

Control of a Flexible Manipulator System with Finite - Time Disturbance Observer

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This paper mainly studies how to control the flexible manipulator effectively, which can drive the flexible manipulator to reach the angular and restrain the vibration of the manipulator simultaneously. Firstly, two disturbance observers based on partial differential equations (PDEs) model are improved to ensure its convergence in finite time. Then, on the basis of the disturbance observer, two active boundary controllers are proposed to achieve the control objective. Finally, the stability and effectiveness of the designed controller are verified by theoretical analysis and simulation.

Keywords: Flexible manipulator; vibration control; finite-time disturbance observer; distributed parameter system (DPS)

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1. Introduction

Nowadays, increasing research interest has been given to flexible manipulators. Compared to traditional rigid robots, flexible manipulators have the advantages of lightweight, high speed, less cost, and safe to operate. Therefore, flexible manipulators have wide range of applications, such as deep space exploration, industrial/factory automation, manufacturing, emergency rescue system, etc [1–3]. However, these benefits come at the cost of flexibility in links or joints. The flexible nature of the robotics results in vibrations on the end effector, which is quite difficult to control. The excess vibrations would lead to the degradation of the system performance, the damage of flexible manipulators, and even serious hazards. Therefore, it is essential to suppress the excessive vibrations of flexible manipulators in practical engineering. This will allow the researchers to design an efficient and effective control method for flexible manipulators.

Compared with the rigid robotic systems that are modeled by ordinary differential equations (ODEs) [4–6], a number of challenges are imposed in research for flexible manipulators that are modeled by PDEs [7–9], includ-

ing modeling dynamics analysis, PDE control design, system optimization, and sensing. Conventional control approaches for flexible manipulator are based on the ODE dynamic models that have been proposed via assumed mode method (AMM) [10, 11], finite difference method (FDM) [12], lumped and finite element method (FEM) [13, 14]. So far, various control methods have been proposed in the control of the rigid robotic systems, such as the neural networks control, sliding mode control, fuzzy logic control and many other control methods. However, these methods can not be directly used in the control of FMS based on PDE model. Therefore, reliable control methods should be attained for vibration control of flexible manipulators.

The model based on PDE can accurately describe the infinite dimensional dynamic characteristics of the flexible manipulator system (FMS) and effectively avoid spillover effects. In [15], based on PDEs-ODEs, the dynamic model of flexible two-link manipulator are built and the control objectives (tracking target angle and suppress vibration) are realized through boundary control method. In [16], iterative learning control is developed for flexible manipulator described by PDE model with input constraint. However,

the influence of disturbance was not considered in literature [15, 16]. The flexible manipulator is easy to be interfered by the outside in the actual work, which makes the system produce mechanical vibration. In order to evaluate the influence of unknown interference on the FMS, scholars have made a lot of research on how to compensate for the influence of external interference. In [17], authors studied the vibration suppression of the flexible beam with nonlinear gap input. On the basis of dynamic PDE model of the flexible beam, boundary controllers with disturbance observer are developed to restrain the vibration of the system and compensate the influence of nonlinear gap. In [18], the active boundary control problem of a kind of axially moving flexible systems is studied. In order to reduce the influence of unknown disturbance on the system, a high order state observer is used to estimate the unmeasurable state variables.

Although the corresponding disturbance observer is proposed to deal with the disturbance in [17, 18], some issues have not been solved yet, and effective controls of are still to be developed. Finite-time convergence is a newly conception as opposed to asymptotic convergence of conventional control. However, the control problem of finite-time convergence are studied usually in ODE systems. There is little research for PDE control problem of flexible manipulators with finite-time convergence. In this paper, we will design two PDE control laws for a FMS with two finite-time disturbance observers. The FMS is modeled as a DPS with infinite dimension. With the proposed controllers, the vibration can be suppressed and the disturbance observer can be converged in finite time. No matter how the external disturbance changes, the disturbance observation error can converge in a finite time and has strong robustness.

2. Dynamic modeling and observer design

2.1. Dynamic modeling

Fig. 1 shows a simple schematic of the FMS in this study. The dynamic analysis of the system is carried out in the XOY global inertial coordinate system. Set $y(x, t) = w(x, t) + x\theta(x, t)$, where $y(x, t)$ denotes the absolute displacement of the FMS, $\theta(x, t)$ represents the elastic vibration of FMS, and $\theta(t)$ is the angular displacement of FMS.

In this study, the FMS is simplified to Euler-Bernoulli beam structure. Then establish a follow-up coordinate system with the center point of the center hub as the origin. Based on this coordinate system, the following expressions of kinetic energy, potential energy, and total virtual work

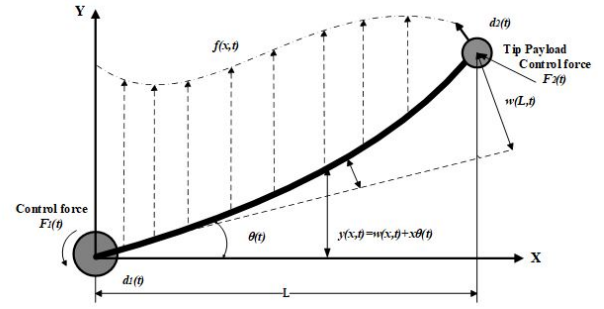


Fig. 1. Structural drawing of FMS.

are listed.

$$E_k(t) = \frac{1}{2} I_h [\dot{\theta}(t)]^2 + \frac{1}{2} \rho \int_0^L [\dot{y}(x, t)]^2 dx + \frac{1}{2} M [\dot{y}(L, t)]^2 \quad (1)$$

$$E_p(t) = \frac{1}{2} EI \int_0^L [\omega''(x, t)]^2 dx + \frac{1}{2} T \int_0^L [\omega'(x, t)]^2 dx \quad (2)$$

$$\delta W(t) = \delta W_d(t) + \delta W_f(t) \quad (3)$$

$$\delta W_d(t) = d_1(t) \delta \theta(t) + d_2(t) \delta y(L, t) + \int_0^L f(x, t) \delta y(x, t) dx \quad (4)$$

$$\delta W_f(t) = F_1(t) \delta \theta(t) + F_2(t) \delta y(L, t) \quad (5)$$

The above equations (1)-(5) are substituted into the Hamilton principle, and the main control equation of FMS can be obtained by variational operation.

$$EI \omega''''(x, t) + \rho \ddot{y}(x, t) + T \omega''(x, t) = f(x, t) \quad (6)$$

where ρ is the mass of the unit length of the FMS, $f(x, t)$ denotes unknown time-varying disturbance, EI stands for the bending stiffness, T is the tension. And the boundary conditions as follows:

$$\omega(0, t) = \omega'(0, t) = \omega''(L, t) = 0 \quad (7)$$

$$M \ddot{y}(L, t) - EI \omega'''(L, t) + T \omega'(L, t) = F_1(t) + d_1(t) \quad (8)$$

$$I_h \ddot{\theta}(t) - EI \omega''(0, t) - T \omega(L, t) = F_2(t) + d_2(t) \quad (9)$$

Where $d_1(t)$, $d_2(t)$ are time variant boundary disturbances added to tip payload and joint respectively, I_h denotes the inertia of the center hub, L represents the length of the manipulator, M is the mass of the payload, $F_1(t)$ and $F_2(t)$ are control inputs for tip payload and joint respectively.

Remark 1: To simplify the formula, the following notations are used:

$$(\Delta)' = \frac{\partial(\Delta)}{\partial s}, (\Delta)'' = \frac{\partial^2(\Delta)}{\partial s^2}, (\Delta)''' = \frac{\partial^3(\Delta)}{\partial s^3}, (\Delta)'''' = \frac{\partial^4(\Delta)}{\partial s^4}, (\dot{\Delta}) = \frac{\partial(\Delta)}{\partial t}, (\ddot{\Delta}) = \frac{\partial^2(\Delta)}{\partial t^2}$$

Assumption 1: We assume that the unknown disturbances $f(x, t)$, $d_1(t)$ and $d_2(t)$ are all bounded, and satisfying $|f(x, t)| \leq \bar{f}$, $|d_1(t)| \leq \bar{d}_1$, $|d_2(t)| \leq \bar{d}_2$, where $\bar{f}$, $\bar{d}_1$, $\bar{d}_2$

are positive numbers. The energy contained in the unknown disturbance of the flexible manipulator's working environment is limited, so it is reasonable to make the above assumptions.

Lemma 1: [19, 20] For a continuous positive definite function $V(t)$, if the following inequality is true:

$$\dot{V}(t) \leq -k_1 V(t) - k_2 V^{k_3}(t) \quad (10)$$

Then the function $V(t)$ will converge to the equilibrium point within a finite time T_f , which is defined as:

$$T_f \leq \frac{1}{k_1(1-k_3)} \ln \frac{K_1 V^{1-k_3}(0) + k_2}{k_2} \quad (11)$$

where $k_1 > 0, K_2 > 0, 0 < k_3 < 1$, all are uncertain numbers.

2.2. Disturbance observer design

In this part, two disturbance observers are designed and the observers can converge in a finite time, which means that the influence of external disturbance can be eliminated in a finite time. First, the forms of the auxiliary functions $\xi_1(t)$ and $\xi_2(t)$ are given as:

$$\eta_1(t) = M\dot{y}(L, t) - \xi_1(t) \quad (12)$$

$$\begin{aligned} \dot{\xi}_1 = & EI\omega'''(L, t) - T\omega'(L, t) + F_1(t) \\ & + k_{11}\eta_1(t) + k_{12}\text{sign}[\eta_1(t)] + k_{13}\eta_1^{\frac{2\pi_{11}-\pi_{12}}{\pi_{12}}} \end{aligned} \quad (13)$$

$$\eta_2(t) = I_h\dot{\theta}(t) - \xi_2(t) \quad (14)$$

$$\begin{aligned} \dot{\xi}_2 = & EI\omega''(0, t) - T\omega'(L, t) + F_2(t) \\ & + k_{21}\eta_2(t) + k_{22}\text{sign}[\eta_2(t)] + k_{23}\eta_2^{\frac{2\pi_{21}-\pi_{22}}{\pi_{22}}} \end{aligned} \quad (15)$$

where $k_{11} - k_{13}$ and $k_{21} - k_{23}$ are unknown normal numbers, $\pi_{11} - \pi_{12}$ and $\pi_{21} - \pi_{22}$ are unknown positive odd numbers, and satisfies $\pi_{i1} < \pi_{i2} < 2\pi_{i1}, i = 1, 2$.

Then two disturbance observers $\hat{\phi}_1(t)$ and $\hat{\phi}_2(t)$ are designed to estimate boundary disturbances. be

$$\hat{\phi}_1 = k_{11}\eta_1(t) + k_{12}\text{sign}[\eta_1(t)] + k_{13}\eta_1^{\frac{2\pi_{11}-\pi_{12}}{\pi_{12}}}(t) \quad (16)$$

$$\hat{\phi}_2 = k_{21}\eta_2(t) + k_{22}\text{sign}[\eta_2(t)] + k_{23}\eta_2^{\frac{2\pi_{21}-\pi_{22}}{\pi_{22}}}(t) \quad (17)$$

According to Assumption 1 and Lemma 1, the above two disturbance observers satisfy the following properties:

Property 1: Let $k_{12-\bar{d}_1} \geq 0$ and $k_{22-\bar{d}_2} \geq 0$, the proposed disturbance observers $\hat{\phi}_1(t)$ and $\hat{\phi}_2(t)$ can converge in $t_1 \geq T_{f1}$ and $t_2 \geq T_{f2}$ respectively. T_{f1}, T_{f2} satisfy the following inequality:

$$T_{f1} \leq \frac{\pi_{12}}{2k_{11}(\pi_{12} - \pi_{11})} \ln \left\{ \frac{k_{11} V_a^{\frac{\pi_{12}-\pi_{11}}{\pi_{12}}}(0) + 2^{\frac{\pi_{11}}{\pi_{12}}} k_{13}}{2^{\frac{\pi_{11}}{\pi_{12}}} k_{13}} \right\} \quad (18)$$

$$T_{f2} \leq \frac{\pi_{22}}{2k_{21}(\pi_{22} - \pi_{21})} \ln \left\{ \frac{k_{21} V_b^{\frac{\pi_{22}-\pi_{21}}{\pi_{22}}}(0) + 2^{\frac{\pi_{21}}{\pi_{22}}} k_{23}}{2^{\frac{\pi_{21}}{\pi_{22}}} k_{23}} \right\} \quad (19)$$

where $V_a(0), V_b(0)$ is the initial value of $V_a(t), V_b(t)$, and $V_a(t), V_b(t)$ will be derived from the proof in Appendix.

Proof: See Appendix A.

Remark 2: From property 1, the designed disturbance observers $\hat{\omega}_1(t)$ and $\hat{\omega}_2(t)$ can estimate the boundary disturbances $d_1(t)$ and $d_2(t)$ when $t_1 \geq T_{fi} (i = 1, 2)$. In addition, according to literature [21, 22], the disturbance error satisfies $|\tilde{\omega}(t) = \omega(t) - \hat{\omega}(t)| \leq E$ when $\forall t \in [0, T_f]$. Combined with Property 1, the time range of this conclusion can be extended to $\forall t \in [0, \infty)$, $|\tilde{\omega}(t)| \leq E$, E denotes a positive number.

3. Controller design and stability analysis

3.1. Boundary controller design

The control objective is to develop controllers to achieve the purpose of suppressing the vibration of the flexible manipulator during operation and achieving precise angular tracking. In order to get the controller equation, the following Lyapunov function are constructed firstly,

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (20)$$

where the $V_1(t), V_2(t)$ and $V_3(t)$ are given as:

$$\begin{aligned} V_1(t) = & \frac{1}{2}\rho \int_0^L [y(x, t)]^2 dx + \frac{1}{2}EI \int_0^L [\omega''(x, t)]^2 dx \\ & + \frac{1}{2}T \int_0^L [\omega'(x, t)]^2 dx \end{aligned} \quad (21)$$

$$V_2(t) = \frac{1}{2}Mu_a^2(t) + \frac{1}{2}(k_2 - k_3)e^2(t) \quad (22)$$

$$V_3(t) = \alpha\rho \int_0^L x\omega'(x, t)y(x, t)dx \quad (23)$$

where α, β are normal numbers, $u_a(t) = y(L, t) + w'(L, t) - w''(L, t)$, θ_d denotes the angular value to be tracked, and $e(t) = \theta(t) - \theta_d$ represents the error of angular tracking.

In order to stabilize the FMS, boundary controller equations are designed as follows:

$$\begin{aligned} F_1(t) = & -EI\omega'''(L, t) + T\omega'(L, t) - M\omega'(L, t) \\ & + M\omega'''(L, t) - k_1 u_a(t) - \hat{\phi}_1(t) \end{aligned} \quad (24)$$

$$z(t) = -k_2 e(t) - k_3 e(t)\hat{\phi}_2(t) \quad (25)$$

where k_1, k_3 are the positive gain values of the controller.

3.2. Analysis of system stability

Lemma 2: The Lyapunov function represented by (20) has upper bound and lower bound as:

$$0 \leq \lambda_1[V_1(t) + V_2(t)] \leq V(t) \leq \lambda_2[V_1(t) + V_2(t)] \quad (26)$$

where λ_1 and λ_2 represent two positive numbers.

Lemma 3: The first time partial derivative of satisfies the inequality as follows:

$$V(t) \leq -\lambda V(t) + C \quad (27)$$

where λ and C represent two positive numbers.

Proof: See Appendix B.

By synthesizing Lemma 2, Lemma 3, Lyapunov direct method and two designed boundary controllers (24)-(25), we can get the following boundedness theorems about FMS.

Theorem 1: In the case of the boundary controllers and the disturbance observers are used in the control of FMS, we can make the following conclusions: 1) The elastic vibration $\omega(x, t)$ and angular error $e(t)$ of the FMS will converge to the set Ω_{11} and Ω_{12} :

$$\Omega_{11} := \left\{ \omega(x, t) \in R \mid |\omega(x, t)| \leq \sqrt{\frac{2L}{T\lambda_1} V(0)e^{-\lambda t} + \frac{C}{\lambda}}, \forall t \in [0, L] \right\} \quad (28)$$

$$\Omega_{12} := \left\{ e(t) \in R \mid |e(t)| \leq \sqrt{\frac{2}{(k_2 - k_3)\lambda_1} (V(0)e^{-\lambda t} + \frac{C}{\lambda})}, \forall t \in [0, \infty) \right\} \quad (29)$$

2) The output states will eventually converge to the small neighborhood of 0:

$$\Omega_{21} := \left\{ \omega(x, t) \in R \mid \lim_{t \rightarrow \infty} |\omega(x, t)| \leq \sqrt{\frac{2LC}{T\lambda_1\lambda}}, \forall x \in [0, L] \right\} \quad (30)$$

$$\Omega_{22} := \left\{ e(t) \in R \mid \lim_{t \rightarrow \infty} |e(t)| \leq \sqrt{\frac{2C}{(k_1 - k_2)\lambda_1\lambda}}, \forall x \in [0, L] \right\} \quad (31)$$

Proof: See Appendix C.

4. Numerical simulations

This part will use the FDM to simulate the output state of the system to verify the effect of the controllers.

The value of parameters $L, M, T, EI, I_h, \rho, \theta_d$ are chosen as 0.8m, 2kg, 3N, 0.8Nm², 0.2kgm², 1kg/m, $\pi/6$ rad respectively. In addition to the above parameters, the initial conditions $\omega(x, 0), \omega(x, 0), \theta(0), \theta(0)$ of the system are all set to zero, and the flexible manipulator vibrates under the action of external disturbance. The external disturbance $f(x, t)$ and $d_1(t), d_2(t)$ are defined as

$$\begin{cases} f(x, t) = [15\sin(10\pi xt) + 3\cos(30\pi xt)]x/10 \\ d_1(t) = 1.5\sin(20\pi t) + 0.5\cos(2\pi t) \\ d_2(t) = \sin(15\pi t) + 1.5\cos(\pi t) \end{cases} \quad (32)$$

This simulation will respectively complete the following two case: 1) The FMS is only affected by external disturbance, setting $F_1(t) = F_2(t) = 0$. 2) Boundary controllers

with disturbance observers designed in the previous section are applied to compare the states of the system without applying the controller. The Simulation output state di-

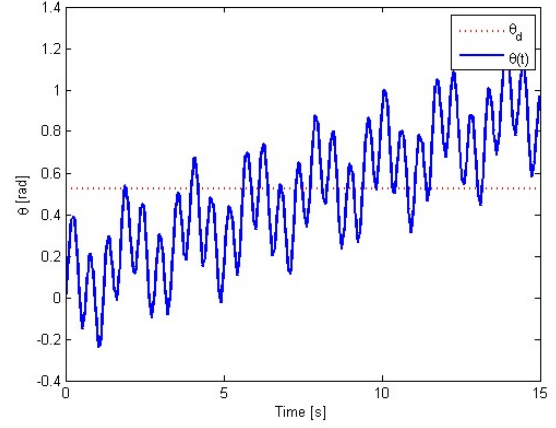


Fig. 2. $\theta(t)$ without control.

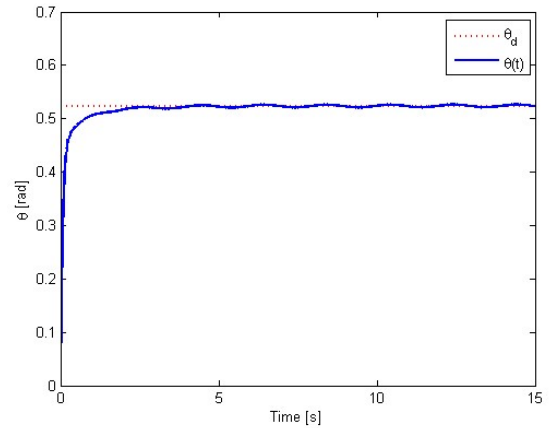


Fig. 3. $\theta(t)$ with proposed control.

agram of case 1 can be seen in Figs. 2, 4, 6. From Figs. 2, 4, 5, the results show that $\omega(x, t)$ and $y(x, t)$ of FMS exhibit periodic vibration under the action of external disturbance, while $\theta(t)$ increases in vibration, which exceeds the expected tracking angular at the end of the manipulator. The movement of the whole FMS is very disordered, which can neither reach the desired angular nor be stable. This situation will affect the normal operation of the FMS.

Next, add the boundary controller (24), (25), first select the controller gain as $k_1 = 120, k_2 = 15, k_3 = 110$, set the disturbance observer parameters as $k_{11} = 65, k_{12} = 125, k_{13} = 90, k_{21} = 70, k_{22} = 110, k_{23} = 80, \pi_{11} = 5, \pi_{12} = 7, \pi_{21} = 5, \pi_{22} = 7$, and then run the simulation program. Figures 3, 5, 7 show the overall vibration, overall motion

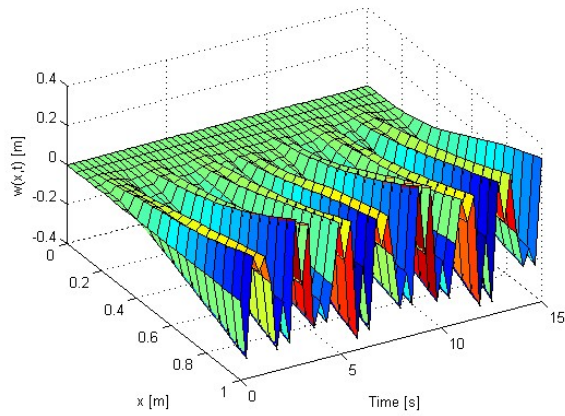


Fig. 4. $w(x,t)$ without control

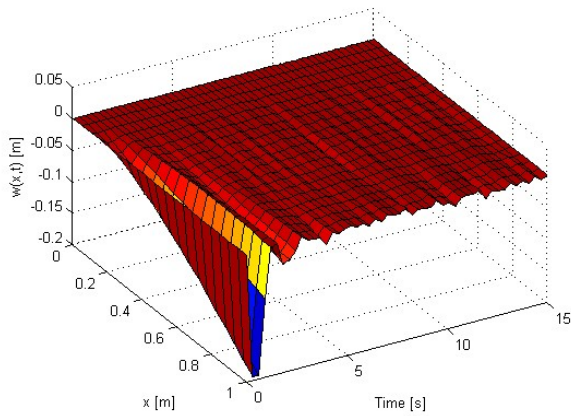


Fig. 5. $w(x,t)$ with proposed control

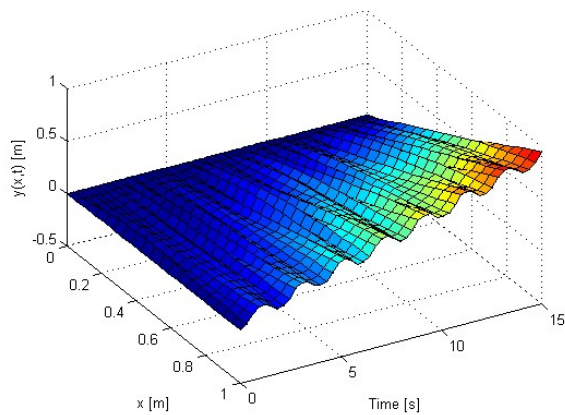


Fig. 6. $y(x,t)$ without control

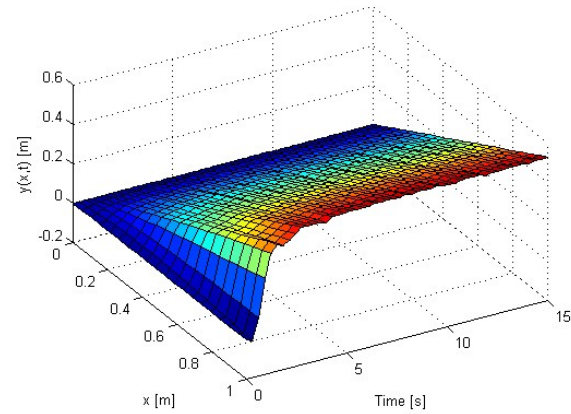


Fig. 7. $y(x,t)$ with proposed control

and angular tracking of the flexible manipulator under the boundary controllers. It can be seen that the chaotic motion of the system has been greatly improved. In Fig. 7, the elastic vibration $w(x,t)$ can be quickly suppressed by controllers. As can be seen in Fig. 7, the displacement $y(x,t)$ converges quickly. Fig. 3 shows that the FMS reaches the target angle θ_d in about 5 seconds, and the system is in a relatively stable state.

In general, the designed boundary controllers can effectively achieve the control purpose of the FMS, and can drive the manipulator to reach the desired angular while suppressing its own elastic vibration.

5. Conclusion

In this paper, the boundary controllers have been proposed to reduce the vibration and track the angular displacement. In addition, two disturbance observers which can converge in finite time are designed to compensate the influence of boundary disturbance. Moreover, the controlled system has been proved to be convergent and stable. Finally, the effectiveness of the constructed boundary control laws with finite-time disturbance observers have been verified by the numerical simulations.

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Appendix A: Proof of property 1

Proof: Two positive definite functions are given in the following forms:

$$V_a(t) = \frac{1}{2}\eta_1^2(t), V_b(t) = \frac{1}{2}\eta_1^2(t) \quad (33)$$

Then we can get $V_a(t)$ and $V_b(t)$:

$$\begin{aligned} V_a &= \eta_1(t)\eta_1(t) \\ &= \eta_1(t)[My(L, t) - \xi_1(t)] \\ &= \eta_1(t)[EI\omega'''(L, t) - T\omega'(L, t) + F_1(t) + d_1(t) - \xi_1(t)] \\ &= \eta_1(t)d_1(t) - k_{11}\eta_1^2(t) - k_{12}|\eta_1(t)| - k_{13}\eta_1(t)\eta_1^{\frac{2\pi_{11}-\pi_{12}}{\pi_{12}}}(t) \\ &\leq -k_{11}\eta_1^2(t) - (k_{12} - D_1)|\eta_1(t)| - k_{13}\eta_1(t)\eta_1^{\frac{2\pi_{11}-\pi_{12}}{\pi_{12}}}(t) \\ &\leq -k_{11}\eta_1^2(t) - k_{13}\eta_1(t)\eta_1^{\frac{2\pi_{11}-\pi_{12}}{\pi_{12}}}(t) \\ &\leq -2k_{11}V_a(t) - 2^{\frac{\pi_{11}}{\pi_{12}}}k_{13}V_a^{\frac{\pi_{11}}{\pi_{12}}}(t) \end{aligned} \quad (34)$$

$$\begin{aligned} V_b &= \eta_2(t)\eta_2(t) \\ &= \eta_2(t)[My(L, t) - \xi_2(t)] \\ &= \eta_2(t)[EI\omega'''(L, t) - T\omega'(L, t) + F_2(t) + d_2(t) - \xi_2(t)] \\ &= \eta_2(t)d_2(t) - k_{21}\eta_2^2(t) - k_{22}|\eta_2(t)| - k_{23}\eta_2(t)\eta_2^{\frac{2\pi_{21}-\pi_{22}}{\pi_{22}}}(t) \\ &\leq -k_{21}\eta_2^2(t) - (k_{22} - D_2)|\eta_2(t)| - k_{23}\eta_2(t)\eta_2^{\frac{2\pi_{21}-\pi_{22}}{\pi_{22}}}(t) \\ &\leq -k_{21}\eta_2^2(t) - k_{23}\eta_2(t)\eta_2^{\frac{2\pi_{21}-\pi_{22}}{\pi_{22}}}(t) \\ &\leq -2k_{21}V_b(t) - 2^{\frac{\pi_{21}}{\pi_{22}}}k_{23}V_b^{\frac{\pi_{21}}{\pi_{22}}}(t) \end{aligned} \quad (35)$$

Therefore, according to Lemma 1, $V_a(t)$ and $V_b(t)$ will converge to equilibrium points in T_{f1} and T_{f2} respectively. Let $k_{12} - \bar{d}_1 \geq 0$ and $k_{22} - \bar{d}_2 \geq 0$, we can get the following equation:

$$\begin{aligned} V_a(t) &= 0, \forall t \geq T_{f1} \\ V_b(t) &= 0, \forall t \geq T_{f2} \end{aligned} \quad (36)$$

Then we can obtain $\eta_1(t) = \eta_2(t) = 0$ when $t \geq T_{f1}$ and $t \geq T_{f2}$. The disturbance observation error is $\varphi_1(t) = \varphi_1(t) - \hat{\varphi}_1(t)$, by substituting the disturbance observer (16) into the disturbance error, we can get

$$\begin{aligned} \omega_1(t) &= \varphi_1(t) - \hat{\varphi}_1(t) \\ &= My(L, t) - EI\omega'''(L, t) + T\omega'(L, t) - F_1(t) - \hat{\varphi}_1(t) \\ &= My(L, t) - EI\omega'''(L, t) + T\omega'(L, t) - F_1(t) - k_{12}\text{sign}[\eta_1(t)] \\ &\quad - k_{13}\eta_1^{\frac{2\pi_{11}-\pi_{12}}{\pi_{12}}}(t) \\ &= My(L, t) - \xi_1(t) = \eta_1(t) \end{aligned} \quad (37)$$

According to the above derivation, we can get the following conclusion: under the condition of satisfying Assumption 1 and $k_{12} - \bar{d}_1 \geq 0$, the disturbance observation error can converge to zero when $t \geq T_{f1}$. In order to solve the inequality (34), the following differential equations need to be obtained:

$$V_a = -2k_{11}V_a(t) - 2^{\frac{\pi_{11}}{\pi_{12}}}k_{13}V_a^{\frac{\pi_{11}}{\pi_{12}}}(t) \quad (38)$$

By solving this differential equation, we can get the following inequalities about T_{f1} .

$$T_{f1} \neq \frac{\pi_{12}}{2k_{11}(\pi_{12} - \pi_{11})} \ln \left\{ \frac{k_{11}V_a^{\frac{\pi_{12}-\pi_{11}}{\pi_{12}}}(0) + 2^{\frac{\pi_{11}}{\pi_{12}}}k_{13}}{2^{\frac{\pi_{11}}{\pi_{12}}}k_{13}} \right\} \quad (39)$$

Similarly, T_{f2} can be obtained as follows:

$$T_{f2} \neq \frac{\pi_{22}}{2k_{11}(\pi_{22} - \pi_{21})} \ln \left\{ \frac{k_{21}V_a^{\frac{\pi_{22}-\pi_{21}}{\pi_{22}}}(0) + 2^{\frac{\pi_{21}}{\pi_{22}}}k_{23}}{2^{\frac{\pi_{21}}{\pi_{22}}}k_{23}} \right\} \quad (40)$$

Appendix B: Proof of lemma 2

Proof: We can obtain the following inequality from $V_3(t)$,

$$V_3(t) \leq \frac{\alpha\rho L}{2} \int_0^L [y(x, t)]^2 + [\omega'(x, t)]^2$$

$dx \leq \alpha_1 V_1(t)$ where $\alpha_1 = \frac{\alpha\rho L}{\min(\rho, T)}$, and α satisfy $0 < \alpha < \frac{\min(\rho, T)}{\rho L}$, then we can get $0 < \alpha_1 < 1$. Set α_2, α_3 satisfy $0 < \alpha_2 = 1 - \alpha_1 < 1, \alpha_3 = 1 + \alpha_1 > 1$ respectively, and then the following expressions can be obtained:

$$0 < \alpha_2 V_1(t) \leq V_1(t) + V_3(t) \leq \alpha_3 V_1(t) \quad (42)$$

Further, the following inequalities can be obtained:

$$0 \leq \lambda_1[V_1(t) + V_2(t)] \leq V(t) \leq \lambda_2[V_1(t) + V_2(t)] \quad (43)$$

where $\lambda_1 = \min(\alpha_2, 1), \lambda_2 = \min(\alpha_3, 1)$ are two positive numbers. Differentiating $V(t)$ in terms of time, bringing in (6) - (9) and combining Remark 2 and Assumption 1, we have:

$$\begin{aligned} V(t) &\leq -(\frac{1}{2}\alpha\rho + \alpha\rho L\delta_1) \int_0^L [y(x, t)]^2 dx \\ &\quad - (\frac{1}{2}\alpha T - \alpha L\delta_3) \int_0^L [\omega'(x, t)]^2 dx \\ &\quad - \frac{3}{2}\alpha EI \int_0^L [\omega''(x, t)]^2 dx - (k_1 - \frac{1}{2}EI - \delta_4)u_a^2(t) - [k_3\delta_5 \\ &\quad + k_3 - \delta_6 + \frac{\alpha\rho L}{\delta_1}]e^2(t) \\ &\quad - \frac{k_3}{\delta_5}e^2(t) - (\frac{1}{2}EI - (T - EI)\delta_1 - \frac{1}{2}\alpha\rho L)[y(L, t)]^2 \\ &\quad - (\frac{1}{2}EI - \frac{|T-EI|}{\delta_1} - EI(1 - \alpha L)\delta_8 - \frac{1}{2}\alpha TL)[\omega'(L, t)]^2 \\ &\quad - (\frac{1}{2}EI - \frac{EI(1-\alpha L)}{\delta_8}[\omega''(L, t)]^2 + \frac{1}{\delta_4}E_1^2 \\ &\quad + \frac{1}{\delta_6}E_2^2 + (\frac{1}{\delta_2} + \frac{\alpha L}{\delta_3}) \int_0^L \bar{f}^2 dx \\ &\leq -\lambda_3[V_1(t) + V_2(t)] + C \end{aligned} \quad (44)$$

Among them, the positive number must meet the following conditions when selecting:

$$\begin{aligned} \sigma_1 &= \frac{1}{2}\alpha\rho + \alpha\rho L\delta_1 - \delta_2 > 0 \\ \sigma_2 &= \frac{1}{2}\alpha T + \alpha\rho L\delta_3 > 0 \\ \sigma_3 &= k_1 - \frac{1}{2}EI - \delta_4 > 0 \\ \sigma_4 &= k_3\delta_5 + k_3 - \delta_6 + \frac{\alpha\rho L}{\delta_1} > 0 \\ \sigma_5 &= \frac{1}{2}EI - (T - EI)\delta_7 - \frac{1}{2}\alpha\rho L > 0 \\ \sigma_6 &= \frac{1}{2}EI - \frac{|T-EI|}{\delta_7} - EI(1 - \alpha L)\delta_8 - \frac{1}{2} > 0 \\ \sigma_7 &= \frac{1}{2}EI - \frac{EI(1-\alpha L)}{\delta_8} > 0 \end{aligned} \quad (45)$$

$$\lambda_3 = \min \left\{ \frac{2\sigma_1}{\rho}, \frac{2\sigma_2}{T}, 3\sigma, \frac{2\sigma_3}{M}, \frac{2\sigma_4}{I_h}, \frac{2k_3}{\sigma_5(k_2 - k_3)} \right\} \quad (46)$$

$$C = \frac{1}{\delta_4} E_1^2 + \frac{1}{\delta_6} E_2^2 + \left(\frac{1}{\delta_2} + \frac{\alpha L}{\delta_3} \right) L \bar{f}^2 \quad (47)$$

Define $\lambda = \lambda_c / \lambda_b$, we can obtain

$$V(t) \leq -(t) + C \quad (48)$$

Appendix C: Proof of theorem 1

Proof: Multiplying (48) by $e^{\lambda t}$, we can obtain the following integral result of the obtained results.

$$V(t) \leq V(0) - \frac{C}{\lambda}$$

$e^{-\lambda t} + \frac{C}{\lambda} \leq V(0)e^{-\lambda t} + \frac{C}{\lambda} \epsilon L_\infty$ (49) which indicates that $V(t)$ is bounded. Combined with $V_1(t)$, we have

$$\begin{aligned} \frac{T}{2L} [\omega(x, t)]^2 &\leq \frac{T}{2} \int_0^L [\omega'(x, t)]^2 dx \leq V_1(t) \leq V_1(t) \\ &+ V_2(t) \leq \frac{V(t)}{\lambda_1} \epsilon L_\infty \end{aligned} \quad (50)$$

In the same way, we can have

$$\frac{k_2 - k_3}{2} e^2(t) \leq V_1(t) \leq V_2(t) \leq \frac{V(t)}{\lambda_1} \epsilon L_\infty \quad (51)$$

Combining (3-66)-(3-68), we can obtain the upper bounds of $\omega(x, t)$ and $e(t)$:

$$|\omega(x, t)| \leq \sqrt{\frac{2L}{L\lambda_1} V(0)e^{-\lambda t} + \frac{C}{\lambda}}, \forall (x, t) \in [0, L] \times [0, \infty) \quad (52)$$

$$|e(t)| \leq \sqrt{\frac{2L}{(k_2 - k_3)\lambda_1} (V(0)e^{-\lambda t} + \frac{C}{\lambda})}, \forall (x, t) \in [0, \infty) \quad (53)$$

When the time t tends to infinity, the following inequality can be obtained

$$\lim_{t \rightarrow \infty} |\omega(x, t)| \leq \sqrt{\frac{2LC}{T\lambda_1}}, \forall x \in [0, L] \quad (54)$$

$$\begin{aligned} \lim_{t \rightarrow \infty} |e(t)| &\leq \sqrt{\frac{2C}{(k_2 - k_3)\lambda_1}} \\ \forall (x, t) \in (0, L) \times [0, \infty), \forall (x, t) \in [0, \infty) \end{aligned} \quad (55)$$

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