

Fractal behaviour of microseismic energy associated with mining-induced tremors

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According to the real-time monitoring data from N1202 island workface of Yuwu coal mine, in Shanxi Province of China, the microseismicity is studied during the actual mining process in the field, the moment magnitude and energy distribution of microseismic events are obtained, the source locations of mining tremors caused by 3 large magnitude microseismic events are determined, and the moment magnitude and energy are calculated. Based on fractal and chaos theory, the mutual information method is introduced to compute the delay time of time series of microseismic energy, the G-P algorithm is introduced to calculate the fractal dimensions of mining tremors with different energy, which is in a declining trend and the lowest on the day of mining tremors. The moment magnitude, energy and fractal behavior of energy under different mining tremors will provide a valuable reference to the early warning of mine dynamic disaster.

Keywords: Mining-induced tremors; Microseismic energy; fractal behavior; Island workface; coal and rock dynamic disasters

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1. Introduction

Microseismic monitoring techniques had been greatly applied in the mining industry [1–6]. The relationship was found between energy and frequency of rockburst [7, 8]. A new method “Seismic Moment Method” was put forward for predicting rockbursts [9]. Based on the quantitative seismology, South African scholars Mendecki et al. [10] presented an early warning method of $M_L > 2.0$ mining tremors in a mine area. According to the regularity of microseismicity, the relationship was discussed between the possible rockburst and the spatial distribution of mining tremors [11].

Recently, different seismic methods and indices were provided for mine dynamic disasters. The local magnitude and energy distribution of mining tremors were investigated occurring over the period 1974–2005 in the Upper

Silesian Coal Basin of Poland [12]. In order to study the rupture processes of fault planes, the double couple and full moment tensor (MT) inversions were performed in the Ruhr region of Germany [13, 14]. Based on the energy distribution of earthquake, a method was found for hazard assessment in coal mine. [15]. A microseismic multi-parameters forecasting approach was presented to determine the disaster caused by crustal stress in tunnels of China [16]. A microseismic multidimensional information pre-warning method was studied for the identification of roof fall in a coalface [17]. A rockburst was investigated based on the multi-parameter precursory characteristics [18]. Two probabilistic methods were provided to assess the temporal and spatial characteristics of microseismicity in a salt mine of France [19].

Since the 1960s, the fractal theory was put forward by

the mathematicians Mandelbrot [20, 21], fractal theory has been widely applied to natural science including earthquake and mining science. For instance, the fractal features exist in the space-time sequences of earthquake events [22–24]. The space-time sequences of earthquake events were investigated in three earthquake prone areas of Italy based on the fractal method [25]. The multifractal structure of time series of events was studied in the seismic region of India [26].

In 1997, the fractal theory of rock mechanics was created [27]. Recently, some researchers showed that the reduction of fractal dimension can be used as the precursory information of rock failure [17, 28]. The evolvement of rock damage had furcation and chaos characters [29]. Microseismic energy was an important parameter describing microseismic events [30]. The fractal characteristics were studied with regard to the time series of acoustic emission energy of coal during the uniaxial compression failure [31, 32]. The fractal behaviour of microseismic energy was investigated in the deep tunnels during the development of immediate rockbursts [33].

From the above studies, it can be seen that there are few researches on the fractal behaviour of time series of microseismic energy associated with mining-induced tremors during the mining process of island workface in the field. Thus, this study will center on specific location, moment magnitude, energy and fractal structure of energy under different mining tremors.

2. Data

2.1. Microseismic real-time monitoring and results

The microseismic monitoring system was arranged at N1202 island workface with a head entry length of 1077.5 m, a tail entry length of 1033 m, an open-off cut length of 294.6 m, and the embedding depth of 480 m~543 m. The average thickness of 3# coal seam is 6.1 m. The microseismic monitoring system is the Engineering Seismology Group (ESG) from Canada. The monitoring system consists of one primary server, three data collection substations and 13 microseismic sensors. The primary server was installed on the ground, three substations were installed in the pump house of mining district, the 7 of 13 microseismic sensors were installed in the coal seam of head entry, and the 6 of 13 were installed in the coal seam of tail entry by drilling method. The monitoring period was from 1 April 2015 to 23 September 2015, when 19637 microseismic events were recorded with magnitudes between -1.6 and 1.0 Mw and energy from 10^2 J to 10^8 J. The monitoring programme is shown in Fig. 1.

2.2. Microseismic events locations and magnitudes results on the day of mining tremors

The 3 microseismic events leading to mining tremors are shown in Fig. 2. The energy of microseismic events are studied for fractal in the zone ± 150 m from the centreline of workface.

3. Fractal dimension calculation methods

Because of different emphasis point, the calculation process of fractal dimension is also different. The box, capacity and correlation dimension are widely used. The correlation dimension can correctly reflect the information of system according to time series [34], so it is used to compute the fractal dimension of energy.

The reconstruction of phase space is the key to the correlation dimension computation. As long as the appropriate delay time and embedding dimension are determined, we can reconstruct a dynamic system with the same topological properties as the original system. Therefore, in this paper, firstly, the delay time is obtained according to the mutual information function, the correlation dimension f_2 is calculated using the Grassberger–Procaccia algorithm (GPA), and then the optimal embedding dimension m is determined based on the saturated dimension. Finally, the fractal dimension is the correlation dimension under the saturated dimension.

3.1. Determination of delay time ξ

In reference to the selection of delay time and embedding dimension, mutual information method is an effective method to estimate the delay time, which not only can be used to study the linear and nonlinear correlation of time series, but also applied to the high dimensional chaotic systems. The recursive algorithm of mutual information was given by Fraser [35].

The process is as follows:

For the time series $\{t_i\}$, $P_t(t_i)$ is defined as the probability that the variable t_i appears, the average information of variable t_i is the information entropy of system, the formula is as follows [35]:

$$Y(T) = - \sum_{i=1}^n P_t(t_i) \ln P_t(t_i) \quad (1)$$

For the two groups of signal $\{t_i, g_j\}$, if the $P_{t,g}(t_i, g_j)$ is defined as the joint probability distribution of variable t_i, g_j , then the joint entropy is:

$$Y(T, G) = - \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} P_{t,g}(t_i, g_j) \ln P_{t,g}(t_i, g_j) \quad (2)$$

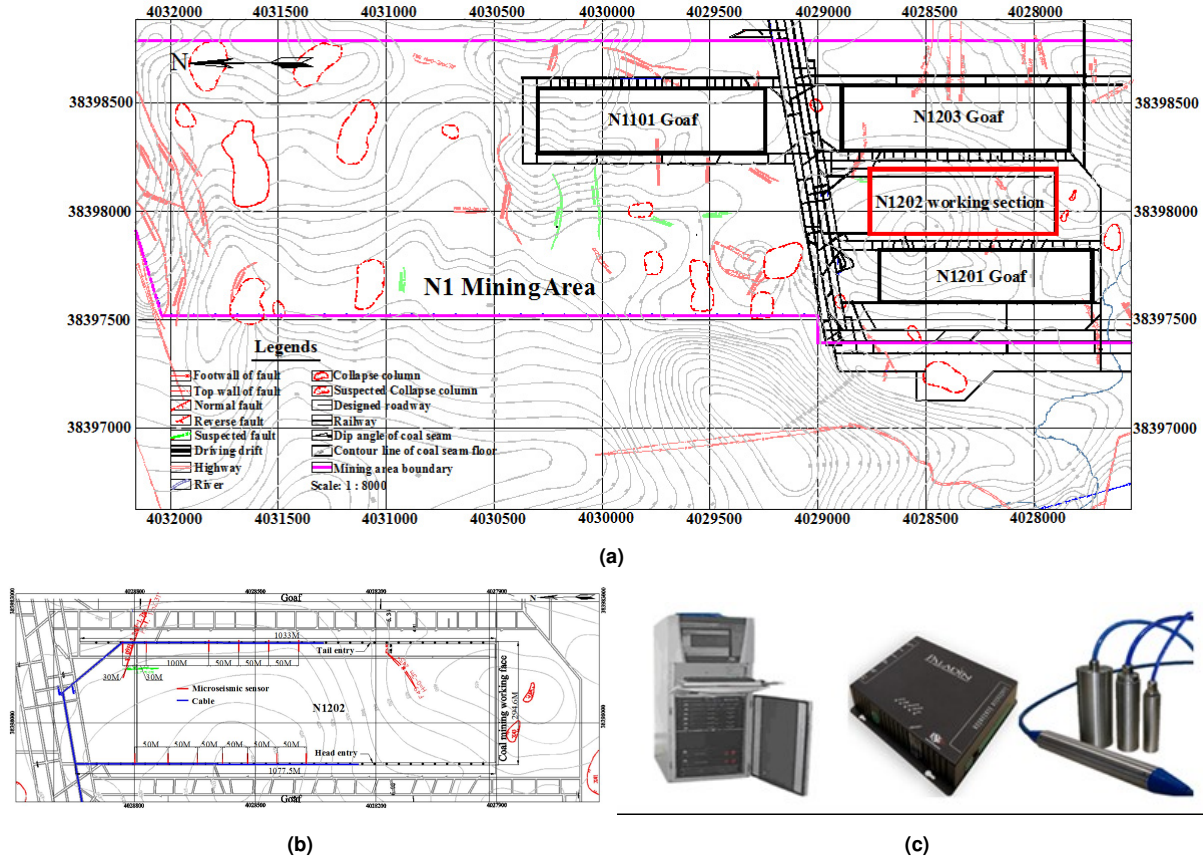


Fig. 1. (a) Underground N1 mining area in Yuwu coal mine, in Shanxi Province of China. (b) Monitored island workface of coal (N1202) and layout of microseismic monitoring system. (c) Microseismic monitoring system: primary server, data collection substation and microseismic sensors.

Where, $[t, g] = [x(s), x(s + \zeta)]$. For the coupled systems $[t, g]$, T is assumed to be t_i , the uncertainty of g is as follows:

$$Y(G|t_i) = -\sum_i P_{s|t} (g_j|t_i) \ln \left[\frac{P_{s|t} (g_j|t_i)}{P_i(t_i)} \right] = -\sum_i \left[\frac{P_{ig}(g_j|t_i)}{P_i(t_i)} \right] \times \ln \left[\frac{P_{ig}(g_j|t_i)}{P_i(t_i)} \right] \quad (3)$$

Where, $P_{g|t} (g_j|t_i)$ is the conditional probability distribution.

Assuming at s time, x is known, the average uncertainty of x at $s + \delta$ time is as follows:

$$Y(G|T) = -\sum_i P_t(t_i) Y(G|T) = -\sum_i P_{t=g} (t_i, g_j) \ln \left[\frac{P_{ig}(t_i, g_j)}{P_t(t_i)} \right] = Y(T, G) - Y(t) \quad (4)$$

As a result, the mutual information for T, G is as follows:

$$I(G, T) = Y(G) - Y(G|T) = Y(G) + Y(t) - Y(T, G) = I(T, G) \quad (5)$$

Therefore, the different delay time ζ can be computed. The calculation is as follows:

$$I(\zeta) = Y(\psi) + Y(\psi_\zeta) - Y(\psi, \psi_\zeta) \quad (6)$$

The results are summarized in Table 1. From the table, we can see that when $I\zeta$ first reaches a minimum value, the delay time ζ of time series of microseismic energy is 2 on 19 April, the ζ is 1 on 17 May, and the ζ is 3 for the rest.

3.2. Correlation dimension calculation

The Grassberger–Procaccia algorithm (GPA) introduced by Grassberger and Procaccia [36] is a classical algorithm of the correlation dimension. It can compute the correlation dimension of time series of chaotic [37].

If according to the original time series, the dynamic characteristics of high dimensional system can not better be reflected, much of behavior information of system will be lost. Only when the original time series is extended to the three-dimensional or higher dimensional phase space, the hidden information can be revealed [34]. On the basis of time interval ζ , the number is taken as the vector component from the original time series, the vector of reconstructed phase space is in accordance with the equation (7) and (8).

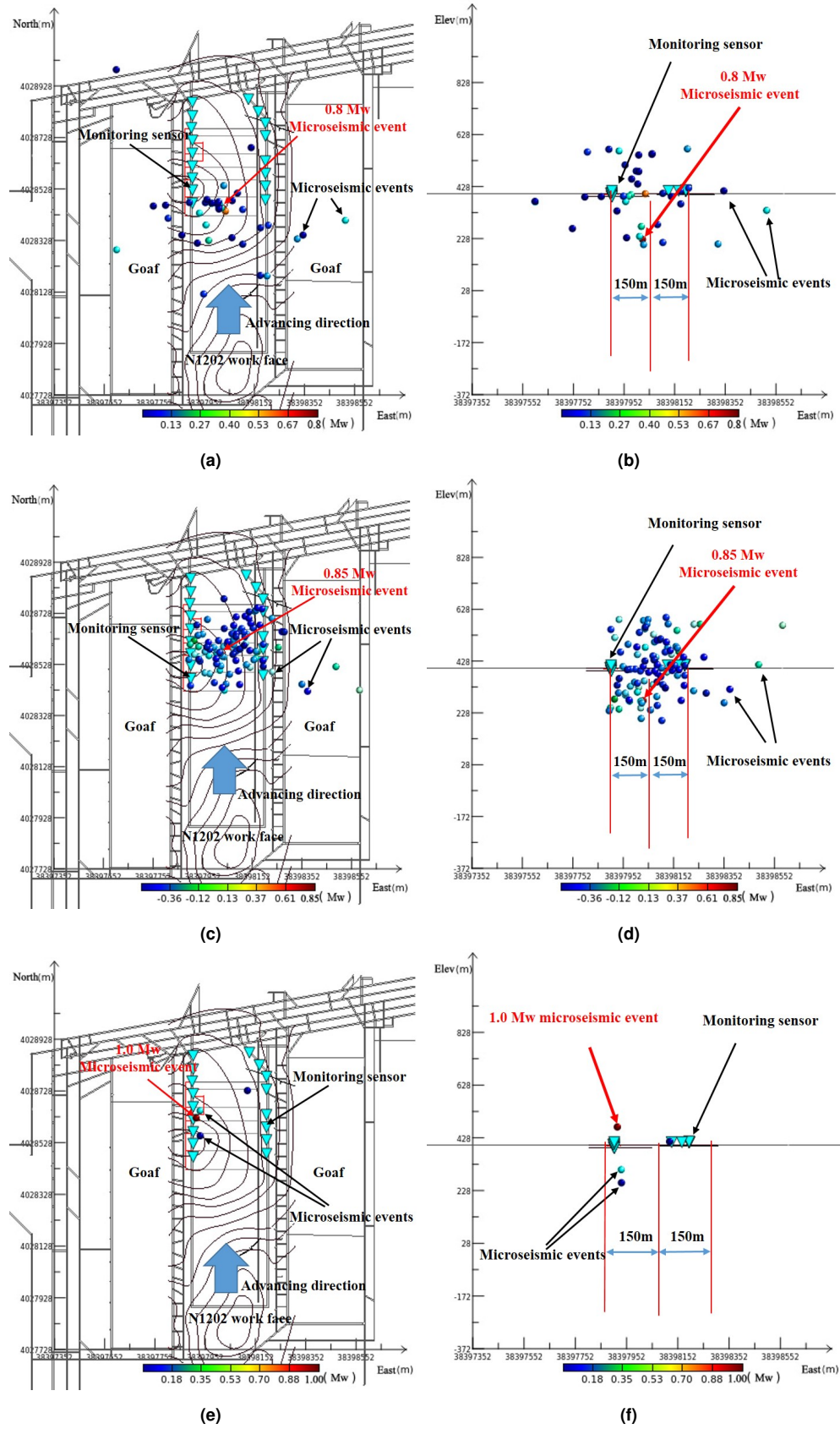
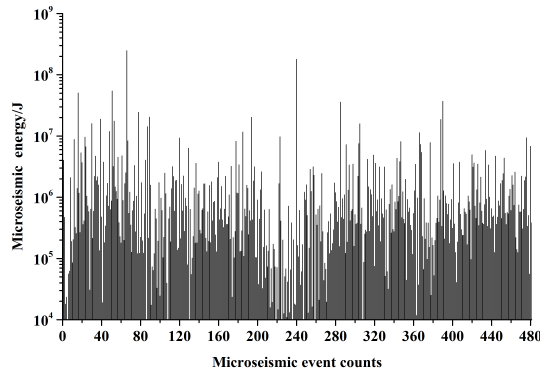
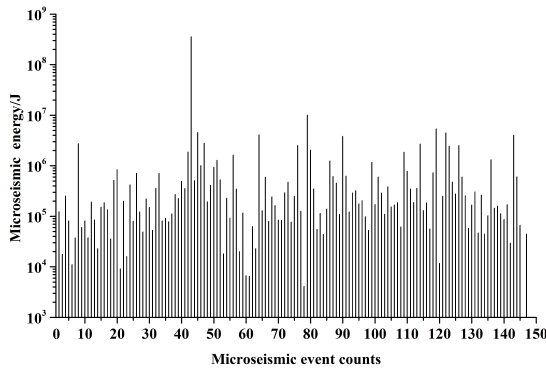
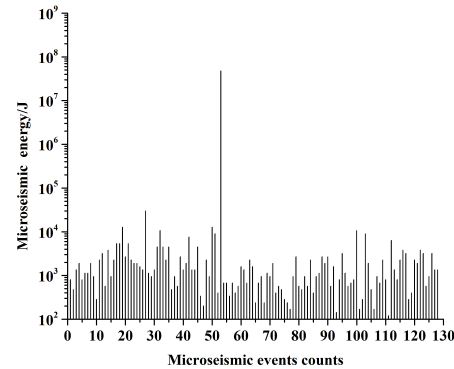


Fig. 2. (a-b) 0.8 Mw mining tremors occurring on 19 April, 2015. (c-d) 0.85 Mw mining tremors occurring on 17 May, 2015. (e-f) 1.0 Mw mining tremors occurring on 3 June, 2015.

Table 1. Calculation results of mutual information under different delay time parameters.

Mining tremors occurrence date	$\xi = 1$	$\xi = 2$	$\xi = 3$	$\xi = 4$	$\xi = 5$	$\xi = 6$
2015/04/19	0.00200	0.00074	0.02697	0.00094	0.00410	0.00323
2015/05/17	0.00031	0.00033	0.00035	0.00038	0.00035	0.00033
2015/06/03	0.02648	0.02093	0.00487	0.13724	0.00633	0.05162

**(a)** 19 April**(b)** 17 May**(c)** 3 June**Fig. 3.** Energy value of microseismicity during the process of mining tremors.

$$\psi_i = [\psi_i, \psi_{i+\xi}, \psi_{i+2\xi}, \dots, \psi_{i+(-1)\xi}] , i = 1, 2, \dots, N \quad (7)$$

$$\psi_i = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{i+(-1)\xi} \end{bmatrix} = \left\{ \begin{array}{l} \psi_1, \psi_{1+\xi}, \psi_{1+2\xi}, \dots, \psi_{1+(-1)\xi} \\ \psi_2, \psi_{2+\xi}, \psi_{2+2\xi}, \dots, \psi_{2+(-1)\xi} \\ \vdots \\ \psi_N, \psi_{N+\xi}, \psi_{N+2\xi}, \dots, \psi_{N+(-1)\xi} \end{array} \right\} , \quad (8)$$

$i = 1, 2, \dots, N$

Where ψ_i is the vector of reconstructed phase space, m

is the embedding dimension, which is the dimension of reconstructed phase space. ξ is the delay time, n is the number of original time series, N is the vector number of reconstructed phase space, $N = n - (m - 1)\xi$.

After the phase space is reconstructed, the correlation dimension can be computed based on the equation (11), (12) and (13). An arbitrary point of the m dimensional phase space is randomly selected as a reference point, the distance of another $N-1$ points to it is calculated. The number of points is counted that fall in the volume element with the center points $\{\psi_i\}_{i=1,2,\dots,N}$. Based on the number of volume element points with as small scalar radius, the

correlation function $\delta_n(v)$ is computed:

$$\delta_m(v) = \frac{\sum_{i \neq j} [Y(v - \|\psi_i - \psi_j\|)]}{N(N-1)} \quad (9)$$

where $Y(\cdot)$ is Heaviside function, which is got by:

$$Y(\psi) = \begin{cases} 0, \psi \leq 0 \\ 1, \psi > 0 \end{cases} \quad (10)$$

The correlation function $\delta_m(v)$ is the distribution probability of distance within v between two points of attractor in the reconstructed phase space.

$$\text{Therefore, } \delta_m(v) = (v/d_{\max})^{f(w,v)}, v \leq d_{\max} \quad (11)$$

Where $f(m, v)$ is a constant related to v and m .

For a certain range of v , the $\delta_m(v)$ and m can form a logarithmic linear relationship.

$$\text{Therefore, } f_2(m, v) = \frac{d \ln \delta_w(v)}{d \ln v} \quad (12)$$

Where, $f_2(m, v)$ is the slope of $\ln \delta_w(v) - \ln v$. When v is close to 0, the correlation dimension f_2 is obtained, which is given by:

$$f_2 = \lim_{v \rightarrow 0} f_2(m, v) \quad (13)$$

4. Results

For the energy of microseismic events (Seen in Fig. 3), the phase space is reconstructed under different embedded dimensions, the curves are drawn, and the correlation dimensions are computed. Finally, the fractal dimension (saturated correlation dimension) is determined. Fig. 4 shows the fractal dimension calculation. From Fig. 4 (a, c, and e), it can be seen that the $\ln \delta_m(v) - \ln v$ variation are shown under different saturated dimensions (m). For mining tremors with different moment magnitudes, the fractal evolution is distinct. But the trend of $\ln \delta_m(v) - \ln v$ curves are similar. When the $\ln v$ is a middle value, the intermediate scale is the systematic feature section, which is a non-scaling interval. The slope of non-scaling interval is the fractal dimension. From Fig. 4 (b, d, and f), it shows that the slope value (f_2) of linear part increases with the increase of embedding dimension (m). When the m is greater than a value, the correlation dimension will not continue to increase. Therefore, the slope value of linear part of double logarithm curve is the fractal dimension under the saturated correlation dimension. The saturated correlation dimension (m) is 11 on 19 April, is 6 on 17 May and is 5 on 3 June. After the saturated correlation dimensions are determined, the slope of non-scaling interval is got. The fractal dimension of microseismic energy is also obtained

before the occurrence of mining tremors (seen in Fig. 5 (a to c)). The fractal dimension of microseismic energy is 0.2725 on 19 April, is 0.1826 on 17 May and is 0.1367 on 3 June.

As shown in Fig. 5 (a to c), there exists the linearity between $\ln \delta_m(v)$ and $\ln v$ for the microseismic energy (i.e. the determination coefficients of R-square exceed 0.98 and the most RMSE (root-mean-square error) values are less than 1×10^{-2}). This means that all of microseismic energy under mining tremors exhibit fractal behaviour.

5. Discussions

The seismic system is a typical dissipative system [10], which exhibits chaotic behaviour [38]. The fractal characteristics research had been carried out on seismicity caused by mining [27, 39–41]. However, there was few data on the fractal dimension of microseismic energy during the coal mining process in the field. Based on fractal and chaos method, Kong et al. [42] used the mutual information method and the GPA to obtain the energy fractal dimension of acoustic emission, but the delay time determined was a direct reference to the previous experimental research in the laboratory. In this study, the delay time of time series of energy are computed during the development of mining tremors with different intensity in the field, which is distinguished from the previous experimental research on acoustic emission in the laboratory.

Feng et al. studied the fractal behaviour of microseismic energy associated with immediate rockbursts, who used the correlation integral function [43] to get the fractal dimension [33]. In this study, the GPA is introduced to obtain the fractal dimensions of energy, which is to reveal the time distribution of microseismic energy. The fractal calculation method is distinguished from the previous method. The results show the fractal dimension is in a declining trend before the occurrence of mining tremors, which is the lowest on the day of mining tremors (seen in Fig. 5 (d)).

6. Conclusions

An energy, moment magnitude, source location and energy fractal characteristics study of microseismic events leading to mining tremors is conducted in a coal mine. The main conclusion of this paper:

- (1) The moment magnitude of 3 microseismic events leading to mining tremors is from 0.8 to 1.0Mw and the energy release of them is more than 10^8 J. The hypocenter locations of 0.8-1.0 Mw microseismic events are determined.
- (2) The delay time ξ of time series of microseismic energy on the day of mining tremors is obtained based on the mutual information method. The fractal dimensions of

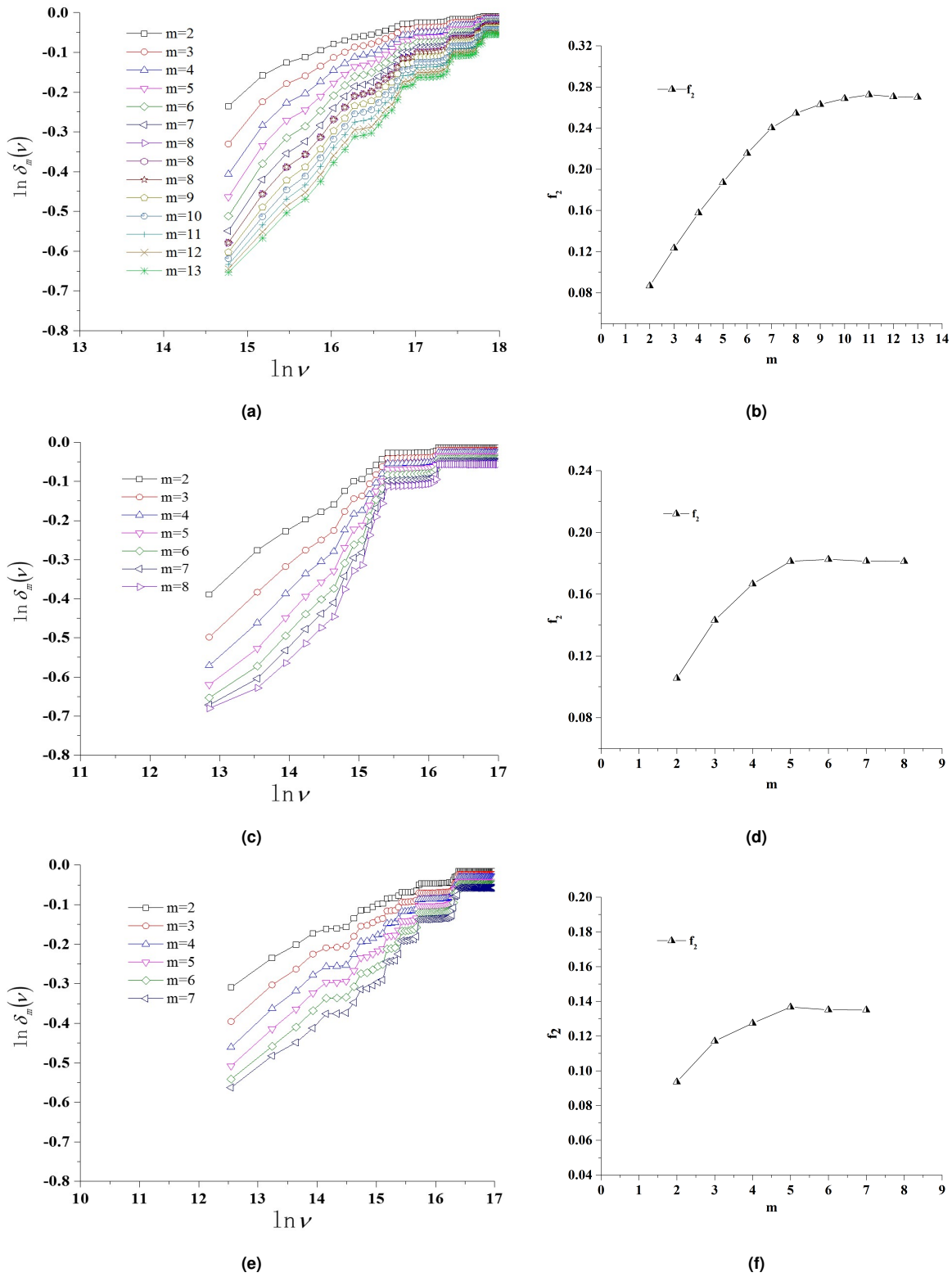


Fig. 4. Fractal dimension calculation of microseismic energy occurring on (a-b) 19 April, (c-d) 17 May, (e-f) 3 June. (a, c, and e) Double logarithmic curve of $\delta_m(v) - v$ under different embedding dimensions (m). (b, d, and f) Relation curve of correlation dimension (f_2) and embedding dimension (m).

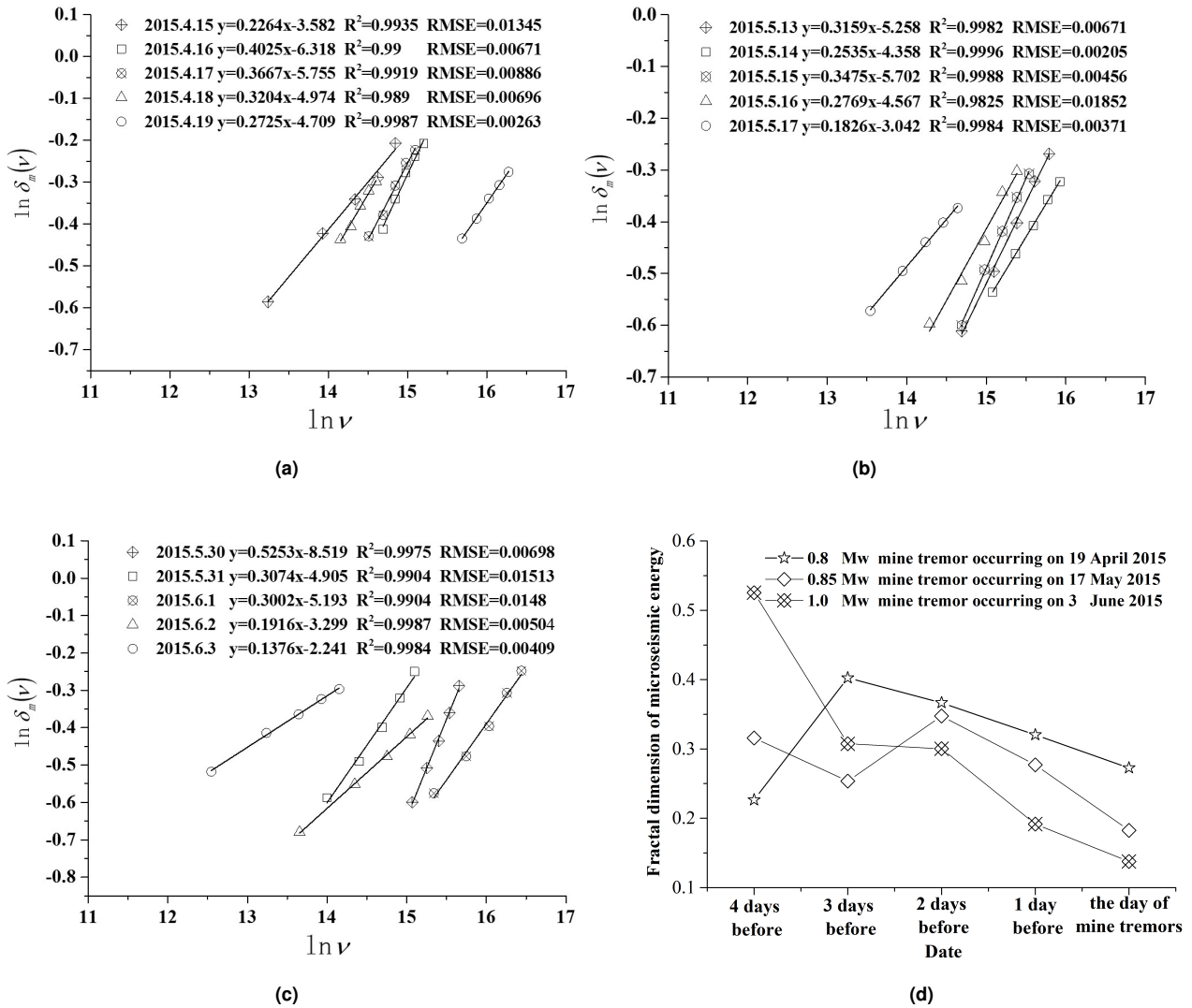


Fig. 5. (a to c) Fractal dimension fitting results of microseismic energy before the occurrence and on the day of mining tremors with different intensity. (d) Fractal dimension trend graph under mining tremors with different energy.

mining tremors with different energy are obtained based on the GPA.

(3) The time distributions of microseismic energy are fractal under mining tremors with different intensity. The fractal dimension is in a declining trend before the occurrence of mining tremors, which is the lowest on the day of mining tremors.

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