Daftardar-Jafari Method For Solving Conformable K (m, P,1) Equation

Guohua Fu* and Xinxin Chang

School of Anyang Institute of Technology, Anyang, 455000, Henan, China

**Corresponding author. E-mail: fgh253@126.com*

Received: Apr. 19, 2022; Accepted: Sep. 04, 2022

The Daftardar-Jafari technique is a powerful solution method for obtaining approximate and exact solutions of some nonlinear partial differential and integral equations. In this paper, the Daftardar-Jafari method is used to establish approximate solution of the conformable K (m, p,1) equation. This equation has been widely used in the applied sciences and engineering. The conformable derivative (CD) was introduced and explored with Khalil et al. We present the figures and tables to compare between the approximate solutions. The approximate results show that the Daftardar-Jafari method is a very efficient and most reliable approach to handle conformable partial differential and integral equations.

Keywords: The Daftardar-Jafari method, CFD, Conformable K (m, p,1) equation.

http://dx.doi.org/10.6180/jase.202308_26(8).0008

1. Introduction

In recent years, differential equations in fractional sense (FDEs) have found significant importance arising from their application in different branches of chemistry, biologist, physics, engineering, economic and other science. Studies show that the behavior of a lot physical systems is deployed on the theory of fractional derivatives. Different important phenomena in chemical and biological processes, electrochemistry, electromagnetic, viscoelasticity, fluid mechanics, signal processing, and control systems are well defined by FDEs. For that we require an affective and reliable approach for the solution of FDEs. Lots of researchers have practical different techniques to study the of FDEs such as the decomposition approach of Adomian [1, 2] , the variational iteration technique [3, 4], the homotopy perturbation and the homotopy analysis methods [5–7], the differential transform [8, 9], Daftardar-Jafari method [10, 11] and so on [12–31]. The Daftardar-Jafari method (DJM) was first proposed in 2006 [32], which was successfully used to set solutions to nonlinear PDEs. It was shown that this new method is efficient to construct analytical solutions approximately. The DJM has been used to solve various nonlinear equations [33–35].

The aim of this work is to expand the application of the DJM to obtain analytical and approximate solution for some conformable K (m, p,1) equation. These equations include conformable K (2,2,1) and K (3,3,1) equations. The conformable K (m, p,1) equation in the following form [36].

$$D_t^{\alpha} v + (v^m)_x - (v^p)_{xxx} + v_{5x} = 0, t > 0, 0 < \alpha \leq 1, \quad (1)$$

where v(x,t) represents the wave profile. Also m,p are positive integer with $m, p \geq 1$. This equation models large discontinuous time scales while forecasting weather or high energy physics in tiny time scales. In the E-infinity theory, time is discontinuous. Thus, fractional models can be the best choice to define such problems [36].

There are many studies for the classical K(m,p,1) such as variational iteration method [37], decomposition approach of Adomian [38], reduced form of the differential transform technique [39] and homogeneous balance method [40]. Also, in [36], by using homotopy perturbation method, authors set solitary wave solutions for time conformable nonlinear K(m,p,1) equations.

The remainder of this study is organized as follows. In Sec. 2, we give brief definition and some significant properties of the CD. In Sec. 3 the DJM introduced. In Sec. 4, using the DJM we establish approximate solution of the conformable K(m,p,1) equation. The conclusions are then given in the in the last Sec. 5.

2. Cd

In this Sec., we summarize fundamental definitions and significant properties of CD used in this present paper.

Khalil et al. [41] have newly defined the fractional integral and derivative named conformable derivative (CD). This newly defined derivative has been used in many different problems [42–47].The α th order CD is given in limit form as

$$D_t^\alpha f(t) = \lim_{\varepsilon \to 0} \frac{f\left(t + \varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon} \quad t > 0, \quad 0 < \alpha \leq 1 \tag{2}$$

Critical properties of the derivative in conformable sense are listed below [41–43].

$$D_t^\alpha \left(t^\delta\right) = \alpha t^{\delta-\alpha}, \forall \delta \in R \tag{3}$$

$$D_t^\alpha (f(t)g(t)) = \left(D_t^\alpha f(t)\right) g(t) + \left(D_t^\alpha g(t)\right) f(t) \tag{4}$$

$$D_t^\alpha \left(\frac{f(t)}{g(t)}\right) = \frac{\left(D_t^\alpha f(t)\right) g(t) - \left(D_t^\alpha g(t)\right) f(t)}{g^2(t)} \tag{5}$$

$$D_t^\alpha (f \circ g)(t) = t^{1-\alpha} g'(t) f'(g(t)) \tag{6}$$

Definition 1. Let $a \geq 0$ and $t \geq a$ and f be a function of t defined on $(a, t]$ with $\alpha \in R$. Then, the fractional integral can be shown as,

$${}_tI_a^\alpha f(t) = \int_a^t \frac{f(x)}{x^{1-\alpha}} dx \tag{7}$$

when the existence of the Riemann integral is satisfied.

3. The DJM

Consider the differential equation with nonlinear parts can be represented in operator form:

$$v(\tilde{x}) - N(v(\tilde{x})) = f(\tilde{x}), \tag{8}$$

Where N represents a nonlinear operator,f is known and $\tilde{x} = (x_1, x_2, \ldots, x_n)$. Assume that the solution of Eq. (8) has a solution in series form:

$$v(\tilde{x}) = \sum_{i=0}^{\infty} v_i(\tilde{x}) \tag{9}$$

Thus, a decomposition can be achieved in the nonlinear operator N as:

$$N\left(\sum_{i=0}^{\infty} v_i\right) = N\left(v_0\right) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} v_j\right) - N\left(\sum_{i=0}^{i-1} v_j\right)\right\} \tag{10}$$

However, by the Eqs. (8) to (10) can be rewritten as:

$$\sum_{i=0}^{\infty} v_i = f + N\left(v_0\right) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} v_j\right) - N\left(\sum_{i=0}^{i-1} v_j\right)\right\} \tag{11}$$

Define the recurrence relation:

$$\begin{cases} \mathrm{v}_0 = \mathrm{f} \\ \mathrm{v}_1 = \mathrm{N}\left(\mathrm{v}_0\right) \\ \mathrm{v}_{\mathrm{m}+1} = \mathrm{N}\left(\mathrm{v}_0 + \ldots + \mathrm{v}_{\mathrm{m}}\right) \\ -\mathrm{N}\left(\mathrm{v}_0 + \mathrm{v}_1 + \ldots + \mathrm{v}_{\mathrm{m}-1}\right) \\ \mathrm{m} = 1, 2, 3, \ldots \end{cases} \tag{12}$$

So:

$$\left(v_1 + v_2 + \cdots + v_{m+1}\right) = N\left(v_0 + v_2 + \ldots + v_m\right), m = 1, 2, 3, \ldots \tag{13}$$

and

$$\sum_{i=0}^{\infty} v_i - N\left(\sum_{i=0}^{\infty} v_i\right) = f \tag{14}$$

The approximate solution in k-terms of Eq. (8) can be written as:

$$v = v_0 + v_1 + v_2 + \ldots + v_{k-1} \tag{15}$$

Proof of the convergence of the DJM can be examined in the [11].

4. Applications

4.1. The DJM for conformable K (2,2,1) equation

Let we take m=2, p=2 in Eq. (1), hence we have

$$D_t^\alpha v + \left(v^2\right)_x - \left(v^2\right)_{xxx} + v_{5x} = 0, t > 0 \tag{16}$$

The exact solution to Eq. (16) when α=1 and subject to the initial data

$$v(x, 0) = \frac{16c - 1}{12}\left(\cosh\left(\frac{1}{4}x\right)\right)^2 \tag{17}$$

Where c is an arbitrary constant and

$$v(x,t) = \frac{16c-1}{12}\cosh^2\left(\frac{ct-x}{4}\right) \tag{18}$$

which is the exact solution. Applying transform on both sides of Eq. (17), we have

$$\frac{\partial}{\partial t}v(x,t) = \frac{1}{t^{1-\alpha}}\left(\begin{array}{c} -2v(x,t)\frac{\partial}{\partial x}v(x,t)+ \\ 6\left(\frac{\partial}{\partial x}v(x,t)\right)\frac{\partial^2}{\partial x^2}v(x,t)+ \\ 2v(x,t)\frac{\partial^3}{\partial x^3}v(x,t)-\frac{\partial^5}{\partial x^5}v(x,t) \end{array}\right), \tag{19}$$

Based onthe DJM, we construct the following result:

$$\begin{gathered} v(x,t) = v_0(x,t) \\ +J\left(\frac{-2v(x,t)\frac{\partial}{\partial x}v(x,t)+6\left(\frac{\partial}{\partial x}v(x,t)\right)\frac{\partial^2}{\partial x^2}v(x,t)+}{t^{1-\alpha}}\right. \\ \left.\frac{2v(x,t)\frac{\partial^3}{\partial x^3}v(x,t)-\frac{\partial^5}{\partial x^5}v(x,t)}{t^{1-\alpha}}\right) \end{gathered} \tag{20}$$

where J is an integral operator. Therefore, the following results are obtained:

$$\left\{\begin{array}{l} v(x,0) = \frac{16c-1}{12}\left(\cosh\left(\frac{1}{4}x\right)\right)^2, \\ N(v) = J\left(\frac{-2v(x,t)\frac{\partial}{\partial x}v(x,t)+6\left(\frac{\partial}{\partial x}v(x,t)\right)\frac{\partial^2}{\partial x^2}v(x,t)+}{t^{1-\alpha}}\right. \\ \left.\frac{2v(x,t)\frac{\partial^3}{\partial x^3}v(x,t)-\frac{\partial^5}{\partial x^5}v(x,t)}{t^{1-\alpha}}\right) \end{array}\right. \tag{21}$$

The iteration solution of Eq. (19) are obtained as follows:

$$\begin{aligned} v_0(x,t) &= \frac{(16c-1)}{12}\left(\cosh\left(\frac{1}{4}x\right)\right)^2 \\ v_1(x,t) &= -\frac{1}{24}\frac{t^{\alpha}(16c-1)\cosh\left(\frac{1}{4}x\right)\sinh\left(\frac{1}{4}x\right)c}{\alpha}, \\ v_2(x,t) &= \frac{1}{192}\frac{c^2t^{2\alpha}\left(32\left(\cosh\left(\frac{1}{4}x\right)\right)^2c-\right.}{\alpha^2} \\ &\quad \frac{\left.2\left(\cosh\left(\frac{1}{4}x\right)\right)^2-16c+1\right)}{\alpha^2} \\ v_3(x,t) &= -\frac{1}{576}\frac{c^3\cosh\left(\frac{1}{4}x\right)\sinh\left(\frac{1}{4}x\right)t^{3\alpha}(16c-1)}{\alpha^3} \end{aligned}$$

The remaining components of $v_k(x,t)$ can be completely determined such that each term is determined by using Eq. (21).

So, we can have the 3th-order approximate solution of Eq. (16) as the following form:

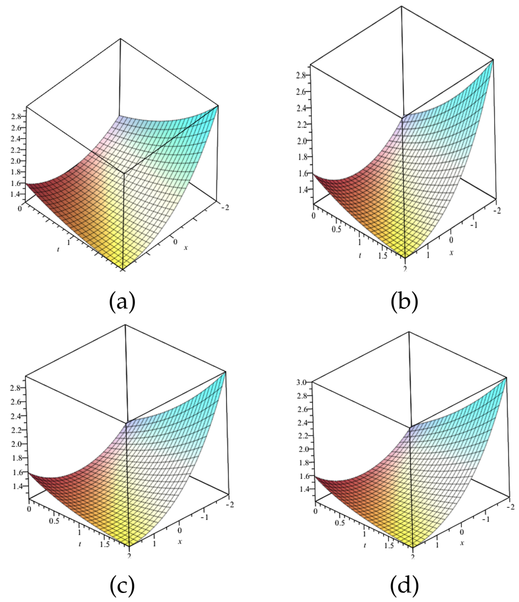

Fig. 1. Solution of Eq. (16) using the 3-iteration of DJM for several values of α when c=1 : (a) exact (α=1) , (b) α=1 , (c)α=0.95 and (d) α= 0.9 .

$$\begin{aligned} \phi_2 &= v_0(x,t)+v_1(x,t)+v_2(x,t)+v_3(x,t) = \\ &\frac{(16c-1)}{12}\left(\cosh\left(\frac{1}{4}x\right)\right)^2 \\ &-\frac{1}{24}\frac{t^{\alpha}(16c-1)\cosh\left(\frac{1}{4}x\right)\sinh\left(\frac{1}{4}x\right)c}{\alpha} \\ &+\frac{1}{192}\frac{c^2t^{2\alpha}\left(32\left(\cosh\left(\frac{1}{4}x\right)\right)^2c\right.}{\alpha^2} \\ &\frac{\left.-2\left(\cosh\left(\frac{1}{4}x\right)\right)^2-16c+1\right)}{\alpha^2} \\ &-\frac{1}{576}\frac{c^3\cosh\left(\frac{1}{4}x\right)\sinh\left(\frac{1}{4}x\right)t^{3\alpha}(16c-1)}{\alpha^3}. \end{aligned} \tag{22}$$

Fig. 1 and Table 1 show solutions of Eq. (16) for different values of α. Table 1 shows the approximate solutions of Eq. (19) for different values of α : $\alpha = 0.9, \alpha = 0.95$ and α=1 using only three iterations of the DJM solution (see also Fig. 2).

4.2. The DJM for conformable K (3,3,1) equation

Let we take m=3, p=3 in Eq. (1), hence we have

$$D_t^{\alpha}v+\left(v^3\right)_x-\left(v^3\right)_{xxx}+v_{5x} = 0, t>0 \tag{23}$$

The exact solution to Eq. (23) when α=1 and subject to the initial condition

Table 1. Solution of Eq. (16) using DJM for different values of α .

x	t	α =0.9	α =0.95	α =1	Exact(α=1)
0.1	0	1.250781412	1.250781412	1.250781412	1.250781412
	0.5	1.269069250	1.265382355	1.262439912	1.262541722
	1	1.329090839	1.320240856	1.312721265	1.314356354
0.6	0	1.278336572	1.278336572	1.278336572	1.278336572
	0.5	1.249790859	1.250089455	1.250676406	1.250781412
	1	1.267984798	1.264030547	1.260875944	1.262541722
0.9	0	1.314356354	1.314356354	1.314356354	1.250781412
	0.5	1.257040494	1.259723496	1.262431657	1.269633188
	1	1.250849839	1.249694958	1.249047377	1.250781412

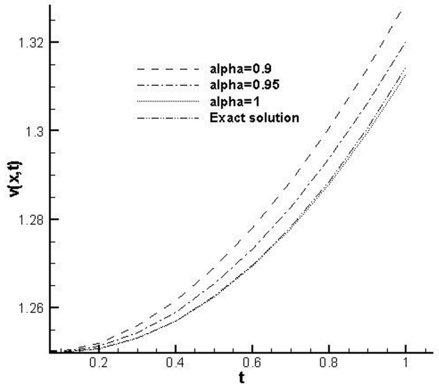

Fig. 2. Solution of Eq. (16) using the 3-iteration of DJM for several values of α when, x=0.1 .

$$v(x,0) = \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right), \tag{24}$$

Where c is an arbitrary constant

$$v(x,t) = \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{ct-x}{3}\right) \tag{25}$$

which is the exact solution. Applying transform on both sides of Eq. (24), we have

$$\begin{aligned}\frac{\partial}{\partial t}v(x,t) &= \frac{1}{t^{1-\alpha}}\left(-3(v(x,t))^2\frac{\partial}{\partial x}v(x,t)+6\left(\frac{\partial}{\partial x}v(x,t)\right)^3\right.\\ &+18v(x,t)\left(\frac{\partial}{\partial x}v(x,t)\right)\frac{\partial^2}{\partial x^2}v(x,t)\\ &\left.+3(v(x,t))^2\frac{\partial^3}{\partial x^3}v(x,t)-\frac{\partial^5}{\partial x^5}v(x,t)\right)\end{aligned} \tag{26}$$

According to the DJM, we construct the following result:

$$\begin{aligned}v(x,t) &= v_0(x,t)\\ &+J\left[\frac{1}{t^{1-\alpha}}\left(-3(v(x,t))^2\frac{\partial}{\partial x}v(x,t)+6\left(\frac{\partial}{\partial x}v(x,t)\right)^3\right.\right.\\ &+18v(x,t)\left(\frac{\partial}{\partial x}v(x,t)\right)\frac{\partial^2}{\partial x^2}v(x,t)\\ &\left.\left.+3(v(x,t))^2\frac{\partial^3}{\partial x^3}v(x,t)-\frac{\partial^5}{\partial x^5}v(x,t)\right)\right]\end{aligned} \tag{27}$$

Where J is an integral operator. Therefore, the following results are obtained:

$$\begin{aligned}v(x,0) &= \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right),\\ N(v) &= J\begin{pmatrix}\frac{1}{t^{1-\alpha}}\left(-\frac{1}{26244}(486c-6)^{3/2}\right.\\ \times\left(\cosh\left(\frac{1}{3}x\right)\right)^2\sinh\left(\frac{1}{3}x\right)\\ +\frac{1}{26244}(486c-6)^{3/2}\left(\sinh\left(\frac{1}{3}x\right)\right)^3\\ \left.-\frac{1}{4374}\sqrt{486c-6}\sinh\left(\frac{1}{3}x\right)\right)\end{pmatrix}\end{aligned} \tag{28}$$

We can obtain the iteration solution of Eq. (27) as follows:

$$\begin{aligned}v_0(x,t) &= \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right),\\ v_1(x,t) &= -\frac{1}{54}\frac{s^{\alpha}\sqrt{486c-6}\sinh\left(\frac{1}{3}x\right)c}{\alpha},\\ v_2(x,t) &= -\frac{1}{472392\alpha^4}\left(\sqrt{486c-6}c^2\right.\\ &\times\begin{pmatrix}81\cosh\left(\frac{1}{3}x\right)c^2t^{4\alpha}-1458\cosh\left(\frac{1}{3}x\right)a^2t^{2\alpha}\\ -324\sinh\left(\frac{1}{3}x\right)ct^{3\alpha}a-\cosh\left(\frac{1}{3}x\right)ct^{4\alpha}\\ +4\sinh\left(\frac{1}{3}x\right)t^{3\alpha}a\end{pmatrix}\left.\vphantom{\sqrt{c}}\right),\end{aligned}$$

The remaining components of $v_k(x,t)$ can be completely determined such that each term is determined by using Eq. (28).

So, we can have the 2th-order approximate solution of Eq. (23) as the following form:

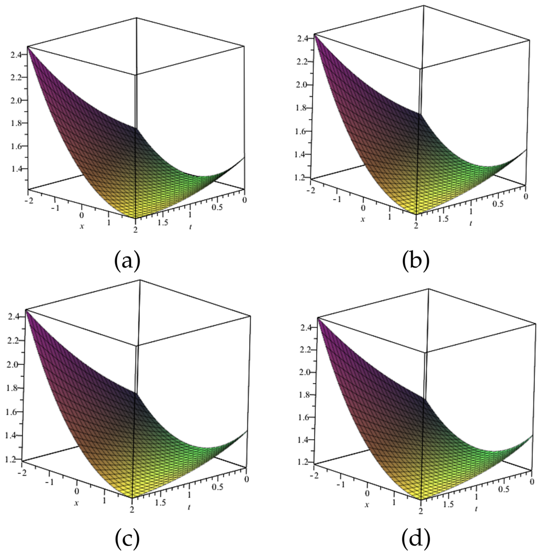

Fig. 3. Solutionof Eq. (23) using the 3-iteration of DJM for several values of α when c=1 : (a) exact (α=1) , (b) α=1 , (c) α=0.95 and (d) α=0.9 .

$$
\begin{aligned}
&\phi_2 = v_0(x,t) + v_1(x,t) + v_2(x,t) \\
&= \sqrt{\begin{aligned}&\frac{81c-1}{54}\cosh\left(\frac{x}{3}\right) - \frac{1}{54}\frac{s^{\alpha}\sqrt{486c-6}\sinh\left(\frac{1}{3}x\right)c}{\alpha} \\ &-\frac{1}{472392\alpha^4}\left(\sqrt{486c-6c^2}\left(81\cosh\left(\frac{1}{3}x\right)c^2t^{4\alpha}\right.\right. \\ &\left.\left.+4\sinh\left(\frac{1}{3}x\right)t^{3\alpha}a\right)\right).\end{aligned}}
\end{aligned} \tag{29}
$$

Fig. 3 and Table 2 show solutions of Eq. (23) for different values of α. Table 2 shows the approximate solutions of Eq. (23) for different values of α: α=0.9,α=0.95 and α=1 using only three iterations of the DJM solution (see also Fig. 4).

5. Conclusions

In this work, we have successfully expand DJM to obtain approximate solution of nonlinear dispersive K(m,n,1) equations with conformable time derivatives. Also we just calculate 2 or 3 terms of DJM. The approximate solutions obtained by the DJM are compared with exact solutions. The result shows that this method is a very formidable, effective and simple mathematical tool for solving the nonlinear conformable differential equations in the fields of science and engineering. The computations associated with examples provided here were performed using Maple 10.

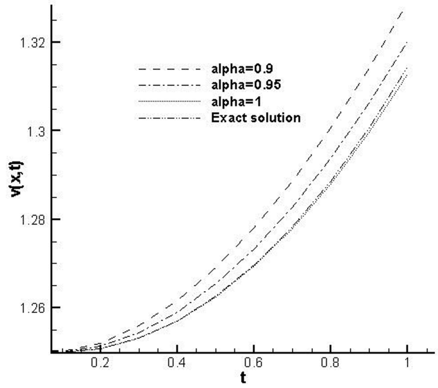

Fig. 4. Solution of Eq. (23) using the 2-iteration of DJM for different values of α when, x=0.1 .

Author Contribution: ***Guohua Fu***: Conceptualization, Methodology, Resources, Writing-original draft, Data curation, Visualization. ***Xinxin Chang***: Writing-review editing, Supervision, Validation, Software, Project administration, Funding acquisition, Software, Investigation.

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Table 2. Solution of Eq. (23) using DJM for different values of α .

x	t	α =0.9	α =0.95	α =1	Exact(α=1)
0.1	0	1.217837502	1.217837502	1.217837502	1.217837502
	0.5	1.227996487	1.227881572	1.230407157	1.233566603
	1	1.272345521	1.270578787	1.276886872	1.284287423
0.6	0	1.241585716	1.241585716	1.241585716	1.241585716
	0.5	1.217837502	1.217719728	1.217200976	1.216926932
	1	1.227996487	1.226177233	1.228818713	1.232125393
0.9	0	1.272345521	1.272345521	1.272345521	1.272345521
	0.5	1.227996487	1.227875433	1.225528561	1.223196855
	1	1.217837502	1.215962572	1.216445807	1.217347508

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