Free Vibration Analysis of Layered Magneto-Electro-Elastic Truncated Conical Shells

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A finite element method is used to analyse the free vibrations of MEE truncated conical shell. Three Boundary Conditions- Clamped-Free (C-F), Simply-Supported (S-S), Clamped-Clamped (C-C) are considered to study the behaviour of magneto-electro-elastic truncated conical shell with constant thickness. The study is made to identify the effect of magnetic and electric coupling on the frequencies of conical shell. Frequency characteristics on conical shells are discussed by varying the semi vertex angles 20^o, 35^o and 50^o. The magneto-electro-elastic truncated conical shells behaviour indicates that as frequency increases with increase in circumferential mode.

Keywords: Magneto-Electro-Elastic (MEE); conical shell; free vibration; axisymmetric finite element; Smart Composites

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1. Introduction

Conical shell plays a major role in various fields likely marine, aerospace, structural etc. Axisymmetric conical shells play a significant role in making missiles, sensors, modern weapons, and types of nozzles. In order to optimize the weight, mechanical strength or displacement, shells with advanced composite materials can be preferred. Magneto-Electro-Elastic (MEE) conical shells have engineering applications such as sensors, actuators and especially in vibration control. [1].

The different parameters of constant thickness truncated conical shell with required boundary conditions studied by Irie et al. [2]. Thambiratnam et al. [3] studied the free vibration of shells by keeping constant thickness using complex optimization technique. Thambiratnam et al. [4] understand the effect of frequency on axisymmetric free vibration conical shell using finite element method. Simply supported condition is used to study the free vibration of rotating truncated conical shell using a method Love's first approximation theory by Lam et al. [5]. Liew et al. [6] analysed the free vibration of conical shell using mesh-free kp-Ritz method. Annigeri et al. [7] have studied the MEE cylindrical shell using semi analytical method. Tornabene et al. [8] analysed the functionally graded shells and plates by first-order shear deformation theory. Firouz-Abadi et al. [9] investigated the vibration behaviour of moderately thick conical shell using higher order shear deformable theory of Jin et al. [10] studied the frequency characteristics of truncated conical shell by developing modified Fourier series method with general boundary conditions. Nejati et al. [11] investigate the parametric study on different volume fraction along thickness of functionally graded truncated conical shell based on generalized differential quadrature. Chandra Mouli et al. [12] have conducted a vibration response study via power law distribution to evaluate the properties of shell in thickness direction. Chuang Wu et al. [13] analysed conical shell for different semi vertex angle, shell thickness using a new method precise integration transfer matrix method. SairaJaved [14] analysed composite laminated conical shell by considering the various parameters shells length ratio, cone angle based on spline approximation method and concluded that frequency varies by varying the materials and geometric properties. Song et al. [15] have studied truncated conical shell with different boundary conditions using power series method. M Vinyas et al.[16] analysed free vibration behaviour of magneto-electro-elastic plates using higher order shear deformation theory. M Vinyas et al.[17] studied on smart magneto-electro-elastic plates using Reddy's third-order shear deformation theory. M Vinyas.[1] understand and review the behaviour of magneto-electro-elastic materials and structures. Research on conical shell vibration under various parameters effect like boundary conditions, cone angle, thickness, radius to height ratio, length to radius ratio has attracted attention in engineering [18, 19].

From the literature view few Researches have focused their study of free vibration on conical shell using analytical, numerical, simulation and standard analysis software's. However it is found that a gap exists for combination of properties such as MEE materials. Hence in Present study frequency behaviour of magneto-electro-elastic truncated conical shell is analysed by semi analytical finite element approach. The smart composite $BaTiO_3$ and $CoFe_2O_4$ were used as piezoelectric and piezomagnetic materials, respectively.

2. Finite Element Modeling of Magneto Electro-Elastic Conical Shell

The constitutive equations for magneto-electro-elastic medium in cylindrical coordinate (r, θ, z) system relating stress σ_j, electric displacement D_j, Magnetic induction B_j, strain S_k, electric field E_k, and magnetic field H_k exhibiting linear coupling between magnetic, electric and elastic fields can be written [7].

$$\sigma_j = C_{jk}S_k - e_{kj}E_k - q_{kj}H_k$$
$$D_j = e_{jk}S_k + \epsilon_{jk}E_k + m_{jk}H_k$$
$$B_j = q_{jk}S_k + m_{jk}E_k + \mu_{jk}H_k$$

where, C_{jk}, ϵ_{jk} and μ_{jk} are elastic, dielectric and magnetic permeability coefficients respectively and e_{kj}, q_{jk} and m_{jk} are the piezoelectric, piezomagnetic and magneto-electric material coefficients respectively. Where S, E and H are independent variables which represent strain, electric field and magnetic field respectively. The material properties and geometry remains same in 'θ' direction. For free vibration studies [7],

$$[K_{eq}]\{u\} + [M]\{\ddot{u}\} = 0$$

where

$$[K_{eq}] = [K_{uu}] + [K_{u\phi}][K_{II}]^{-1}[K_I] + [K_{u\psi}]([K_{IV}]^{-1})[K_{III}]$$
$$K_{eq\_reduced}] = [K_{uu}] + [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{u\phi}]^T + [K_{u\psi}][K_{\psi\psi}]^{-1}[K_{u\psi}]$$
$$[K_{eq\_\phi\phi}] = [K_{uu}] + [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{u\phi}]^T$$
$$[K_{eq\_\psi\psi}] = [K_{uu}] + [K_{u\psi}][K_{\psi\psi}]^{-1}[K_{u\psi}]^T$$

$[K_{eq}]$ = Stiffness matrix for fully coupled magneto-electro-elastic material

$[K_{eq-reduced}]$ = Stiffness matrix neglecting magnetoelectric coupling

$[K_{eq-\phi\phi}]$ = Stiffness matrix considering piezoelectric effect

$[K_{eq-\psi\psi}]$ = Stiffness matrix considering piezomagnetic effect

2.1. Stacking Sequence for Layered Conical Shell

The analysis is conducted for the two alternate stacking sequences (B/F and F/B) of smart materials $BaTiO_3$ (B) and $CoFe_2O_4$ (F). Here B/F represents a two layered conical shell with the inner shell material is B and outer shell material is F. The density for both piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ is 5730 kg m^{-3}[7].

The geometry of the axisymmetric MEE conical shell is shown in the Fig.1, where h represent the thickness of the cone, a and b represents radius of large and small base of the cone respectively, H is the cone height and θ is the semi-vertex angle of the cone. r and z are the axis of cylindrical coordinate system. R_i represents the distance of internal surface from the central axis.

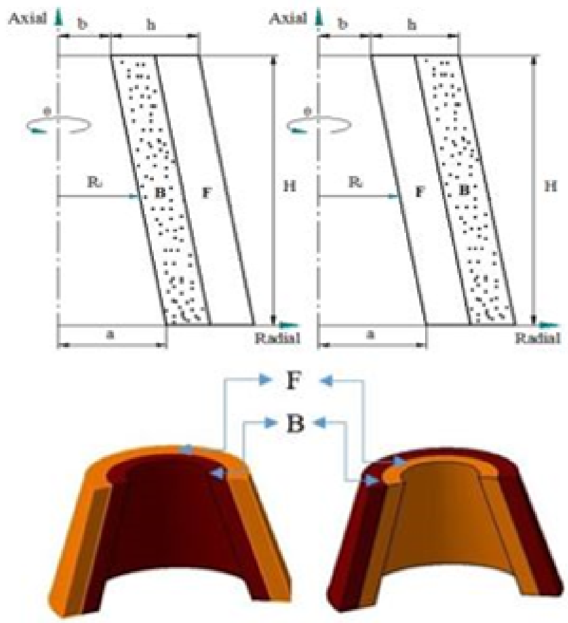

Fig. 1. Two layered MEE conical shell for different stacking of B/F and F/B

Free vibration studies of layered magneto-electro-elastic conical shell is conducted for three boundary conditions, viz., Clamped-Free (C-F) u = v = w = ϕ = ψ = 0, at z = 0, Simply-Supported (S-S) v = w = ϕ = ψ, at z = 0 z = H, Clamped-Clamped (C-C) u = v = w = ϕ = ψ = 0, at z = 0, z = H, using three noded triangular elements with three degree of freedom (dof) per node

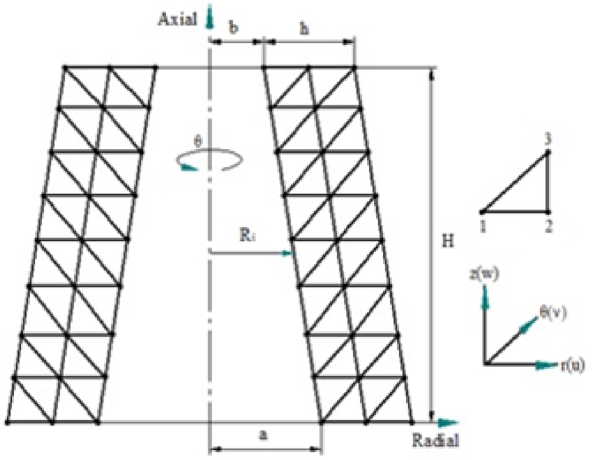

Fig. 2. Finite element discretization using triangular elements.

3. Modeling and Material Properties

Free vibration studies of conical shell by varying semi vertex cone angle (θ), cone height (H) with constant thickness (h), base radius (a) and top radius (b).

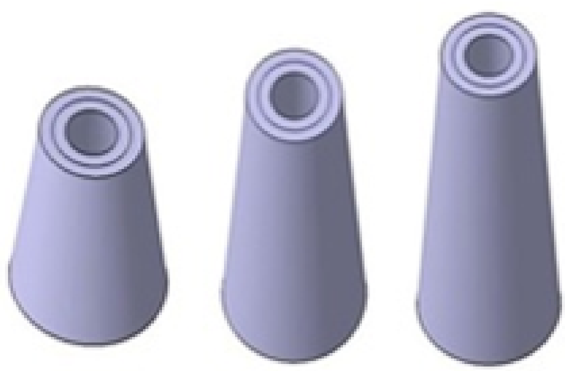

Fig. 3. Isometric views of MEE Conical shells for θ = 20^o, 35^o and 50^o

The dimensions of layered MEE conical shell used in the study are base radius (a = 1m), top radius (b = 0.5m), constant thickness (h = 0.1m) by varying semi vertex angle (θ) and cone height (H). The different shell heights obtained by changing the semi-vertex angles θ = 20^o, 35^o and 50^o for both stacking sequence. The elements in radial and axial directions are selected for better aspect ratio.

Table 1. Elements for semi vertex angle (θ)

Sl. No.	Semi Vertex angle	Elements in Radial direction	Elements in Axial direction	Total Number of Elements
1	20^o	4	58	464
2	35^o	4	34	272
3	50^o	4	26	208

The notations used are

ω_{uu} = Frequency in rad/sec by using [K_{uu}]

ω_{eq} = Frequency in rad/sec by using [K_{eq}]

$\omega_{eq-reduced}$ = Frequency in rad/sec by using [$K_{eq-reduced}$]

$\omega_{\psi\psi}$ = Frequency in rad/sec by using [$K_{\psi\psi}$]

$\omega_{\phi\phi}$ = Frequency in rad/sec by using [$K_{\phi\phi}$]

Table 2. Material properties of smart composite [7]

Here C_{ij} in 109 N/m^2, e_{ij} in C/m^2, εij in 10^{-9} C/Vm, q_{ij} in N/A m, μ_{ij} in 10^{-4} N s^2/C^2 and m_{ij} in 10^{-12} N s/VC

	F	B
C_{11}	286	166
C_{12}	173	77
C_{13}	170	78
C_{33}	269.5	162
C_{44}	45.3	43
e_{31}	0	-4.4
e_{33}	0	18.6
e_{15}	0	11.6
ε_{11}	0.08	11.2
ε_{33}	0.093	12.6
μ_{11}	-5.9	0.05
μ_{33}	1.57	0.1
q_{31}	580	0
q_{33}	700	0
q_{15}	560	0
m_{11}	0	0
m_{33}	0	0

4. Result discussions

4.1. Validation

The developed computer code for MEE conical shell finite element analysis is validated with the studies by Thambiratnam et al. [4] and the results of the validation show good agreement.

4.2. B/F stacking for different semi-vertex angles θ = 20^o, 35^o and 50^o having three boundary conditions.

4.2.1. Studies on layered MEE conical shell for C-F condition.

The study conducted on C-F boundary condition. In table 3 to 5, MEE conical shell considering the bottom is free and top is clamped. Table 6 to 8 values obtained for MEE conical shell considering the bottom is clamped and top is

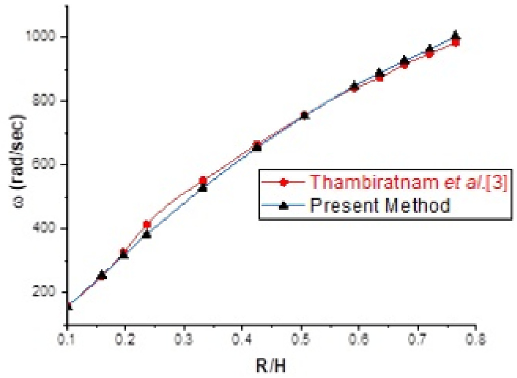

Fig. 4. Validation of the Frequency for different R/H ratio (θ=40^o).

Table 3. Frequency in rad/sec, θ = 20^o

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1224	1234	1234	1223	1235
2	831.8	838.9	838.9	830.5	840.1
3	1290	1294	1294	1289	1294
4	2222	2228	2228	2221	2228
5	3410	3420	3420	3410	3420
6	4808	4821	4821	4806	4822
7	6389	6406	6406	6387	6408
8	8133	8154	8154	8130	8157
9	10020	10050	10050	10020	10050
10	12040	12070	12070	12030	12070

Table 4. Frequency in rad/sec, θ = 35^o

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1982	1988	1988	1979	1991
2	1394	1397	1397	1391	1401
3	1497	1500	1500	1494	1502
4	2179	2186	2186	2178	2188
5	3208	3220	3220	3207	3221
6	4462	4480	4480	4462	4481
7	5895	5918	5918	5894	5919
8	7479	7508	7508	7478	7509
9	9195	9230	9230	9193	9232
10	11030	11070	11070	11030	11070

free. The frequency values in table 6 to 8 are predominantly high compared with the frequency values in table 3 to 5.

4.2.2. Studies on layered MEE conical shell for S-S condition.

4.2.3. Studies on layered MEE conical shell for C-C condition.

In table 9 to 11 the study is carried for magneto-electro-elastic conical shell considering Simply-Supported (S-S) boundary condition. Table 12 to 14 values obtained for magneto-electro-elastic conical shell considering Clamped-Clamped (C-C) boundary condition. The frequency values in table 9 to 11 are predominantly less compared with the values in table 12 to 14.

Table 5. Frequency in rad/sec, θ = 50^o

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	2144	2146	2146	2137	2153
2	1622	1618	1618	1613	1626
3	1602	1600	1600	1595	1607
4	2022	2026	2026	2018	2030
5	2773	2784	2784	2770	2787
6	3754	3773	3773	3752	3775
7	4909	4935	4935	4907	4937
8	6239	6239	6239	6203	6241
9	7621	7663	7663	7619	7665
10	9143	9191	9191	9140	9194

Table 6. Frequency in rad/sec, θ = 20^o

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	2637	2656	2656	2635	2658
2	1758	1772	1772	1755	1775
3	2636	2641	2641	2632	2645
4	4017	4021	4021	4009	4028
5	5581	5586	5586	5572	5595
6	7344	7350	7350	7333	7361
7	9282	9289	9289	9269	9302
8	11370	11380	11380	11360	11390
9	13590	13600	13600	13570	13610
10	15920	15930	15930	15900	15950

Table 7. Frequency in rad/sec, θ = 35^o

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	4078	4093	4093	4076	4096
2	2856	2862	2862	2851	2867
3	3169	3176	3176	3163	3182
4	4530	4544	4544	4522	4552
5	6238	6255	6255	6225	6267
6	8101	8121	8121	8085	8137
7	10090	10120	10120	10070	10130
8	12200	12230	12230	12180	12250
9	14420	14450	14450	14400	14480
10	16740	16780	16780	16710	16810

Table 8. Frequency in rad/sec, θ = 50^o

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	4109	4123	4123	4104	4128
2	3229	3228	3228	3221	3237
3	3340	3343	3343	3330	3353
4	4386	4400	4400	4373	4413
5	5898	5920	5920	5879	5939
6	7641	7669	7669	7615	7696
7	9533	9567	9567	9499	9601
8	11540	11580	11580	11500	11620
9	13630	13690	13690	13590	13730
10	15810	15880	15880	15770	15930

Table 9. Frequency in rad/sec, θ = 20°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1503	1515	1515	1502	1517
2	1918	1919	1919	1913	1923
3	2490	2487	2487	2484	2493
4	3657	3653	3653	3650	3660
5	5166	5161	5161	5158	5169
6	6894	6888	6888	6885	6897
7	8801	8794	8794	8790	8805
8	10860	10850	10850	10850	10870
9	13060	13050	13050	13040	13060
10	15370	15360	15360	15350	15380

Table 10. Frequency in rad/sec, θ = 35°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1216	1219	1219	1215	1221
2	2610	2620	2620	2603	2627
3	3294	3291	3291	3283	3302
4	4313	4305	4305	4301	4317
5	5700	5691	5691	5687	5704
6	7359	7351	7351	7345	7365
7	9212	9206	9206	9198	9221
8	11210	11210	11210	11200	11220
9	13340	13330	13330	13320	13350
10	15570	15560	15560	15550	15580

Table 11. Frequency in rad/sec, θ = 50°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	839.3	841.4	841.4	838.5	842.3
2	2400	2402	2402	2393	2409
3	3372	3363	3363	3356	3380
4	4295	4279	4279	4273	4301
5	5442	5427	5427	5419	5451
6	6825	6814	6814	6801	6838
7	8402	8396	8396	8378	8421
8	10130	10130	10130	10110	10160
9	11990	11990	11990	11960	12020
10	13940	13950	13950	13920	13980

Table 12. Frequency in rad/sec, θ = 20°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	3959	3977	3977	3956	3981
2	3058	3069	3069	3053	3074
3	3193	3198	3198	3186	3206
4	4156	4159	4159	4147	4168
5	5611	5616	5616	5602	5625
6	7350	7356	7356	7339	7367
7	9283	9290	9290	9270	9303
8	11370	11380	11380	11360	11390
9	13590	13600	13600	13570	13610
10	15920	15930	15930	15900	15950

4.3. F/B stacking for different semi-vertex angles θ = 20°, 35° and 50° having three boundary conditions.

4.3.1. Studies on layered MEE conical shell for C-F condition.

The study conducted on C-F boundary condition. In table 15 to 17, MEE conical shell considering the bottom is free and top is clamped. Table 18 to 20 values obtained for MEE conical shell considering the bottom is clamped and top is free. The frequency values in table 18 to 20 are predomi-

Table 13. Frequency in rad/sec, θ = 35°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	5714	5797	5797	5700	5811
2	5114	5171	5171	5098	5186
3	5148	5187	5187	5131	5205
4	5783	5810	5810	5765	5829
5	6919	6941	6941	6900	6960
6	8441	8463	8463	8422	8483
7	10250	10270	10270	10230	10300
8	12270	12300	12300	12250	12320
9	14450	14480	14480	14420	14510
10	16750	16790	16790	16720	16820

Table 14. Frequency in rad/sec, θ = 50°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	6811	6865	6865	6780	6896
2	6613	6651	6651	6581	6685
3	6706	6733	6733	6671	6768
4	7154	7176	7176	7119	7212
5	7951	7974	7974	7916	8009
6	9073	9100	9100	9038	9136
7	10480	10520	10520	10450	10550
8	12140	12180	12180	12100	12220
9	14000	14060	14060	13960	14100
10	16030	16100	16100	15990	16140

Table 15. Frequency in rad/sec, θ = 20°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1206	1226	1226	1205	1226
2	814.2	837.1	837.1	813.4	837.8
3	1299	1311	1311	1298	1312
4	2251	2261	2261	2250	2262
5	3461	3472	3472	3460	3473
6	4882	4895	4895	4881	4896
7	6491	6506	6506	6489	6507
8	8264	8282	8282	8262	8284
9	10180	10200	10200	10180	10210
10	12230	12250	12250	12230	12260

Table 16. Frequency in rad/sec, θ = 35°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1959	1981	1981	1958	1982
2	1362	1390	1390	1361	1391
3	1483	1508	1508	1482	1508
4	2194	2212	2212	2193	2213
5	3247	3263	3263	3246	3264
6	4525	4544	4544	4525	4545
7	5983	6005	6005	5982	6006
8	7593	7619	7619	7592	7620
9	9336	9367	9367	9335	9368
10	11200	11230	11230	11190	11230

Table 17. Frequency in rad/sec, θ = 50°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	2126	2143	2143	2125	2144
2	1597	1613	1613	1596	1615
3	1586	1603	1603	1584	1605
4	2024	2041	2041	2023	2042
5	2794	2811	2811	2793	2812
6	3793	3812	3812	3792	3814
7	4966	4988	4988	4965	4989
8	6279	6306	6306	6278	6308
9	7713	7745	7745	7711	7746
10	9250	9287	9287	9248	9289

Table 18. Frequency in rad/sec, θ = 20°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	2623	2650	2650	2622	2651
2	1733	1765	1765	1731	1767
3	2631	2650	2650	2630	2651
4	4016	4043	4043	4013	4045
5	5606	5635	5635	5601	5639
6	7397	7428	7428	7392	7434
7	9364	9396	9396	9358	9402
8	11480	11510	11510	11470	11520
9	13730	13760	13760	13720	13770
10	16090	16120	16120	16080	16130

Table 19. Frequency in rad/sec, θ = 35°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	4064	4082	4082	4064	4082
2	2809	2841	2841	2808	2842
3	3146	3172	3172	3144	3174
4	4537	4563	4563	4534	4565
5	6267	6302	6302	6264	6306
6	8160	8201	8201	8154	8207
7	10180	10230	10230	10170	10230
8	12320	12370	12370	12310	12380
9	14560	14620	14620	14560	14630
10	16910	16970	16970	16900	16980

Table 20. Frequency in rad/sec, θ = 50°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	4126	4140	4140	4121	4145
2	3198	3218	3218	3196	3220
3	3309	3331	3331	3306	3333
4	4376	4398	4398	4372	4402
5	5904	5932	5932	5898	5938
6	7666	7700	7700	7657	7708
7	9580	9621	9621	9569	9632
8	11610	11660	11660	11600	11670
9	13730	13790	13790	13720	13800
10	15940	16000	16000	15920	16020

nantly high compared with the frequency values in table 15 to 17.

Table 21. Frequency in rad/sec, θ = 20°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1518	1542	1542	1516	1544
2	1886	1918	1918	1884	1919
3	2461	2487	2487	2460	2489
4	3661	3677	3677	3658	3680
5	5196	5209	5209	5192	5213
6	6950	6961	6961	6944	6966
7	8883	8893	8893	8876	8899
8	10970	10980	10980	10960	10990
9	13190	13200	13200	13180	13210
10	15530	15530	15530	15520	15540

Table 22. Frequency in rad/sec, θ = 35°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	1233	1246	1246	1232	1248
2	2603	2633	2633	2599	2636
3	3275	3305	3305	3271	3309
4	4313	4338	4338	4309	4343
5	5730	5749	5749	5724	5755
6	7417	7433	7433	7410	7440
7	9294	9311	9311	9286	9319
8	11320	11340	11340	11310	11350
9	13460	13480	13480	13450	13490
10	15710	15730	15730	15700	15740

Table 23. Frequency in rad/sec, θ = 50°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	851.2	856.2	856.2	850.2	857.2
2	2413	2427	2427	2408	2432
3	3372	3390	3390	3363	3400
4	4302	4319	4319	4291	4330
5	5467	5484	5484	5455	5496
6	6872	6888	6888	6858	6902
7	8471	8490	8490	8456	8506
8	10220	10250	10250	10210	10260
9	12100	12120	12120	12080	12140
10	14070	14100	14100	14050	14120

4.3.2. Studies on layered MEE conical shell for S-S condition.

4.3.3. Studies on layered MEE conical shell for C-C condition.

Table 24. Frequency in rad/sec, θ = 20°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	3923	3940	3940	3921	3941
2	2974	3013	3013	2972	3014
3	3119	3164	3164	3117	3166
4	4130	4168	4168	4127	4171
5	5629	5662	5662	5624	5666
6	7401	7433	7433	7396	7439
7	9365	9397	9397	9358	9403
8	11480	11510	11510	11470	11520
9	13730	13760	13760	13720	13770
10	16090	16120	16120	16080	16130

In table 21 to 23 the study is carried for MEE conical shell

Table 25. Frequency in rad/sec, θ = 35°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	5659	5721	5721	5653	5728
2	5029	5105	5105	5023	5111
3	5064	5148	5148	5057	5154
4	5730	5811	5811	5723	5818
5	6910	6982	6982	6903	6990
6	8477	8542	8542	8469	8550
7	10330	10390	10390	10320	10390
8	12380	12440	12440	12370	12450
9	14590	14650	14650	14580	14660
10	16920	16980	16980	16910	16990

Table 26. Frequency in rad/sec, θ = 50°

Frequency	ω_{uu}	ω_{eq}	$\omega_{eq-redu}$	$\omega_{\psi\psi}$	$\omega_{\phi\phi}$
1	6750	6821	6821	6727	6844
2	6547	6621	6621	6527	6642
3	6644	6721	6721	6624	6741
4	7107	7185	7185	7088	7204
5	7928	8004	8004	7910	8023
6	9077	9152	9152	9060	9170
7	10520	10590	10590	10500	10610
8	12200	12270	12270	12180	12290
9	14090	14160	14160	14070	14180
10	16140	16220	16220	16130	16230

considering S-S boundary condition. Table 24 to 26 values obtained for MEE conical shell considering C-C boundary condition. The frequency values in table 21 to 23 are predominantly less compared with the values in table 24 to 26.The values tabulated in the table 3 to 26 there is negligible changes between ω_{eq} and $\omega_{eq-redu}$.

5. Conclusions

- The study is performed for two layered truncated MEE axisymmetric truncated conical shell. Two stacking with B/F and F/B considered.

- The effect of piezoelectric phase is to increase while piezomagnetic effect is to decrease the structural frequency of the system for both stacking sequence.

- In the table 3 to 8 and table 15 to 20 for both B/F and F/B stacking, it is clearly says that when the truncated conical shell bottom is clamped and top is free, the piezoelectric and piezomagnetic effect is high compared with respect to the condition when conical shell bottom is free and top is clamped.

- For both B/F and F/B stacking, at Semi vertex angle θ = 20 °, it is clearly says that when the truncated conical shell bottom is clamped and top is free, the average piezoelectric and piezomagnetic effect is high around 42 percent compared with respect to the condition when conical shell bottom is free and top is clamped.

- The frequency behaviour analysed for B/F and F/B stacking smart composite truncated conical shell considering circumferential modes such that lowest frequency occurs at 2nd circumferential mode.

- In B/F and F/B stacking, it is clearly says that when the truncated conical shell ; Bottom is free and top is clamped condition and Bottom is clamped and top is free condition there is no much appreciable changes in piezoelectric and piezomagnetic effect.

- In B/F and F/B stacking, for Clamped-Clamped boundary condition the effect of piezoelectric and piezomagnetic is predominantly high when comparing with Simply-Supported boundary condition.

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