonic load of the tunnel invert are compared. The literature [15] obtained the prediction formula of subway vibration load by studying the influence of different factors on wave propagation, including the properties of foundation soil, track material, lining shape, sub-way model, structure form and other factors. The literature [16] comprehensively considered the impact of wheel-track contact, including train vibration load and wheel axle load when the train wheel contacts the track and moves. The literature [17] conducted field tests on a certain line of the Shanghai subway to study the ground vibration caused by subway operation. The results show that the ground vibration is mainly caused by the unevenness of the contact surface between the train wheels and the track when the train is running. Through the finite element analysis of the building-soil-tunnel model, the literature [18] found that the vibration frequency of buildings caused by the vibration load generated by subway operation is low and mainly concentrated in the range of 0-10Hz, and is related to the train speed. The literature [19] compared the environmental noise assessment rules with the environmental protection standard rules, and pointed out the

deficiencies in the formulation of various standards and some problems that need to be dealt with differently in the implementation process, and put forward the similarities and differences of environmental vibration indicators caused by train vibration in different standards and different regions. The literature [20] analyzed the environmental pollution caused by the vibration of the subway by setting up reference points along a certain subway line for field testing. Moreover, according to the measured data obtained from the re-search, the attenuation law of vibration in different frequency bands is analyzed, and the mass method and relative displacement method are used to analyze the multi-point excitation environment vibration of large-scale buildings.

2.1. Theory of dynamics and finite element modeling

According to D'Alembert's principle, the differential equation of motion of the tunnel-soil system under earthquake action is:

$$[M]\ddot{u} + [C]\dot{u} + [K]u = -[M]\ddot{u}_g \quad (1)$$

In the formula, $[M]$ is the mass matrix of the system, $[C]$ is the damping matrix of the system, $[K]$ is the stiffness matrix of the system, $\ddot{u}$ is the acceleration vector of the system, $\dot{u}$ is the velocity vector of the system, u is the displacement vector of the system, and $\ddot{u}_g$ is the input acceleration vector of the bedrock surface.

If it is assumed that the time differential element of the seismic acceleration function is Δt , the incremental form of the differential equation of the system motion under the action of the earthquake from time t to time $t + \Delta t$ is expressed as:

$$[M]\Delta\ddot{u}(t) + [C(t)]\Delta\dot{u}(t) + [K(t)]\Delta u(t) = -[M]\Delta\ddot{u}_g(t) \quad (2)$$

When $[C(t)]$ and $[K(t)]$ in the formula do not change with time, such a numerical calculation method is called an elastic time history analysis method. Eq. 2 The basic principle of the solution is to use different approximate methods to determine $\Delta\ddot{u}(t)$, $\Delta\dot{u}(t)$ and $\Delta u(t)$ the relationship between them. Generally, $\Delta u(t)$ is used to represent $\Delta\ddot{u}(t)$ and $\Delta\dot{u}(t)$. In this way, the result of $t + \Delta t$ can be completely obtained from the results of t and Δt , and used as the initial data for the next step until the solution is completed. Commonly used methods for solving dynamics include midpoint acceleration method, Wilson θ method and Newmark β method.

Due to the existence of damping, a system subjected to some kind of excitation cannot continue to oscillate, but gradually decay and disappear over time. Simply put, damping is the factor that makes the system motion stop.

As an important parameter of dynamic calculation, damping has a great influence on the final calculation result. According to the energy dissipation mechanism, damping is generally divided into material damping, structural damping and fluid damping. This includes the friction between the system and the outside world and the internal resistance of the medium. Mathematically, for the convenience of expression, damping is generally divided into viscous damping and complex damping. Although damping exists objectively in reality, it is difficult for many current models to accurately determine the mechanism of system energy dissipation. In the calculation, the viscous damping is generally determined by field measurement.

Rayleigh damping is widely used in engineering. Rayleigh damping is to use mass matrix and stiffness matrix to express the damping matrix in the dynamic equation as:

$$[C] = \alpha[M] + \beta[K] \quad (3)$$

In the formula, α is the damping coefficient related to the structural quality, and β is the proportional coefficient related to the structural stiffness.

In the external excitation, low-order modes are easily excited because they occupy most of the effective mass of the system. Usually, the first mode shape is selected to determine the damping in the analysis and calculation, then the proportional constant can be expressed as:

$$\alpha = \zeta_1 \omega_1 \quad (4)$$

$$\beta = \frac{\zeta_1}{\omega_1} \quad (5)$$

Among them, ω_1 is the first natural circular frequency of the structure, and ζ_1 is the first mode damping ratio of the structure. The value of ω_1 is obtained through modal analysis, and the first-order damping ratio of the structure in engineering can be obtained through experiments.

The Fourier transform is a linear integral transform, which can transform the signal between the time domain and the frequency domain. It plays an extremely important role in signal processing such as noise removal, filtering and image edge extraction, and physics such as heat transfer, vibration, sinusoidal voltage and current generated by alternating current, and describing the trajectory of planets, the periodic changes of the earth atmosphere, and so on.

If $f(x)$ has a period of $2l$ and satisfies Dirichlet's theorem of convergence, then:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{l} x + b_n \sin \frac{n\pi}{l} x \right) \quad (6)$$

$$a_n = \frac{1}{l} \int_{-l}^l f(x) \cos \frac{n\pi}{l} x dx, \quad n = 0, 1, 2, 3, \dots \quad (7)$$

$$b_n = \frac{1}{l} \int_{-l}^l f(x) \sin \frac{n\pi}{l} x dx, n = 0, 1, 2, 3, \dots \quad (8)$$

After that, we briefly introduce the derivation of Fourier series. Just as the Taylor formula uses each derivative of a function as the most essential feature of the function as a coefficient and describes the function in the form of a polynomial, it is necessary to understand why a function can be fully expressed by a trigonometric function. First, an arbitrary function can be decomposed into the sum of an odd function and an even function, and the sine function and the cosine function have exactly this property.

Second, since the period of $\cos \frac{n\pi}{l} x$, $\sin \frac{n\pi}{l} x$ is $2l$, $\cos \frac{n\pi}{l} x$, $\sin \frac{n\pi}{l} x$ ($n = 0, 1, 2, 3, \dots$) is a function of period $2l$. The third is to determine the coefficients a_n and b_n . We know that the trigonometric function system $\{1, \cos \frac{n\pi}{l} x, \sin \frac{n\pi}{l} x, n = 0, 1, 2, 3, \dots\}$ is complete and orthogonal in the interval $[-l, l]$.

If n functions $\varphi_i(t)$, $n = 0, 1, 2, 3, \dots$ form an arbitrary orthogonal function space at (t_1, t_2) , then the arbitrary function signal can be linearly superimposed by these orthogonal function sets, namely:

$$f(t) = \sum_{i=1}^{\infty} C_i \varphi_i(t) \quad (9)$$

$$C_i = \frac{\int_{t_1}^{t_2} f(t) \varphi_i(t) dt}{\int_{t_1}^{t_2} \varphi_i^2(t) dt} \quad (10)$$

It is easy to deduce a_n and b_n as:

$$a_n = \frac{\int_{-l}^l f(x) \cos \frac{n\pi}{l} x dx}{\int_{-l}^l \cos^2 \frac{n\pi}{l} x dx} \quad (11)$$

$$b_n = \frac{\int_{-l}^l f(x) \sin \frac{n\pi}{l} x dx}{\int_{-l}^l \sin^2 \frac{n\pi}{l} x dx} \quad (12)$$

$$= \frac{1}{l} \int_{-l}^l f(x) \sin \frac{n\pi}{l} x dx, n = 0, 1, 2, 3, \dots$$

According to Euler's formula, it is easy to get the complex exponential form of the Fourier series:

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{jn \frac{n\pi}{l} x}, n = 0, 1, 2, 3, \dots \quad (13)$$

$$c_n = \frac{1}{2l} \int_{-l}^l f(x) e^{jn \frac{n\pi}{l} x} dx, n = 0, 1, 2, 3, \dots \quad (14)$$

We set:

$$\omega_n = \frac{n\pi}{l}, n = \dots, -3, -2, -1, 0, 1, 2, 3, \dots \quad (15)$$

It can be seen that ω_n changes by jump.

$$\Delta\omega_n = \omega_{n+1} - \omega_n = \frac{\pi}{l} \quad (16)$$

Through this process, it is easy to obtain the frequency spectrum of the periodic function.

For Fourier series expansion, we must first ensure that the function is a periodic function. However, many signals encountered in engineering practice are non-periodic sampling signals. In this way, it is necessary to regard the aperiodic function as a periodic function when the period T approaches and transforms from positive infinity.

$$\Delta\omega_n = \omega_{n+1} - \omega_n = \frac{\pi}{l} \rightarrow \infty, \omega_n = \omega \quad (17)$$

When the Fourier series of the periodic function is extended to aperiodic, there is:

$$f(x) = \lim_{l \rightarrow \infty} \sum_{n=-\infty}^{+\infty} c_n e^{i\omega_n x} \quad (18)$$

$$= \lim_{\Delta\omega_n \rightarrow 0} \sum_{n=-\infty}^{+\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\xi) e^{-i\omega_n \xi} d\xi \right] e^{i\omega_n x} \Delta\omega_n$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} f(\xi) e^{-i\omega_n \xi} d\xi \right] e^{i\omega_n x} d\omega \quad (19)$$

Therefore, the Fourier transform of a continuous function can be expressed as:

$$F(\omega) = \int_{-\infty}^{+\infty} f(\xi) e^{-i\omega \xi} d\xi \quad (20)$$

3. Data simulation analysis

On the basis of the above analysis, the simulation of the finite element method proposed in this paper in subway tunnel construction is analyzed, and the actual situation of some projects is compared with the simulated value of the method in this paper. First of all, this paper carries out the simulation of the horizontal displacement of the pile, as shown in Table 1 and Fig. 1.

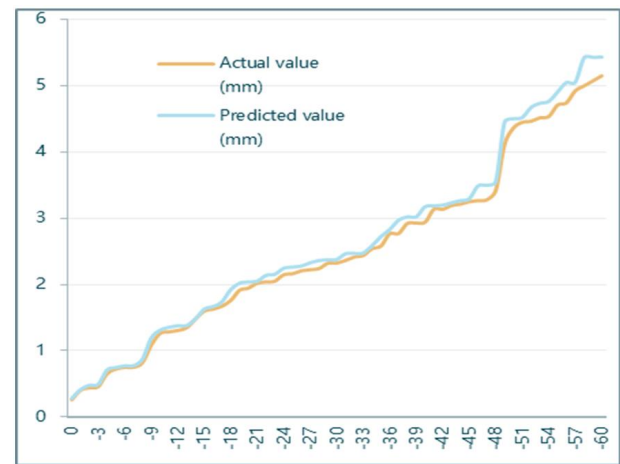


Fig. 1. Statistical diagram of the simulation value of pile horizontal displacement (mm).

Table 1. Statistical table of simulated values of pile horizontal displacement (mm).

Pile depth (m)	Actual Value (mm)	Predicted value (mm)	Pile depth (m)	Actual Value (mm)	Predicted value (mm)
0	0.261	0.283	-31	2.363	2.464
-1	0.405	0.416	-32	2.414	2.469
-2	0.440	0.479	-33	2.437	2.475
-3	0.458	0.494	-34	2.540	2.586
-4	0.653	0.711	-35	2.580	2.723
-5	0.724	0.746	-36	2.764	2.828
-6	0.750	0.773	-37	2.768	2.969
-7	0.752	0.776	-38	2.920	3.019
-8	0.821	0.872	-39	2.927	3.021
-9	1.085	1.190	-40	2.939	3.172
-10	1.262	1.310	-41	3.134	3.186
-11	1.285	1.354	-42	3.135	3.198
-12	1.306	1.379	-43	3.191	3.232
-13	1.347	1.380	-44	3.212	3.264
-14	1.478	1.481	-45	3.248	3.293
-15	1.598	1.628	-46	3.264	3.487
-16	1.628	1.667	-47	3.278	3.497
-17	1.673	1.737	-48	3.416	3.549
-18	1.759	1.923	-49	4.113	4.449
-19	1.911	2.020	-50	4.359	4.499
-20	1.946	2.038	-51	4.445	4.517
-21	2.019	2.050	-52	4.462	4.670
-22	2.039	2.137	-53	4.514	4.733
-23	2.049	2.154	-54	4.533	4.760
-24	2.146	2.245	-55	4.705	4.903
-25	2.162	2.263	-56	4.740	5.047
-26	2.206	2.279	-57	4.920	5.050
-27	2.222	2.327	-58	4.996	5.418
-28	2.241	2.360	-59	5.071	5.427
-29	2.319	2.371	-60	5.147	5.429
-30	2.323	2.376			

From the above analysis results, the finite element method proposed in this paper performs well in predicting the horizontal displacement of piles in subway tunnel construction. Next, this article simulates the change of the steel support axial force with the excavation depth, and the results are shown in Table 1 and Fig. 2.

From the above analysis, the finite element method proposed in this paper can play a certain role in subway tunnel construction and has certain effects.

4. Conclusion

The subway construction allows the land to be used better, and the underground space construction has been further developed. At the same time, it also promotes the research

and development of foundation pit engineering. The numerical simulation analysis method of foundation pit engineering has been greatly improved, and it has been used in engineering practice to assist and guide construction. The traditional foundation pit support design method chooses fewer calculation parameters, so it has simple experience and cannot accurately express the complex characteristics of the soil, and it cannot reflect the dynamic change process of foundation pit excavation. The excavation process of the foundation pit will not only cause lateral deformation of the retaining structure and cause the surface soil to sink, but also cause the stress release at the bottom of the pit, the uplift of the soil and the vertical displacement of the retaining structure. The finite element method provides

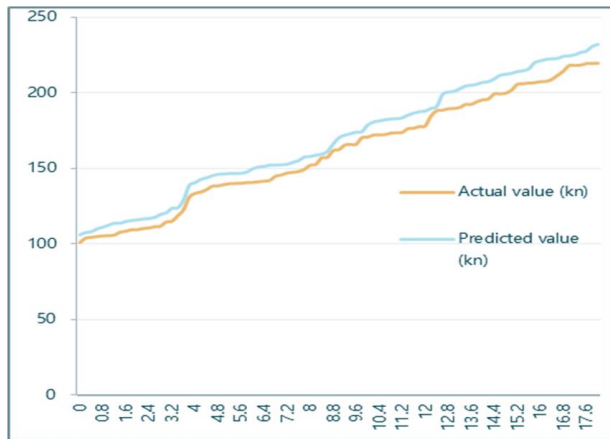


Fig. 2. Statistical diagram of changes in steel support axial force with excavation depth.

new ideas and methods for simulating excavation and support of foundation pits. Finally, this article analyzes the simulation effect of the finite element method proposed in this article in subway tunnel construction, and compares the actual situation of some projects with the simulated value of this method. The research results show that the model constructed in this paper has a certain effect.

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