

The Influence of Temperature on Magnetic Quantum Effects in Semiconductor Structures

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A theory is constructed of the temperature dependence of quantum oscillation phenomena in narrow-gap electronic semiconductors, taking into account the thermal smearing of Landau levels. Oscillations of longitudinal electrical conductivity in narrow-gap electronic semiconductors at various temperatures are studied. An integral expression is obtained for the longitudinal conductivity in narrow-gap electronic semiconductors, taking into account the diffuse broadening of the Landau levels. A formula is obtained for the dependence of the oscillations of longitudinal electrical conductivity on the band gap of narrow-gap semiconductors. The calculation results are compared with experimental data.

Keywords: Oscillations of electronic heat capacity; oscillations of magnetic susceptibility and oscillations of electrical conductivity; cyclotron effective mass

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1. Introduction

It is known that, using quantum oscillation phenomena it is possible to determine the basic physical quantities (longitudinal conductivity, magnetic susceptibility, thermoelectric power and other transport phenomena) in electronic and nanoscale semiconductors. In particular, oscillations of longitudinal electrical conductivity and oscillations of magnetic susceptibility provide valuable information on the energy spectra of free electrons in electronic semiconductor structures. In a strong magnetic field, the longitudinal conductivity is determined using the following expression [1]:

$$\begin{aligned}\sigma_{zz} &= -\frac{e^2}{2\pi^2 m} \hbar \omega_c \sum_N \int_{\hbar \omega_c/2}^{\infty} k_Z 2\tau_N(E) \frac{\partial f_0(E)}{\partial E} dk_Z \\ &= -\frac{(2m)^{1/2} e^2}{\pi^2 \hbar^3} \hbar \omega_c \int_{\hbar \omega_c/2}^{\infty} \sum_N \left[E - \left(N + \frac{1}{2} \right) \hbar \omega_c \right]^{1/2} \tau_N(E) \frac{\partial f_0(E)}{\partial E} dE\end{aligned}\quad (1)$$

Here, N - is the number of Landau levels, ω_c - is the cyclotron frequency, $\tau_N(E)$ - relaxation time, E - is the energy of a free electron in a quantizing magnetic field. $\frac{\partial f_0(E)}{\partial E}$ - the energy derivative of the Fermi-Dirac function, takes on the character of a delta function at low temperatures.

From formula 1 it is seen that the effective mass is a constant, so this expression is applicable only for the parabolic dispersion law. However, if the dispersion law is non-parabolic (Kane's dispersion law), then the effective mass is strongly dependent on energy ($m^*(E)$). It is known that, just in narrow-gap electronic semiconductors, the effective mass depends on the energy ($m^*(E)$) [2-4]. Recently, many experiments performed on oscillations of longitudinal electrical conductivity and oscillations of magnetic susceptibility in narrow electron gap semiconductors [5-8]. In these works, quantum oscillation phenomena at a constant temperature studied. However, until now, the theory of temperature dependence has not been developed for these processes in narrow-gap electronic semiconductors. The study of quantum oscillation phenomena associated with equilibrium and nonequilibrium quantities allows us to identify new properties of massive, low-dimensional, and electronic semiconductors. Such values include longitudinal magnetic susceptibility, electronic heat capacity, thermodynamic potential, electrical conductivity, and others. In a quantizing magnetic field and at low temperatures, such quantities oscillate. All quantum oscillation

phenomena depend on the spectral density of energy states in semiconductors. The spectral density of states in semiconductors determined by the energy spectrum of electrons and holes. As experiments show, the density of states depends on temperature. The temperature dependence explained by thermal broadening of discrete levels in the sample. As shown in [9, 10], the density of states at low temperatures from a continuous spectrum turns into a discrete one. This is because at low temperatures the thermal broadening of the discrete levels decreases, disappears, and the continuous spectrum turns into discrete levels. The temperature dependence of the spectral density of states in a quantizing magnetic field was considered in [9, 10]. It is shown that with increasing temperature, the density of states in a strong field turns into a continuous spectrum of the density of states of electrons in the absence of a magnetic field. In this case, with increasing temperature, in the collision of electrons, the thermal motion smears the discrete Landau levels and turns them into a continuous spectrum of density of states. Oscillations of the longitudinal magnetic susceptibility have observed in wide-gap and narrow-gap semiconductors at constant temperatures in works [11–14]. Also, in these works the temperature dependence of the oscillation amplitude of the longitudinal magnetic susceptibility was considered in a strong magnetic field. However, in the above works, a concrete theory of oscillations of the longitudinal magnetic susceptibility in narrow-gap semiconductors, taking into account the temperature dependence of the spectral density of states, was not constructed.

The aim of this work is to construct a theory of the temperature dependence of the oscillations of longitudinal electric conductivity and oscillations of the magnetic susceptibility in narrow-gap electron semiconductors, taking into account the thermal broadening of the Landau levels.

2. Determination of the Temperature Dependence of the Shubnikov-de Haas Oscillations in Narrow-gap Semiconductors

2.1. Dependence of Oscillations of Longitudinal Electrical Conductivity on the Band Gap in Narrow-gap Electronic Semiconductors

Let us consider oscillations of longitudinal electrical conductivity in narrow-gap semiconductors. In a quantizing magnetic field, the electron energy of the conduction band is determined by the following expression [1]:

$$E_{N\pm} = -\frac{E_g}{2} + \frac{1}{2} \sqrt{E_g^2 + 4E_g \left[\left(N + \frac{1}{2}\right) \hbar \omega_c + \frac{\hbar^2}{k_z} 2m_n \pm \frac{g_0 \mu_B H}{2} \right]} \quad (2)$$

Where E_N - is the electron energy of the conduction band in a quantizing magnetic field with a nonparabolic dispersion law. E_g - is the band gap of narrow-gap semiconductors. We define k_z from formula 2, excluding spin. From here, we find k_z^2 :

$$k_z^2 = \frac{2m}{\hbar^2} \left[\frac{E_N^2}{E_g} + E_N - \left(N + \frac{1}{2}\right) \hbar \omega_c \right] \quad (3)$$

From 3, we determine the wave function along the Z axis with the nonparabolic dispersion law:

$$k_z = \frac{\sqrt{2m}}{\hbar^2} \sqrt{\left[\frac{E_N^2}{E_g} + E_N - \left(N + \frac{1}{2}\right) \hbar \omega_c \right]} \quad (4)$$

Differentiating formula 4 we obtain the following expression:

$$dk_z = \frac{\sqrt{2m}}{\hbar^2} \frac{\frac{2E_N}{E_g} + 1}{2 \sqrt{\left[\frac{E_N^2}{E_g} + E_N - \left(N + \frac{1}{2}\right) \hbar \omega_c \right]}} \quad (5)$$

Substituting 3 and 5 into 1, we determine the expression for the longitudinal conductivity in narrow-gap electronic semiconductors:

$$\sigma_{zz} = -\frac{(2m)^{\frac{1}{2}} e^2}{\pi^2 \hbar^3} \cdot \hbar \omega_c \cdot \int_{\hbar \omega_c/2}^{\infty} \sum_N \left(\frac{2E_N}{E_g} + 1 \right) \left[\frac{E_N^2}{E_g} + E_N - \left(N + \frac{1}{2}\right) \hbar \omega_c \right]^{1/2} \tau_N(E) \frac{\partial f_0(E)}{\partial E} dE \quad (6)$$

As can be seen from formula 6, if $E_g \rightarrow \infty$ then $\frac{2E_N}{E_g} \rightarrow 0$ and $\frac{E_N^2}{E_g} \rightarrow 0$, then formula 6 goes over to 1. Now, let us analyse the longitudinal conductivity oscillations for various narrow-gap electronic semiconductors with a nonparabolic dispersion law. Formula 6 allows you to graphically analyse the dependence of $\sigma_{zz}(E, H, T, E_g)$. In Figure 1 is shows the dependence of the oscillation longitudinal conductivity on the strong magnetic field in InSb. Here, $T=1$ K, $E_g = 0.234$ eV [15] and the number of Landau levels in the conduction band is $N=10$. As can be seen from this figure, with increasing magnetic field induction, the amplitudes of oscillations of the longitudinal conductivity increase. It can also be seen from the figure that the amplitude of the conductivity oscillation is 10. Each oscillation of the amplitude of the longitudinal conductivity corresponds to one discrete Landau level.

With the help of formula 6, we compare the oscillations of the longitudinal electrical conductivity for various values of the band gap. In Figure 2 oscillation phenomena are presented for InSb and InAs at a constant temperature. Here, $T=4$ K, $E_g=0.234$ eV [15] for InSb, $E_g=0.414$ eV [15] for InAs and the number of Landau levels in the conduction band is equal to $N=12$. As can be seen from this figure, with

an increase in the band gap, one can observe a downward movement of the oscillation graph. For example, longitudinal electrical conductivity oscillations at $E_g=0.234$ eV, $B=0.5$ T, $T=1$ K is equal to $\sigma_{zz} = 0.266 \frac{1}{\Omega m \cdot sm}$. Longitudinal conductivity at $E_g=0.414$ eV, $B=0.5$ T, $T=1$ K is equal to $\sigma_{zz} = 0.246 \frac{1}{\Omega m \cdot sm}$ (Figure 2). It follows that with the help of the band gap of narrow-gap semiconductors at constant temperatures, it is possible to control the oscillations of longitudinal electrical conductivity.

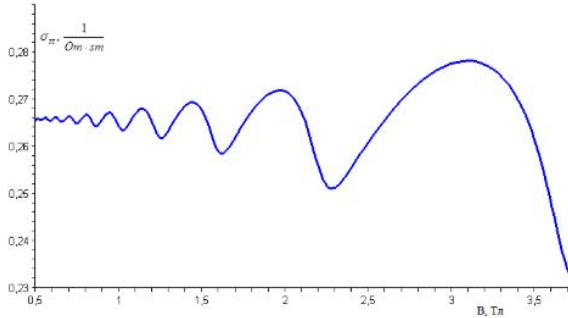


Fig. 1. Longitudinal electrical conductivity oscillations in InSb at $T=1$ K, calculated by formula 6

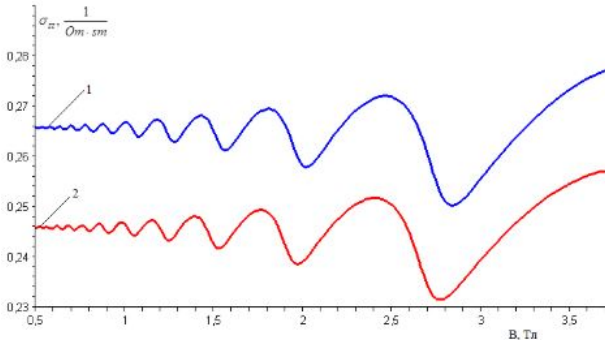


Fig. 2. Longitudinal electrical conductivity oscillations in narrow-gap semiconductors at $T=1$ K, calculated by formula 6. 1- for InSb; 2 - for InAs.

Thus, from Figure 2, a strong dependence of the longitudinal electrical conductivity oscillations on the band gap in narrow-gap semiconductors is seen. But, as can be seen from formula 1, for a spectrum with a parabolic dispersion law, the longitudinal electrical conductivity oscillations do not depend on the band gap.

2.2. Temperature Dependence of Longitudinal Conductivity Oscillations in Narrow-gap Electronic Semiconductors

Let us consider the temperature dependence of the longitudinal electric conductivity oscillations in narrow-gap

electronic semiconductors. The graphs in Figure 1 and Figure 2 are obtained at low temperatures and strong magnetic fields. In this case, the Landau levels are manifested sharply and the thermal broadening is very weak. The broadening of the discrete levels is described by the derivative of the Fermi-Dirac energy distribution function ($\frac{\partial f(E, \mu, T)}{\partial E}$). To take into account the temperature dependence of the longitudinal conductivity oscillations, we expand the $\sigma_{zz} = (E, H, T, E_g)$ in the derivative of the function Fermi-Dirac distribution ($\frac{\partial f(E, \mu, T)}{\partial E}$). Then the longitudinal conductivity oscillations will depend on the temperature. As is known, the band gap of semiconductors is highly dependent on temperature $E_g(T)$ [15, 16]. The temperature dependence of the band gap of semiconductors can be determined using the empirical relation of Varshni [15, 16] or the analytical expression of Feng [16] and other relations. For example, the empirical relations of Varshni have the following form [15, 16]:

$$E_g(T) = E_g(0) - \frac{\alpha_1 T^2}{\alpha_2 + T} \quad (7)$$

Here, $E_g(0)$ - is the width of band gap at $T = 0$ K, α_1 and α_2 are empirical parameters. The energy derivative of the Fermi-Dirac function is determined by the following expression:

$$\frac{\partial f_0(E, \mu, T)}{\partial E} = -\frac{1}{KT} \frac{\exp(\frac{E-\mu}{kT})}{(\exp(\frac{E}{\mu}) + 1)^2} \quad (8)$$

Hence, substituting 7 and 8 in 6, we obtain the temperature dependence of the oscillations of the longitudinal conductivity in narrow-gap semiconductors in the presence of a strong magnetic field:

$$\begin{aligned} \sigma_{zz}(E, H, T, E_g(T)) &= \frac{(2m)^{\frac{1}{2}} e^2}{\pi^2 \hbar^3} \hbar \omega_c \cdot \int_{\hbar \omega_c/2}^{\infty} \sum_N \tau_0 E^\tau \left(\frac{2E_N}{E_g(0) - \frac{\alpha_1 T^2}{\alpha_2 + T}} + 1 \right) \\ &\left[\frac{E_N^2}{E_g(0) - \frac{\alpha_1 T^2}{\alpha_2 + T}} + E_N - (N + \frac{1}{2}) \hbar \omega_c \right]^{1/2} \frac{\partial f_0(E, \mu, T)}{\partial E} dE \end{aligned} \quad (9)$$

Thus, it becomes possible to calculate the longitudinal conductivity oscillations in narrow-gap semiconductors at various temperatures. We plot graphic the $\sigma_{zz} = (E, H, T, E_g)$ dependences with the help formula 9. In Figure 3 shows the oscillations of the longitudinal conductivity in InSb at temperatures $T=1$ K, 25 K, and 77 K. Here, the magnetic field induction varies in the interval $B=0.5$ T $\div$ 3 T. At a temperature of $T=1$ K, $kT=0.086$ meV and $\hbar \omega_c = 10$ meV, $\frac{\hbar \omega_c}{kT} = 0.12 \cdot 10^3 = 120$. In this case, the thermal smearing does not occur. Thus, the longitudinal conductivity oscillations are observed at $kT \ll \hbar \omega_c$ temperatures. It can be seen from Figure 3, that at a temperature of 77 K the

amplitudes of the longitudinal electrical conductivity oscillations are practically no noticeable and coincide with $\sigma_{zz} = (E, H, T, E_g)$ in the absence of a magnetic field.

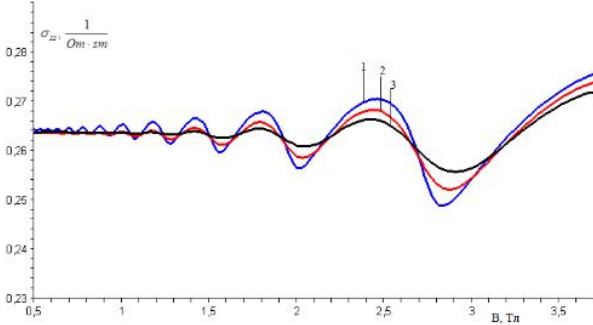


Fig. 3. On temperature dependence of longitudinal conductivity oscillations in InSb, calculated by formula 9. 1-T=1K; 2-T=25 K; 3-T=77 K.

3. Calculation of de Haas-van Alphen Oscillations in Narrow-gap Semiconductors at Various Temperatures

Let us consider the temperature dependence of de Haas-van Alphen oscillations (the magnetic susceptibility oscillations) in narrow-gap semiconductors taking into account the temperature dependence of the density of states. for narrow-gap semiconductors, the spectral density of states is determined by the following expression [10]:

$$N_s(E, H) = \frac{(m)^{3/2}}{(2)^{1/2} \pi^2 \hbar^3} \frac{\hbar \omega_c}{2} \sum_{N=0}^{N_{max}} \frac{\frac{2E}{E_g} + 1}{\sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \hbar \omega_c}} \quad (10)$$

$N_s(E, H)$ - spectral density of energy states with non-parabolic dispersion law. integrating formula 10, we obtain the total number of quantum states per unit volume:

$$N(E, H) = \int N_s(E, H) dE = \frac{m^{3/2}}{(\pi \hbar)^2} \cdot \frac{eH}{c} \sum_{N=0}^{N_{max}} \sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \frac{\hbar e H}{mc}} \quad (11)$$

In quantizing magnetic fields, the free energy of electrons without taking into account spin is expressed in terms of the total number of the quantum state in the following form [1, 14]:

$$F(E, H, T) = n(\mu) - \frac{m^{3/2}}{(\pi \hbar)^2} \cdot \frac{eH}{c} \int \sum_{N=0}^{N_{max}} \sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \frac{\hbar e H}{mc}} \cdot \frac{1}{(1 + \exp(\frac{E - \mu}{kT}))^{-1}} dE \quad (12)$$

Where, n - is the concentration of charge carriers, μ - is the fermi level. differentiating 12 with respect to h and we find $\frac{dF(E, H, T)}{dH}$:

$$\begin{aligned} \frac{dF(E, H, T)}{dH} &= \frac{dn(\mu)}{dH} - \frac{m^{3/2}}{(\pi \hbar)^2} \cdot \int \sum_{N=0}^{N_{max}} \frac{d \left(\frac{eH}{c} \sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \frac{\hbar e H}{mc}} \right)}{dH} \cdot \frac{1}{(1 + \exp(\frac{E - \mu}{kT}))} dE \\ &= - \frac{em^{3/2}}{c(\pi \hbar)^2} \cdot \int \sum_{N=0}^{N_{max}} \left(\sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \frac{\hbar e H}{mc}} - \frac{(N + \frac{1}{2}) \frac{\hbar e H}{mc}}{2 \cdot \sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \frac{\hbar e H}{mc}}} \cdot \frac{1}{(1 + \exp(\frac{E - \mu}{kT}))} \right) dE \end{aligned} \quad (13)$$

Differentiating 13 again with respect to h , we obtain $\frac{d^2F(E, H, T)}{dH^2}$:

$$\begin{aligned} \chi(E, H, T) &= \frac{d^2F(E, H, T)}{dH^2} = \\ &= - \frac{e^2 m^{3/2}}{4 \hbar (c \pi)^2} \cdot \int \sum_{N=0}^{N_{max}} \frac{(N + \frac{1}{2})^2 \frac{3 \hbar e H}{mc} - 4(N + \frac{1}{2}) \left(\frac{E^2}{E_g^2} + E \right)}{\left(\sqrt{\frac{E^2}{E_g^2} + E - (N + \frac{1}{2}) \frac{\hbar e H}{mc}} \right)^3} \cdot \frac{1}{(1 + \exp(\frac{E - \mu}{kT}))} dE \end{aligned} \quad (14)$$

Here, $\chi(E, H, T)$ - are magnetic susceptibility oscillations or de Haas-van Alphen oscillations for narrow-gap semiconductors. Thus, using formula 14, one can calculate the temperature dependences of the magnetic susceptibility oscillations in narrow-gap electronic semiconductors. Now consider the numerical calculations using the computer program Maple. Using formula (14), we construct a graph of the dependence of the magnetic susceptibility oscillations on the strong magnetic field strength in $n - Bi_2Te_{2.85}Se_{0.15}$ (Figure 4). Here, $E_g(0) = 0.18 eV$ [17], $T=2K$, magnetic field strength at $B = 0.1 \div 3T$ (or $H = 1 \div 30 kOe$). From Figure 4 it follows that with an increase in the magnetic field induction, the oscillation amplitude of the magnetic susceptibility increases significantly.

At low temperatures, the discrete Landau levels manifest themselves sharply and the thermal broadening of the discrete levels is not felt. Thermal broadening of levels in a strong magnetic field leads to smoothing of discrete levels. Thermal broadening is taken into account with the help of $\chi(E, H, T)$ - functions.

In Figure 4 are shows graphs of magnetic susceptibility in three-dimensional space for $n-bi_2te_{2.85}se_{0.15}$. Here, the dependences of the magnetic susceptibility oscillations on the strong magnetic field and temperature are obtained. with increasing temperature, the discrete landau levels begin to smooth out, and with $kT \sim \hbar \omega$, the magnetic susceptibility oscillations gradually disappear

As can be seen from Figure 4, at sufficiently high $kT > \hbar \omega$ temperatures, the $\chi(E, H, T)$ oscillations smoothed into continuous magnetic susceptibility spectra will not sense the magnetic field. At a temperature of 60K, the oscillations are practically no noticeable. Thus, it can be seen

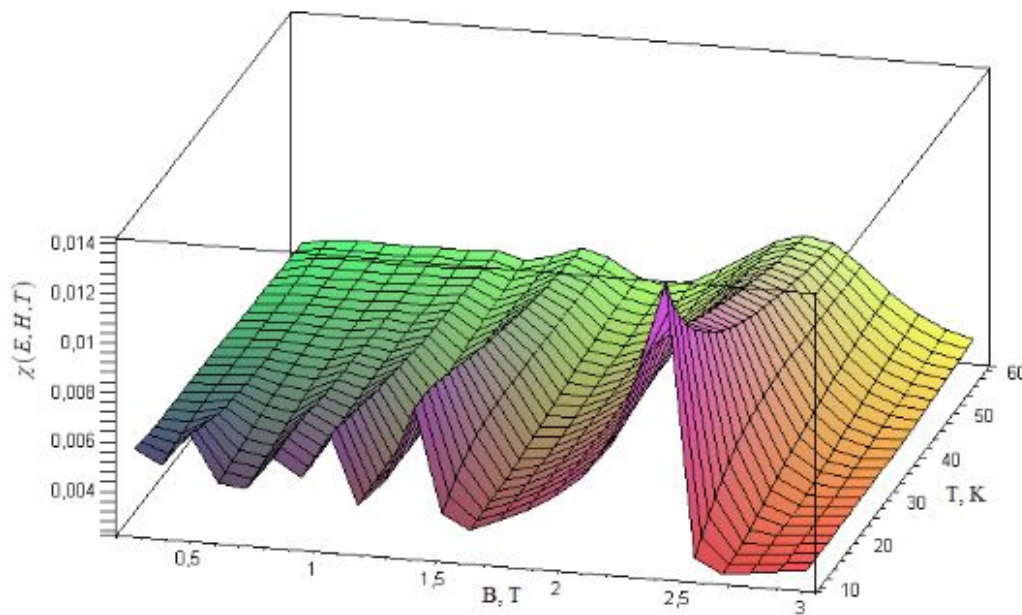


Fig. 4. Dependence of the de Haas-van Alphen oscillations on temperature and magnetic field in $n\text{-Bi}_2\text{Te}_{2.85}\text{Se}_{0.15}$, calculated by formula 14

from graphs that the peaks of the Landau levels decrease with increasing temperature. This shows that a decrease in the height of the discrete Landau levels appears with allowance for the thermal broadening of the energy levels of the allowed zone.

4. Comparison of Theory with Experimental Results

Let us analyze the quantum oscillations phenomena for specific semiconductors in a quantizing magnetic field. Figure 5 and 7 shows quantum magnetic effects at the temperature of measurements $T = 2\text{ K}$ in $p\text{-Bi}_{2-x}\text{Fe}_x\text{Te}_3$ [18]. Within experimental accuracy, the oscillations density of states is a continuous, steadily increasing smooth function of energy. Oscillations in the quantum effects are not determined in high temperature (Fig. 7). This indicates that $\omega c > kT$. Let us expand this representation of the density of states in series with respect to formula 17. In the expansion, the initial term should be represented by the ideal $R_{zz}(E, H)$ from formula 17. However, unlike the ideal case, we are going to take into account the dependence of the electron mass on energy. We will position the $R_{zz}(E, H)$ peaks according to the values of the energy multiple to c ; however, we consider the height of peaks depending on energy. Then, choosing the coefficients in the sum before $R_{zz}(E, H)$, we fit the theoretical values of $R_{zz}(E, H)$ to experimental ones with the adjustable parameter being exactly the coefficient at $R_{zz}(E, H)$. Further on, we will lower the temperature

in formula 17. Starting with a temperature close to 20 K, the smooth curve starts to oscillate and the Landau levels become distinct (Fig. 5 and Fig. 7). Thus, at a temperature corresponding to $kT > \omega c$, the Landau levels start to be distinctly stand out as separate peaks of $R_{zz}(E, H)$. In a high magnetic field, the continuous spectrum of the energy states becomes highly deformed and is transformed into oscillating lines; as the temperature is increased, the discrete lines become broadened and the densities of the energy states transform into a continuous spectrum. Experimentally, quantum magnetic effects depend on the energy, temperature, and effective mass. The effective mass depends on energy, while c depends on the effective mass. Thus, the distance between peaks changes as the energy is increased. At a constant effective mass, the dependence of the density of states on the energy has the form $R_{zz} \sim \rho$ (density of states). If, as the energy of electrons is increased, their effective mass becomes larger, the distance between Landau levels decreases and the curve describing the density of states shifts to higher densities of states and shifts upward along the axis of the density of states. In contrast, if $m(E)$ decreases as the energy is increased, the curve representing the density of states shifts downward in energy along the axis of the quantum oscillations phenomena.

It is known that in the case of the density of states Landau quantization is a periodic function of the magnetic field. This leads to oscillations Shubnikov – de Haas and de Haas – van Alphen are periodic functions in the strong magnetic

field. The de Haas-van Alphen effect in magnetic semiconductors observed in the work [18]. Were received of the magnetic susceptibility oscillations in p – $\text{Bi}_{2-x}\text{Fe}_x\text{Te}_3$ were obtained at $T=2\text{K}$, $x=0$ [18] and $E_g(0)=0.2\text{eV}$ [17]. In figure 7 submitted the theoretical and experimental graph for p – $\text{Bi}_{2-x}\text{Fe}_x\text{Te}_3$ ($x=0$) and at $T=2\text{K}$. A theoretical graph obtained using formula (14). As can be seen in observed figure the amplitude of the Landau levels on the theoretical curve is much higher than on the experimental graph. With the help formulas 14, one can plot graph the magnetic susceptibility oscillations for p – $\text{Bi}_{2-x}\text{Fe}_x\text{Te}_3$ at various temperatures. In figure 6 shows the oscillations of the magnetic susceptibility in p – $\text{Bi}_{2-x}\text{Fe}_x\text{Te}_3$ at temperatures $T=2\text{K}$, 20K , 40K , 60K , and 77K . From figure 7 we can see that at high temperatures, the oscillation amplitudes erode, and a strong magnetic field is not felt. this is due to the fact that the thermal broadening of the Landau levels is enhanced at high temperatures.

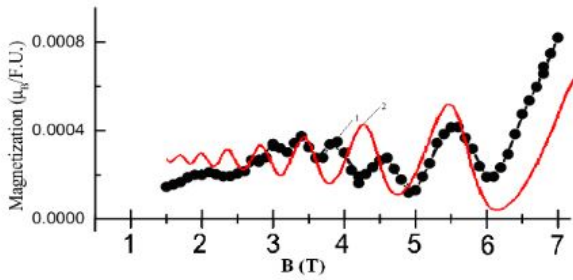


Fig. 5. De Haas-van Alphen oscillations in p – $\text{Bi}_{2-x}\text{Fe}_x\text{Te}_3$ at $T=2\text{ K}$ and $x=0$; 1-experiment [18], 2-theory, calculated by formula 14

$$\frac{A_\chi(T_2)}{A_\chi(T_1)} = \frac{T_2}{T_1} \frac{\text{sh}\left(\frac{2\pi^2 k T_1}{h \frac{eB}{m_c^*}}\right)}{\text{sh}\left(\frac{2\pi^2 k T_2}{h \frac{eB}{m_c^*}}\right)} \quad (15)$$

As is known from the literature, using changes in the amplitude $A_\chi(T)$ of the magnetic susceptibility oscillations at various temperatures, it is possible to determine the cyclotron effective mass in narrow-gap semiconductors. Naturally, the amplitude of the magnetic susceptibility strongly depends on the cyclotron effective mass and is determined by the following expression [19, 20]:

As can be seen from formula (15), these expressions are transcendental equations for the variable m_c^* . If, $T_1 = 2T_2$ then from these equations it is not difficult to find a simple

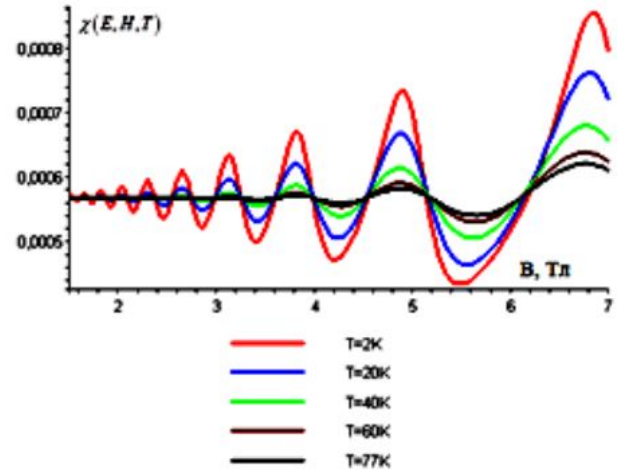


Fig. 6. Temperature dependence of the magnetic susceptibility oscillations in p – $\text{Bi}_{2-x}\text{Fe}_x\text{Te}_3$ at $x=0$, calculated by formula 14.

analytical expression m_c^* for as a function of $\chi(E, H, T)$:

$$\begin{aligned} \frac{A(T_2)}{A(T_1)} &= \frac{T_2}{2T_1} \frac{\text{sh}\left(2 \frac{2\pi^2 k T_2}{h \omega_c}\right)}{\text{sh}\left(\frac{2\pi^2 k T_2}{h \omega_c}\right)} = \frac{1}{2} \frac{2 \text{sh}\left(\frac{2\pi^2 k T_2}{h \omega_c}\right) \text{ch}\left(\frac{2\pi^2 k T_2}{h \omega_c}\right)}{\text{sh}\left(\frac{2\pi^2 k T_2}{h \omega_c}\right)} \\ &= \text{ch}\left(\frac{2\pi^2 k T_2}{h \omega_c}\right) = \text{ch}\left(\frac{2m_c^* \pi^2 k T_2}{h e B}\right) \quad (16) \\ \text{arch}\left[\frac{A(T_2, B)}{A(T_1, B)}\right] &= \frac{2m_c^* \pi^2 k T_2}{h e B} \\ \Rightarrow m_c^* &= \frac{e h B}{2\pi^2 k T_2} \text{arch}\left[\frac{A(T_2, B)}{A(T_1, B)}\right] \end{aligned}$$

Thus, using expressions 14 and 16, we can explain the experimental results in narrow-gap semiconductors. In particular, for $\text{Mg}_2\text{Si}_{0.3}\text{Sn}_{0.7}$, the cyclotron effective mass is $0.5m_0$ (m_0 is the mass of a free electron) [21].

Let us analyze the longitudinal conductivity oscillations of specific narrow-gap electronic materials in a quantizing magnetic field. For a unit volume of semiconductors, the following condition is satisfied:

$$\begin{aligned} R_{zz}((E, H, T, E_g(T))) &\approx \rho_{zz}((E, H, T, E_g(T))) \\ &= \frac{1}{\sigma_{zz}(E, H, T, E_g(T))} \quad (17) \end{aligned}$$

Here, the R_{ZZ} is the longitudinal magnetoresistance. In Figure 7, the results of theoretical calculations are compared with experimental data for Bi_2Se_3 [8] at a measurement temperature of $T=4.2\text{K}$, $E_g(T) = 0.15\text{eV}$ [20] and in the magnetic field induction range $B=0\div 32\text{T}$. The theoretical curve for $R_{zz}((E, H, T, E_g(T)))$ is obtained with the help formula 17. As can be seen from this figure, discrete Landau levels are not observed in the range of the magnetic field induction $B=5\div 10\text{T}$. in the experimental graph. But, the oscillations of the longitudinal magnetoresistance in the theoretical curve are manifested precisely in this interval of

magnetic field induction. Using formula 17, we can calculate the oscillations of the longitudinal magnetoresistance in Bi_2Se_3 at various temperatures. As can be seen from Figure 7, the theoretical curve and experimental data are in good agreement.

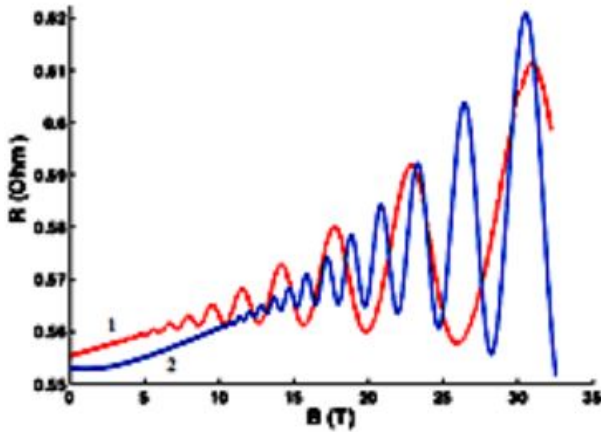


Fig. 7. Magnetoresistance oscillations in Bi_2Se_3 at $T=4.2K$. 1-theory calculated by formula 17; 2-experiment [8].

5. Discussion and conclusion

Based on the study, the following conclusion can be made: A theory is developed of the temperature dependence of the longitudinal conductivity and magnetic susceptibility oscillations in narrow-gap semiconductors, taking into account the thermal smearing of Landau levels. A new analytical expression obtained for the dependence of the longitudinal conductivity oscillations on the band gap of narrow-gap semiconductors. A new method is obtained for the temperature dependence of the amplitude of quantum oscillation phenomena, taking into account the thermal broadening of the Landau levels, and using this method it becomes possible to explain some experimental results.

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