Models For The Obnoxious Facility Dispersion Problem

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Received: Sep. 29, 2023; Accepted: Mar. 04, 2024

Conventional facility location problems usually disperse facilities regardless of whether they are desirable. A recent variant emerged as the obnoxious facility dispersion problem that considers the clients influenced by the facilities. The only formulation for this problem is the obnoxious p-median problem, which resembles the p-median facility dispersion problem. Still, alternative models exist. This study presents and investigates several possible models for the obnoxious facility dispersion problem. Given that the models exhibit respective facility dispersion patterns, this study proposed to compare the models using the entropy of facility distribution.

Keywords: Obnoxious facility, Facility dispersion, 0-1 programming

http://dx.doi.org/10.6180/jase.202501_28(1).0019

1. Introduction

Economic development gives rise to facilities with various functions, capabilities, and facets. Considering costs, security, and quality of service, dispersing facilities from each other has become a critical issue, referred to as the facility dispersion problem. Kuby [1] presented models for dispersing facilities before Erkut and Neuman [2] reviewed models for locating undesirable facilities. Erkut [3] attempted to solve the discrete p dispersion problem using a heuristic approach. Erkut and Neuman [4] further presented four mathematical models with respective objective functions to form the MaxMinMin, MaxSumMin, MaxMinSum, and MaxSumSum models. Lei and Church [5] recently proposed a unified model that can replicate the four models of Erkut and Neuman [4] by changing specific parameters.

The above studies focused only on dispersing facilities from each other. However, facility dispersion should also consider the clients and those near the facilities, especially when facilities have potentially unwelcome effects. An obnoxious facility is associated with undesirable effects on the surroundings, such as nuclear power plants, incinerators, oil refineries, etc. Moon and Chaudhry [6] examined the relationship between clients and desirable/ undesirable facilities and suggested managing this part separately from the relationship among facilities. Recent studies, including Alumur and Kara [7], Batta et al. [8], and Cappanera et al. [9], also suggested that models can be impractical without considering the effects of obnoxious facilities on the clients or surrounding communities. For another example, public awareness of the detrimental impacts of global warming has sparked significant interest in renewable energy sources, making it a pertinent topic in facility location studies. Researchers like Karimulla, Ravi, et al. [10] have explored cost minimization strategies for energy production using renewable sources. Additionally, Bigler [11] has addressed the challenge of locating obnoxious facilities, such as wind turbines, which can create noise pollution, highlighting the complexity of site selection in such cases, an example of the obnoxious p-median problem (OpMP) modelled by Lin and Chiang [12].

Labbé et al. [13] presented a model for dispersing obnoxious facilities from their clients and referred to it as the obnoxious p-median problem (OpMP). Because the OpMP is highly time-consuming, Belotti et al. [14] presented a branch-and-cut method for solving the problem. Also, Colmenar et al. [15], Herrán et al. [16], Lin and Guan [17], and Mladenović et al. [18] have presented respective heuristic approaches for the OpMP. Maximizing the sum of the minimum distance from each client to the facilities, the OpMP resembles the MaxSumMin problem of Erkut and Neuman [4]. In this regard, the current study presents four additional models for dispersing obnoxious facilities on the basis of the studies of Erkut and Neuman [4] and Lei and Church [5]. Numerical examples showed that the models exhibit diverse facility dispersing patterns. Therefore, the current study devised an entropy index to measure the models' facility dispersion performance.

The paper's organization is as follows. Section 2 reviews the models for the nonobnoxious facility dispersion problem. Section 3 presents the obnoxious models: the obnoxious MaxMinMin, MaxMinSum, MaxSumMin, MaxSumSum, and unified model. Section 4 compares the models using random numeric examples. Section 5 provides a simple constraint to remedy the facility aggregation problem, and finally, Section 6 concludes the paper and acknowledgment.

2. Facility dispersion models

The objective functions of Erkut and Neuman [4] consist of two tiers. The lower tier aggregates the distance from one facility to the other facilities. The higher tier then maximizes an aggregation of the lower-tier measure. The aggregation function in the two tiers can be sum or min. The combinations of two tiers and two aggregation functions form four models: the MaxSumMin, MaxMinMin, MaxSumSum, and MaxMinSum models.

Let J denote the set of candidate sites. The $0-1$ variable $y_j = 1$ indicates that site j is selected to locate a facility; otherwise, $y_j = 0$. Notably, "facility" and "selected site" are synonyms, especially regarding models. The MaxSumMin model can be formulated as follows to maximize the sum of Z_j, the minimum distances from facility j to the other facilities.

Model: MaxSumMin.

$$\text{Max} \sum_{j \in J} Z_j \tag{1}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z_j \leq M y_j \quad \forall j \in J \tag{3}$$

$$Z_j \leq d_{jk} + M\left(1 - y_k\right) \quad \forall j, k \in J \tag{4}$$

$$y_j \in \{0, 1\}$$

Constraint (2) requires that the number of selected sites equals p. Constraint (3) stipulates that $Z_j = 0$ if site j has no facility. The term d_{jk} in constraint (4) denotes the distance between sites j and k, and M is a very large positive number. d_{jk} is an upper bound for Z_j when $y_k = 1$. Moon and Chaudhry [6] referred to this model as the p-defense problem.

The MaxMinMin model is as follows.

Model: MaxMinMin.

$$\text{Max } Z \tag{5}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z \leq d_{jk} + M\left(1 - y_j\right) + M\left(1 - y_k\right) \quad \forall j < k \in J \tag{6}$$

$$y_j \in \{0, 1\}$$

The notation MaxMinMin means that the objective is to maximize the minimum of the minimal distances from each facility to the other facilities, namely, to maximize the minimal distance between any two facilities. Constraint (6) specifies that d_{jk} is an upper bound for Z_j when $y_j = y_k = 1$. Shier [19] referred to this model as the p dispersion problem.

The MaxSumSum model maximizes the sum of the distances between all facility pairs, referred to as the p-dispersion sum model by Kuby [1]. The model is as follows.

Model: MaxSumSum.

$$\text{Max} \sum_{j \in J} Z_j \tag{1}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z_j \leq M y_j \quad \forall j \in J \tag{3}$$

$$Z_j \leq \sum_{k \in J} d_{jk} y_k \quad \forall j \in J \tag{7}$$

$$y_j \in \{0, 1\}$$

Constraint (3) and (7) together specifies that the sum of the distances from facility j to the other facilities is an upper bound for Z_j.

The MaxMinSum model maximizes the minimum distance sum from each facility to the other facilities and can be formulated as follows.
Model: MaxMinSum.

$$\text{Max } Z \tag{5}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z \leq \sum_{k \in J} d_{jk} y_k + M\left(1 - y_j\right) \quad \forall j \in J \tag{8}$$

$$y_j \in \{0,1\}$$

Constraint (8) stipulates that the sum of the distances from facility j to the other facilities is an upper bound for Z when site j is selected; otherwise, Z is irrelevant to site j. Lei and Church [5] noted that this model is much less studied than other dispersion models.Based on the method of Ogryczak and Tamir [20] for minimizing the sum of the K largest functions, Lei and Church [5] proposed a unified model that can replicate the four models of Erkut and Neuman [4] and beyond. The model of Lei and Church [5] can be formulated as follows:
Model: Unified Model.

$$\text{Max } Kt - \sum_{j \in J} u_j \tag{9}$$

Subject to:

$$\sum_{j \in J} y_j = p, \tag{2}$$

$$t - u_j \leq q_j \quad \forall j \in J \tag{10}$$

$$q_j = (L+1)s_j - \sum_{k \in J} z_{jk} \quad \forall j \in J \tag{11}$$

$$s_j - z_{jk} \leq d_{jk} + M\left(2 - y_j - y_k\right) \quad \forall j,k \in J \tag{12}$$

$$u_j, z_{jk} \geq 0, y_j \in \{0,1\}$$

Associated with auxiliary variables t and u_j, K determines the number of smallest partial sums at the higher tier. K ranges from 1 to $|J|$.$K = 1$ equals the min function, while $K = |J|$ represents the sum function. u_j is non-zero if facility j is associated with one of the K smallest partial sums; otherwise, $u_j = 0$.

Similarly, associated with auxiliary variables s_j and z_{jk}, L determines the number of smallest partial sums at the lower tier. L ranges from 1 to $|J| - 1$.$L = 1$ equals the min function, while $K = |J| - 1$ represents the sum function. Like u_j, z_{jk} is non-zero when facility k is involved in the $L+1$ closest facility to facility j; otherwise, $z_{jk} = 0$. Notably, the 1 of $L+1$ in constraint (11) represents facility j itself because the zero distance to itself is included.

3. Obnoxious facility dispersion models

The OpMP of Labbé et al. [13] might be the first to focus on the impact of facilities on their clients. Notably, although Labbé et al. used the term client and assumed that the facilities serve clients, those affected by facilities can be others than clients. With J denoting the set of facilities, let I denote the set of clients, $I \cap J = \varnothing$. The OpMP model can then be formulated as follows.
Model: OpMP

$$\text{Max} \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \tag{13}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$y_j + \sum_{k \in S_{ij}} x_{ik} \leq 1 \quad \forall i \in I, \forall j \in J \tag{14}$$

$$x_{ij} \leq y_j \quad \forall i \in I, \forall j \in J \tag{15}$$

$$y_j, x_{ij} \in \{0,1\}, \forall i \in I, \forall j,k \in J$$

Each client and facility pair is associated with a distance d_{ij}. The $0-1$ variable $x_{ij} = 1$ indicates that client i is served by facility j; otherwise, $x_{ij} = 0$. Each client is assumed to be served by its nearest facility. For each site j, a set S_{ij} records the sites whose distance to client i exceeds d_{ij}, namely,

$$S_{ij} = \left\{k \in J \mid \left(d_{ik} > d_{ij}\right) \vee \left(d_{ik} = d_{ij} \wedge k > j\right)\right\} \tag{16}$$

Constraint (14) stipulates that $x_{ij} = 0$ when $y_j = 1$, namely, facilities in S_{ij} cannot serve client i. The effect of S_{ij} is that every client can only be served by its nearest facility. Constraint (15) stipulates that $x_{ij} = 0$ when $y_j = 0$, that is, unselected sites serve no clients. Consequently, the model OpMP maximizes the sum of the minimum distances from the clients to their respective nearest facilities.

The OpMP involves considerable $0-1$ variables to indicate which facility serves which client. By eliminating the $0-1$ variable x_{ij}, Chiang and Lin [21] presented a compact model for the OpMP and showed its efficiency. In

the current study, the compact model is referred to as the obnoxious MaxSumMin model, denoted as OMaxSumMin, to distinguish it from the MaxSumMin model of Erkut and Neuman [4]. The OMaxSumMin model can be formulated as follows.
Model: OMaxSumMin.

$$\text{Max} \sum_{i \in I} Z_i \tag{17}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z_i \leq d_{ij} + M\left(1 - y_j\right) \quad \forall i \in I \wedge \forall j \in J \tag{18}$$

$$y_j \in \{0, 1\}$$

Furthermore, Lin and Chiang [12] presented an even more efficient formulation for the OMaxSumMin model.

Given the similar OMaxSumMin model to the MaxSumMin model, other similar models can be formulated based on the framework of Erkut and Neuman [4]. The OMaxMinMin model can be formulated as follows.
Model: OMaxMinMin.

$$\text{Max } Z \tag{19}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z \leq d_{ij} + M\left(1 - y_j\right) \quad \forall i \in I \wedge \forall j \in J \tag{20}$$

$$y_j \in \{0, 1\}$$

Constraint (20) stipulates that d_{ij} is an upper bound for objective Z if site j is selected.

As a result, Z is bounded by the minimum of all d_{ij} with $y_j = 1$.

Next, the OMaxSumSum model is as follows to maximize the distance sum from all clients to all facilities.
Model: OMaxSumSum.

$$\text{Max} \sum_{i \in I} Z_i \tag{17}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z_i \leq \sum_{j \in J} d_{ij} y_j \quad \forall i \in I \tag{21}$$

$$y_j \in \{0, 1\}$$

Constraint (21) bounds Z_i by the distance sum from client i to all facilities.

The OMaxMinSum model can be formulated as follows to maximize the minimum distance sum from a client to the facilities.
Model: OMaxMinSum.

$$\text{Max } Z \tag{19}$$

Subject to:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$Z \leq \sum_{j \in J} d_{ij} y_j \quad \forall i \in I \tag{22}$$

$$y_j \in \{0, 1\}$$

Constraint (22) requires that each client's distance sum to all facilities be an upper bound for objective Z.

Finally, the unified obnoxious model can be formulated as follows.
Model: O-Unified Model.

$$\text{Max } Kt - \sum_{i \in I} u_i \tag{23}$$

Subject to:

$$t - u_i \leq q_i \quad \forall i \in I \tag{24}$$

$$q_i = Ls_i - \sum_{j \in J} z(i, j) \quad \forall i \in I \tag{25}$$

$$s_i - z(i, j) \leq d(i, j) + M\left(1 - y_j\right) \quad \forall i \in I, j \in J \tag{26}$$

$$\sum_{j \in J} y_j = p \tag{2}$$

$$u_i, z(i, j) \geq 0, y_j \in \{0, 1\}, \forall i \in I, j \in J$$

The unified model of Lei and Church [5] also has an obnoxious version: the O-Unified model. With constraints (24) and (25), the upper tier of the O-Unified model deals with the K largest partial sums of distances from individual clients to the facilities. At the lower tier, constraint (26) determines the L largest distances from a client to the facilities. Because the closest facilities to a facility always include the facility itself; therefore, the coefficient of s_j in constraint (11) needs to be $L + 1$. However, in the obnoxious version, constraint (25) determines the closest facilities to a client rather than a facility. The coefficient of s_i is L. Thus, one can obtain intermediate solutions by adjusting K and L apart from those obtained using the four obnoxious models.

Existing studies have focused exclusively on the OpMP, namely, the OMaxSumMin model. The other obnoxious models have never been investigated. Meanwhile, Appendix A shows that the OMaxMinMin and the OMaxSumSum are trivial problems that can be easily solved with sorting. Furthermore, the models exhibit distinct facility dispersion patterns critical for facility dispersion.

4. Numerical tests

The obnoxious models use various objective functions to disperse facilities from their clients. However, the status of facility dispersion is also critical. This section investigates the facility dispersion characteristics of the four fundamental obnoxious models. The unified model is excluded from the comparison because it renders featureless blended outcomes.

This study used four classes of test problems with $|I| = 100, 200$ and $|J| = 50, 100$, respectively. Thirty random cases were generated for each class by randomly locating clients and facilities in a square of 10000 by 10000 . The cardinality p was set to 10 for all problems. The problems were solved using Gurobi 8.1.1 on a personal computer with an Intel Core i7 CPU and 16 GB RAM. The main objective of this study is to analyze potential differences among the four model solutions based on experimental outcomes. To achieve this goal, the scope of the data instances was limited to sizes that allowed the exact model approaches to generate outcomes. Previous research by Chiang and Lin [21] revealed that for larger instances, the time complexity increases significantly for larger instances, sometimes leading to an inability to find an optimal solution within the 24-hour timeout constraint. Consequently, this study utilized instances with the setout parameter values, to ensure a sufficient sample size for statistical hypothesis comparison. It is worth noting that the development of efficient metaheuristic algorithms could enable the testing of larger data instances in future investigations.

This study devised an entropy metric to gauge the degree of facility dispersion. When the problem landscape is partitioned into grids, the entropy of facility dispersion can be defined as follows.

$$\varepsilon = \begin{cases} -\sum_k p_k \ln(p_k) & p_k > 0 \\ 0 & p_k = 0 \end{cases} \tag{27}$$

where p_k is the portion of facilities located in grid k. Notably, the p_k is a different notation from the cardinality p. High entropy signifies high facility dispersion. However, grids too few cause multiple facilities to be located in one grid, resulting in narrowly ranged low entropies. On the other hand, when grids are plenty, high entropies can easily achieve because most grids have only one or zero facilities. Both situations are unbeneficial for comparing models. Therefore, the grid number had better be close to the cardinality p. Sixteen grids appeared suitable in this study when p was set to 10 . The entropy maximum is $-10 \times 0.1 \times \ln(0.1) \approx 2.302585$ when each grid has zero or precisely one facility.

The application of the principle of maximum entropy to represent the performance of a specified problem model is not novel, for example, Singh, Tiwari, et al. [22] applied the principle of maximum entropy to measure the performance of queueing systems using generalized entropy. This illustrates a relevant application of maximum entropy principles in problem modeling and performance evaluation. Table 1 shows the mean entropies and associated standard deviations of the four obnoxious models, also presented with box and whisker plots in Figs. 1 to 4. The results exhibit a consistent pattern: the OMaxMinMin model's box is at the highest, while the OMaxSumMin model's box is at the lowest. Specifically, the OMaxMinMin model performed best to disperse facilities evenly over the grids. In contrast, the OMaxSumMin model performed the worst for tending to gather facilities in a few grids.

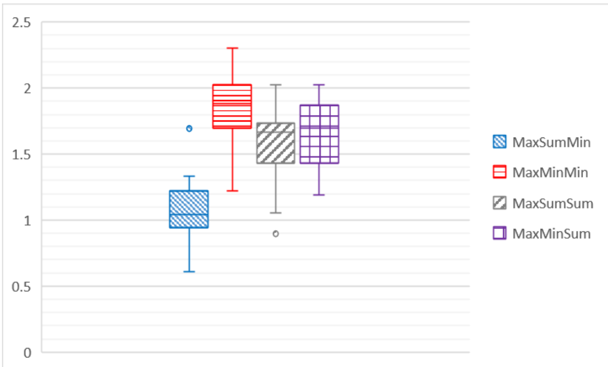

Fig. 1. Box and whisker plot for the 100-client-50-facility problem

A pairwise t-test was used to verify whether the entropy differences were statistically significant. Table 2 shows the p-values of the paired t-tests. The p-values were all under 0.05, signifying that the obnoxious models are significantly different.

Fig. 5 illustrates the typical facility scattering pattern of the obnoxious models. The grey dots are candidate sites. The strong repelling force of the OMaxSumMin model

Table 1. Mean entropy with standard deviation (in bracket) by the obnoxious models

Client	Facility	OMaxSumMin	OMaxMinMin	OMaxSumSum	OMaxMinSum
100	50	1.021(0.268)	**1.832**(0.238)	1.567(0.2801)	1.654(0.224)
100	100	0.412(0.321)	**1.673**(0.305)	1.171(0.2841)	1.366(0.197)
200	50	1.073(0.213)	**1.948**(0.142)	1.518(0.2577)	1.655(0.202)
200	100	0.599(0.236)	**1.760**(0.229)	1.251(0.2490)	1.413(0.111)

Table 2. P-values of paired t-test

Client		Facility					
		50			100		
		OMaxMinMin	OMaxSumSum	OMaxMinSum	OMaxMinMin	OMaxSumSum	OMaxMinSum
100	OMaxSumMin	0.000	0.000	0.000	0.000	0.000	0.000
	OMaxMinMin		0.000	0.003		0.000	0.000
	OMaxSumSum			0.025			0.002
200	OMaxSumMin	0.000	0.000	0.000	0.000	0.000	0.000
	OMaxMinMin		0.000	0.000		0.000	0.000
	OMaxSumSum			0.001			0.002

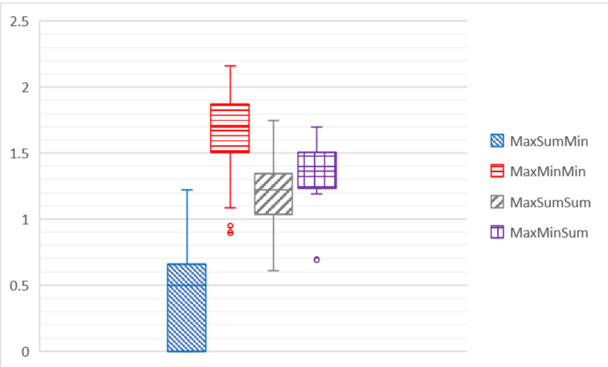

Fig. 2. Box and whisker plot for the 100-client-100-facility problem

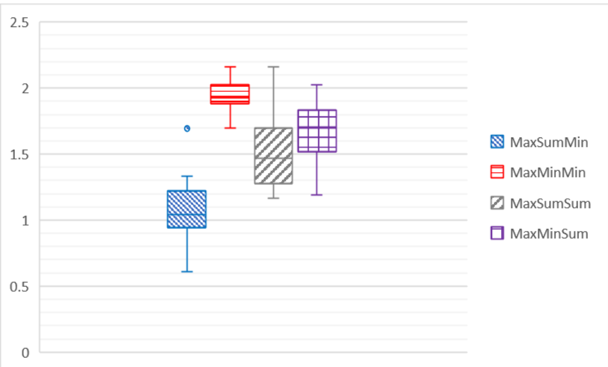

Fig. 3. Box and whisker plot for the 200-client-50-facility problem

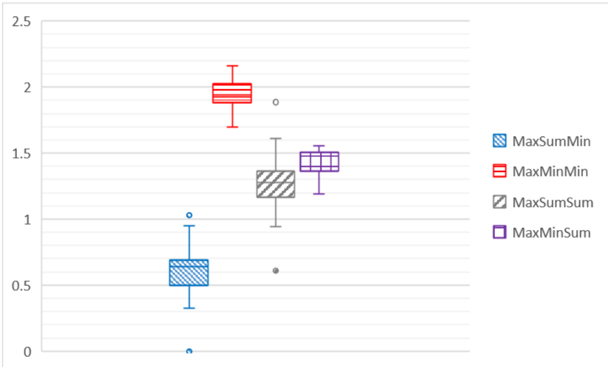

Fig. 4. Box and whisker plot for the 200-client-100-facility problem

pushed the facilities to one corner. The OMaxSumSum and OMaxMinSum models had weaker repelling force and pushed the facilities to the corners. The last two models had entropies higher than the OMaxSumSum model. However, these dispersion outcomes are unfavorable. The OMaxMinMin model performed best in distributing the facilities evenly over the landscape and thus had the highest entropy. However, none of the models can prevent facilities from local aggregation.

Erkut and Neuman [4] have noted that the MaxMinMin and MaxSumMin models chose sites at both the edges and the middle of the landscape. However, the OMaxMinSum and OMaxSumSum models prefer sites close to the boundaries or the corners. However, the existence of clients seemed to change the model behavior. The OMaxMinMin model remains the best, whereas the OMaxSumMin model turns out to be the worst. Meanwhile, the OMaxMinSum and OMaxSumSum models remained to locate facilities at the corners.

5. A remedy to facility aggregation

The objective functions of the obnoxious models focus only on the distances between clients and facilities, causing them to have different degrees of facility aggregation. A remedy,

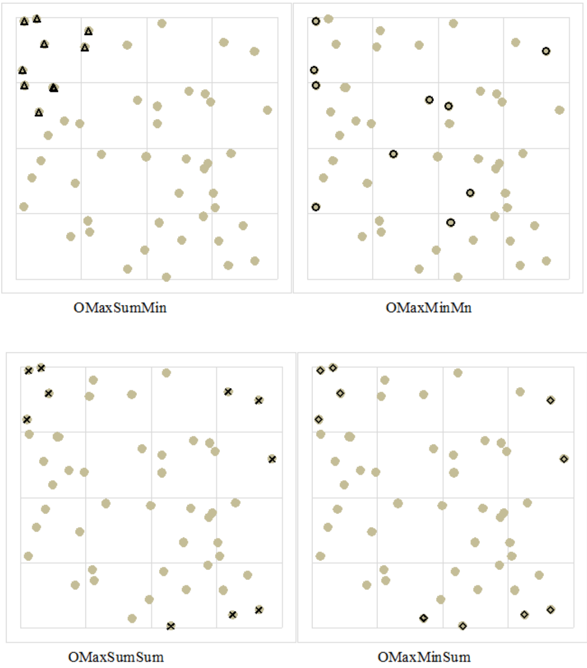

Fig. 5. Typical facility scattering patterns by various models

such as an additional objective function, might be necessary to prevent the facilities from aggregating.

Colmenar et al. [23] and Sánchez-Oro et al. [24] have recently attempted to use heuristic algorithms to solve a bi-objective OpMP. The first is the OMaxSumMin objective, and the second is the MaxSumMin objective. However, they did not explain why the second objective is the MaxSumMin. The first and second objectives, respectively, have at least four options. There exist at least 16 combinations for the biobjective problem. Furthermore, multi-objective $0-1$ programming problems are known to be difficult and time-consuming. The issue needs further in-depth studies.

Instead, a simple approach is to impose a distance constraint between facilities, such as the following.

$$d_{jk} + M\left(1-y_j\right) + M\left(1-y_k\right) \geq D \quad j < k \wedge \forall j,k \in J \tag{28}$$

In numerical examples to test its effects, the D was set to 1000,2000 , and 3000 , considering the landscape of the test problems. Table 3 shows the mean entropies by the obnoxious models with constraint (28). As expected, the facility dispersion increased with D. However, the solutions by the models become indifferent when a large D highlights the relationship between facilities more than between clients and facilities. Notably, some test problems were infeasible with $D = 3000$. Overall, the OMaxMinMin still performed the best.

Meanwhile, Table 4 shows the mean computation time by the obnoxious models with constraint (28). Although no longer trivial, the OMaxMinMin and the OMaxSumSum were still the most efficient, whereas the OMaxSumMin remained the least efficient. A considerable D value diminished the computation time because the solution spaces were reduced. Constraint (28) helps to retain a certain level of facility dispersion. Nevertheless, the D value depends on the decision-maker's careful determination.

6. Conclusions

This study investigates a class of models for the obnoxious facility dispersion problem that aims to disperse facilities from their clients, which differs from classic facility location problems. The models were compared in terms of their facility-dispersing capability. An entropy metric is devised to gauge facility dispersion. Preliminary results show that the OMaxSumMin model, the most studied in the literature, demonstrated disappointing performance in dispersing facilities. Meanwhile, the OMaxMinMin model performed the best.

To summarize, this study formulated new and novel models for different objectives of minimize the overall obnoxiousness of the clients, by maximizing the distances between the selected facilities from the clients. Among the four different facility-client dispersion models formulated in the study, only the OpMP has been formulated and studied intensively in the literature. Since there are no previous studies comparing these four models, this work could make a significant contribution to the field.However, simply dispersing facilities away from clients causes facilities to aggrege, which is impractical and undesirable. A remedy measure is essential for preventing the facilities from aggregating. Multi-objective formulations can be helpful, but at the same time, they are highly complicated. Further in-depth studies are required to verify their applicability. This study adopted a simple approach to avoid facility aggregation: imposing a minimum distance constraint D between facilities. Additional numerical tests show that the constraint helps to increase facility dispersion. However, the models can be infeasible, and the solutions become indifferent when D is overly large. The OMaxMinMin still performed the best, whereas the OMaxSumMin remained the worst performer. The only drawback is that D depends on the decision-maker's careful determination.

The importance of this research lies in understanding the potential issues that may arise when using different models in practice. The structures of optimal facility dispersion generated by these four different mathematical models

Table 3. Mean entropy with standard deviation by the obnoxious models with a facility distance constraint

Client	Facility	*D*	OMaxSumMin	OMaxMinMin	OMaxSumSum	OMaxMinSum
100	50	1000	1.463(0.179)	2.019(0.139)	1.886(0.147)	1.911(0.179)
		2000	2.059(0.117)	2.233(0.087)	2.181(0.122)	2.196(0.087)
		3000	2.291(0.039)	2.297(0.028)	2.303(0.000)	2.297(0.028)
100	100	1000	1.278(0.125)	**1.996**(0.195)	1.644(0.185)	1.682(0.186)
		2000	1.910(0.127)	2.192(0.099)	2.077(0.180)	2.063(0.176)
		3000	2.298(0.025)	2.298(0.025)	2.289(0.042)	2.289(0.042)
200	50	1000	1.429(0.168)	**2.050**(0.140)	1.851(0.180)	1.867(0.183)
		2000	2.063(0.144)	2.201(0.096)	2.176(0.116)	2.195(0.113)
		3000	2.303(0.000)	2.303(0.000)	2.303(0.000)	2.303(0.000)
200	100	1000	1.299(0.157)	2.019(0.157)	1.721(0.161)	1.728(0.162)
		2000	2.173(0.096)	2.148(0.125)	1.932(0.082)	2.107(0.120)
		3000	2.293(0.035)	2.298(0.025)	2.298(0.025)	2.293(0.035)

Table 4. Mean computation time (s) with standard deviation by the obnoxious models

Client	Facility	*D*	OMaxSumMin	OMaxMinMin	OMaxSumSum	OMaxMinSum
100	50	0	6.50(1.90)	0.01(0.03)	0.00(0.00)	0.04(0.05)
		1000	11.00(4.41)	0.00(0.00)	0.00(0.00)	0.04(0.05)
		2000	24.09(9.83)	0.05(0.00)	0.00(0.00)	0.01(0.03)
		3000	0.02(0.04)	0.00(0.02)	0.00(0.00)	1.06(0.85)
100	100	0	13.17(7.28)	0.10(0.00)	0.00(0.00)	0.05(0.06)
		1000	153.93(102.81)	0.10(0.00)	0.00(0.02)	0.11(0.03)
		2000	758.86(566.29)	0.10(0.00)	0.00(0.00)	0.11(0.03)
		3000	37.84(18.30)	0.11(0.03)	0.00(0.00)	0.10(0.00)
200	50	0	24.55(13.00)	0.02(0.04)	0.00(0.00)	0.05(0.05)
		1000	33.84(17.48)	0.10(0.00)	0.00(0.00)	0.05(0.05)
		2000	68.18(31.24)	0.10(0.00)	0.00(0.00)	0.05(0.05)
		3000	4.79(3.32)	0.10(0.00)	0.00(0.00)	0.09(0.04)
200	100	0	48.02(26.67)	0.10(0.02)	0.00(0.00)	0.11(0.08)
		1000	463.80(317.82)	0.20(0.00)	0.00(0.00)	0.14(0.08)
		2000	2727.21(1433.82)	0.20(0.00)	0.00(0.00)	0.12(0.05)
		3000	112.40(58.25)	0.20(0.02)	0.00(0.00)	0.24(0.09)

are very different as observed on the typical facility scattering patterns in Fig. 5. The OMaxSumMin model presents a solution where factories are concentrated in a specific offset area. In contrast, the QMaxMinMin model's solution is to search for open areas to place factories. The solutions of the OMaxSumSum and the OMaxMinSum models involve allocating factories in the peripheral open areas. These different patterns reflect the various strategies and outcomes that may arise when optimizing facility dispersion. This can help users of these models avoid the shortcomings of the models. This study further validates their performance and explore the facility-facility dispersion aspect each model.

This study also proved that the OMaxMinMin and OMaxSumSum models are trivial problems without the facility distance constraint. Notably, although the OMaxMinMin model performed the best, it is not necessarily the best choice for all decision-makers. Model selection is always up to the decision-maker.

Based on the observations made in improving the facility-facility dispersion for the facility-client dispersion solutions, it is evident that these two objectives are not complementary and optimizing in terms of one property will lead to compromises in the other. This can be a significant challenge in designing effective solutions, therefore, future studies to understand and manage such conflicts is crucial for developing robust and balanced solutions is a most in the dispersion of obnoxious facilities, therefore multi-objectives optimization models and suitable heuristic algorithms to effectively solving the problem is needed.

Acknowledgments

This work was supported by the National Science and Technology Council, Taiwan. Grand: MOST 109-2410-H-182-013

Appendix A

Unlike the non-obnoxious models, the obnoxious models focus on the distances between determinate clients and undetermined facilities. This fundamental difference leads the OMaxSumSum and the OMaxMinMin models to be trivial problems.

Proposition 1. *The OMaxMinMin model is a trivial problem.*

Proof of Porposition 1: Let $m_j = \min\left\{d_{ij}, i \in I\right\}$*. Constraint (20) stipulates that* $Z \leq m_j$ *when* $y_j = 1$*; otherwise, Z is unbounded by* m_j *when* $y_j = 0$*. Because* m_j *is determinate, the optimal objective value for the OMaxMinMin model would be the p largest* m_j*.* **QED**

Proposition 2. *The OMaxSumSum model is a trivial problem.*

Proof of Porposition 2: Let $Z_j = \sum_{i\in I} d_{ij}y_j$*. Then* $Z_j = \sum_{i\in I} d_{ij}$ *if* $y_j = 1$*; otherwise,* $Z_j = 0$*. Constraint (22) stipulates that* $Z \leq \sum_{j\in J} Z_j y_j$*. Because* Z_j *is determinate, the optimal objective value for the OMaxSumSum model would be the sum of the p largest* Z_j*.* **QED**

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