A New Method For Ranking Interval-valued Intuitionistic Trapezoidal Fuzzy Sets

S.N. Murty Kodukulla* and V. Sireesha

Department of Mathematics, GITAM (Deemed to be University), Visakhapatnam, India

*Corresponding author. E-mail: snmurtyk@gmail.com

Received: Jun. 30, 2024; Accepted: Aug. 10, 2024

An Interval-valued intuitionistic trapezoidal fuzzy set (IVITrFS) is a powerful tool for modelling uncertainty. The ranking of IVITrFS plays a vital role in fuzzy set theory to compare and analyze the given information. An IVITrFS is a special type of Intuitionistic Fuzzy Set (IFS) and interval-valued intuitionistic fuzzy set (IVIFS) with a consecutive domain of real numbers. The existing ranking methods are good at ranking, but there are some cases in which the existing methods fails to rank effectively, and hence there is a need for a new ranking method. With this objective, the proposed ranking method is derived in this study. In this paper, we proposed a new method for ranking IVITrFS from a geometric point of view by defining the improved score function using the concept of centroids. The comparative results shows that the proposed method is innate and effective, very useful to computational Intelligence, decision-making, predictive system analysis, and performance analysis.

Keywords: Rankin of fuzzy sets; interval-valued intuitionistic trapezoidal fuzzy sets(IVITrFS); improved score function

http://dx.doi.org/10.6180/jase.202506_28(6).0019

1. Introduction

The fuzzy set theory was introduced by Zadeh in 1965. Intuitionistic fuzzy sets were proposed by Atanassov after he generalized the concept of a fuzzy set. Chang and Zadeh investigated fuzzy logic and their arithmetic operations first, and it is well-established that fuzzy numbers can be ranked and that fuzzy numbers can be used to make decisions in an expected and unexpected situation.

The trapezoidal intuitionistic fuzzy number (TrIFN) was first suggested by Wang in 2008. The fields IFSs and IVIFSs are discernible sets, similar to regular fuzzy sets. Additionally, the discrete form of IFN is extended to the continuous form by the domains TIFN, TrIFN, and IVITrFN. The membership, non-membership function and hesitation degree are the characteristics of an IVITrFS, which effectively describes the decision-making information with various dimensions and units.

Compared to the TIFN and TrIFN, and IVTIFS is better at capturing ambiguity and uncertainty. It is important to note that IVITrFSs are significant in both scientific research and practical applications. IVITrFSs was first described by Wan along with their mathematical operations. Novel score and accuracy expectation functions for IVITrFSs were defined by Wu and Liu in 2013.

When dealing with intuitionistic fuzzy decision-making and Intuitionistic Fuzzy clustering, the systematic ranking of intuitionistic fuzzy numbers plays a crucial task. A trapezoidal intuitionistic fuzzy number (TrIFN) is defined by Wang and Zhang, followed by an interval number ranking method that transforms the ranking of TrIFNs into an interval number ranking.

The ranking of IFNs was introduced by Mitchell in addition to Wang and Zhang, as a functional categorical with a generalization of Chen and Hwang's scoring for a new ranking IFNs system, Nayagam et al. proposed a new technique for scoring IFNs of a special type. Rank-based decision-making is an essential component of many applications involving interval-valued INFs. Score and performance accuracy functions were proposed by Xu and Chen. An interval-valued intuitionistic fuzzy number is also ranked according to an accuracy function proposed by Ye.

Score functions and accuracy functions cannot provide enough information about alternatives for ranking interval valued intuitive fuzzy numbers in a few instances. A Ratio Ranking method for solving multi attribute functional decision-making problems was developed by Li et.al using triangular INFs and the value index and undefined index. In this paper, we have simplified the concept of TRIFNs so that they are easier to understand and have a natural interpretation, even though TRIFNs have been established previously.

Centroid is the balancing point of any geometric representation. Several researchers proposed various ranking methods by using centroids in fuzzy sets, IFS, IVIFS, and in many other domains. In this research article, we are mainly focused to propose a new ranking method for IVITrFS from a geometric viewpoint, improved score function of IVITrFSs, by using the concept of centroids.

2. Literature survey

Zadeh [1] invented the fuzzy set theory, which is currently utilized to characterize a wide variety of applications across a wide range of fields. In fuzzy sets, the degree to which an element participates in the set is denoted by a number in the interval [0,1], whereas the degree to which it does not participate is denoted by the complement of that element. In spite of this, the concept runs counter to human intuition when put into reality.

Since that time, a number of other researchers have carried out considerable and in-depth study on fuzzy set theory, and a great many different kinds of fuzzy sets have been developed. Atanassov and Atanassov [2] presented the idea of the Intuitionistic Fuzzy Set (IFS), which is a generalization of what is known as the fuzzy set [3]. According to Bustince and Burillo [4], Gau and Buehrer [5] first proposed the idea of a nebulous set, which is conceptually comparable to an IFS. Atanassov and Atanassov [6] created the concept of Interval-Valued Fuzzy Set (IVFS) by generalizing the IFS. This concept combines the IFS concept with the notion of interval valued fuzzy set (IVIFS), which was first introduced by Atanassov and Atanassov [6]. IFSs and IVIFSs have been put to use in a wide variety of fields, including optimization, decision-making, supplier selection, and investment alternatives, to name a few of these applications' many additional applications. One particular example of a fuzzy set is one that contains fuzzy numbers.

An Intuitionistic fuzzy number, also known as an IFN, is a generalization of fuzzy numbers [7] that appears to sufficiently explain a quantity that is unknown. Along the same lines as fuzzy numbers, Shu et al. [8] devised an algorithm for intuitionistic fuzzy fault tree analysis and proposed a triangular intuitionistic fuzzy number (TIFN). The design of the four arithmetic operations over the TIFNs in Shu et al. [8] contained a number of errors, which Li [9] identified and then fixed. Wang [10] presented Intuitionistic Trapezoidal Fuzzy Numbers (TrIFN), in which membership and non-membership values are intervals rather than exact numbers, and Intuitionistic Trapezoidal Fuzzy Numbers (ITFN) were also presented. The weighted arithmetic average operator of IVTIFSs, in addition to a couple of its operational concepts, were described by Wan [11].

There is a method that ranks IVTrIFSs based on the score function as well as the accuracy function, and it is explained here. The interval-valued trapezoidal intuitionistic fuzzy geometric aggregation operations that Wu and Liu [12] developed were applied to Multi Attribute Group Decision Making IVITrFSs.

An IVITrFS is essentially a powerful tool for describing the information relevant to these three facets, but it does so by specifying three characteristic functions: the membership function, the non-membership function, and the hesitation degree. These functions are listed in the previous sentence. In terms of its capacity to communicate ambiguity and uncertainty, IVITrFS is more effective than both TIFN and TrIFN. The IVITrFS is able to provide a description of information relevant to decision-making that incorporates various dimensions and units. Therefore, IVITrFSs are extremely important for both theoretical and applied study in the scientific community. Ranking was developed for IVITrFSs by Wan [11], Wu and Liu [12], who also investigated arithmetic and geometric aggregation operators for them. The fundamental goal of this study is to offer a new ranking approach for IVITrFSs that is based on an easy-to-understand, uncomplicated, and effective score function. Konwar et al. [13] defined some new contraction mappings and establish fixed point theorems in a complete intuitionistic fuzzy normed linear spaces. MurtyKodukulla et al. [14] proposed a modified Euclidean distance measure for IVITrFS to the existing Euclidean distance measure and proposed the Jaccard distance measure. Konwar and Debnath [15] introduced the notion of continuous linear operators in IFnNLS and

Furthermore, they establish the uniform continuity theorem and Banach's contraction principle in an IFnNLS. Debnath [16] introduced the notion of lacunary ideal convergence in intuitionistic fuzzy normed linear spaces as a variant of the notion of ideal convergence. Mallika and Sireesha [17] proposed a new ranking method based on the Dice similarity metric and an IVTrIF TOPSIS method is proposed.

Ranking is a challenging task, and every ranking method has its own significance and applicability. Several researchers proposed various ranking methods, and there is no unique ranking method that is suitable for all kinds of applications. The existing ranking methods are good at ranking, but there are some cases in which the existing methods fails to rank effectively, one of the cases is discussed in comparison between ranking methods (Example 4.2), and hence there is a need for a new ranking method. With this objective, the proposed ranking method is derived in this study. Further numerical examples were discussed with a comparative analysis to show the effectiveness of the proposed method over various existing ranking methods.

The structure of this paper is as follows. In section 3, the IVITrFS definition and existing ranking methods are presented. In section 4, a novel IVITrFS ranking method based on an improved score function is proposed and a comparative analysis is presented to demonstrate that the proposed method is more efficient than existing methods.

3. Preliminaries

Here we briefly define the interval-valued intuitionistic trapezoidal fuzzy set.

3.1. Definition: Interval-valued Intuitionistic Trapezoidal Fuzzy Set (IVITrFS)

Let A be an IVITrFS; its membership function is given by

$$
\begin{aligned}
m_A^U(\mathrm{x}) &= \frac{x-a}{b-a} \quad m_A^U && \text{for a} \leq \mathrm{x} \leq \mathrm{b} \\
&= m_A^U && \text{for b} \leq \mathrm{x} \leq \mathrm{c} \\
&= \frac{d-x}{d-c} m_A^U && \text{for c} < \mathrm{x} \leq \mathrm{d} \\
&= 0 \text{ otherwise.}
\end{aligned}
$$

$$
\begin{aligned}
m_A^L(\mathrm{x}) &= \frac{x-a}{b-a} \quad m_A^L && \text{for a} \leq \mathrm{x} \leq \mathrm{b} \\
&= m_A^L && \text{for b} \leq \mathrm{x} \leq \mathrm{c} \\
&= \frac{d-x}{d-c} m_A^L && \text{for c} < \mathrm{x} \leq \mathrm{d} \\
&= 0 && \text{otherwise .}
\end{aligned}
$$

Its non-membership function is given by

$$
\begin{aligned}
n_A^U(\mathrm{x}) &= \frac{b-x+n_A^U(\mathrm{x}-\mathrm{a})}{b-a} && \text{for a} \leq \mathrm{x} < \mathrm{b} \\
&= n_A^U && \text{for b} \leq \mathrm{x} \leq \mathrm{c} \\
&= \frac{x-c+n_A^U(\mathrm{d}-\mathrm{x})}{d-c} && \text{for c} < \mathrm{x} \leq \mathrm{d} \\
&= 0 && \text{Otherwise.} \\
n_A^L(\mathrm{x}) &= \frac{b-x+n_A^L(\mathrm{x}-\mathrm{a})}{b-a} && \text{for a} \leq \mathrm{x} < \mathrm{b} \\
&= n_A^L && \text{for b} \leq \mathrm{x} \leq \mathrm{c} \\
&= \frac{x-c+n_A^L(\mathrm{d}-\mathrm{x})}{d-c} && \text{for c} < \mathrm{x} \leq \mathrm{d} \\
&= 0 && \text{Otherwise.}
\end{aligned}
$$

Where

$$
\begin{aligned}
&0 \leq m_A^L \leq m_A^U \leq 1 \\
&0 \leq n_A^L \leq n_A^U \leq 1 \\
&0 \leq m_A^U + n_A^U \leq 1 \\
&0 \leq m_A^L + n_A^L \leq 1 \\
&\mathrm{a, b, c, d} \in \mathrm{R}
\end{aligned}
$$

then $\mathrm{A} = \left([\mathrm{a,b,c,d}]; \left[m_A^L, m_A^U\right]; \left[n_A^L, n_A^U\right]\right)$ is called an Interval-valued Intuitionistic Trapezoidal Fuzzy set IVITrFS.

3.2. Existing Ranking Methods

Here, we briefly reviewed the definitions of score and accuracy functions and also the rankings of IVITrFSs from the literature.

3.2.1. *Wan [11]*

Wan [11] developed a ranking method for interval-valued trapezoidal intuitionistic fuzzy sets based on score and accuracy functions. For any interval-valued trapezoidal intuitionistic fuzzy set $\tilde{A}$ the score and accuracy functions are defined as:

Score function: $S(\tilde{A}) = \left[\frac{a+b+c+d}{4}\right]\left[\frac{m_{\tilde{A}}^L+m_{\tilde{A}}^U-n_A^L-n_A^U}{2}\right]$

Accuracy function: $H(\tilde{A}) = \left[\frac{a+b+c+d}{4}\right]\left[\frac{m_A^L+m_{\tilde{A}}^U+n_A^L+n_A^U}{2}\right]$

The set with the highest score for a specific IVITrFS is preferred the most (ranked the best). Compare using accuracy if scores are equal. The most popular set is the one with the highest accuracy. They are deemed equal if both the accuracy and score are the same.

3.2.2. *Wu and Liu [12]*

Wu and Liu [12] developed a ranking method for interval-valued intuitionistic trapezoidal fuzzy sets based onscore expected function and accuracy expected function function.

The score expected function is $S(\tilde{A}) = \frac{S_X(\tilde{A})}{2}\left[\frac{a+b+c+d}{2}\right]$

Where $S_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^L + m_{\tilde{A}}{}^U - n_{\tilde{A}}^L - n_{\tilde{A}}^U}{2}\right]$ is the score function

The accuracy expected function is

$$H\left(\widetilde{A_x}\right) = \frac{H_X\left(\widetilde{A_x}\right)}{2}\left[\frac{a+b+c+d}{2}\right]$$

where $H_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^L + m_{\tilde{A}}^U + n_{\tilde{A}}^L + n_{\tilde{A}}^U}{2}\right]$ is the accuracy function

The set with the highest score for a specific IVITrFS is ranked highly. When scores are the same, compare accuracy. The most popular set is the one with the highest accuracy. They are deemed equal if both the accuracy and score are the same. It should be emphasized that Wu and Liu [12], and Wan [11].

3.2.3. *Veeramachaneni and Kandikonda [18]*

Veeramachaneni and Kandikonda [18] defined the Value index and Ambiguity index for ranking of IVITrFSs

Value index:

$$V(\tilde{A}) = \left[\frac{a+2b+2c+d}{12}\right]\left[1 + S_X(\tilde{A}) - H_X(\tilde{A})\right] \text{ and}$$

Ambiguity index:

$$A(\tilde{A}) = \left[\frac{(d-a)-2(b-c)}{12}\right]\left[1 + S_X(\tilde{A}) - H_X(\tilde{A})\right]$$

Where $S_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^L + m_{\tilde{A}}^U - n_{\tilde{A}}^L - n_{\tilde{A}}^U}{2}\right]$ is the score function

$H_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^L + m_{\tilde{A}}{}^U + n_{\tilde{A}}^L + n_{\tilde{A}}^U}{2}\right]$ is the accuracy function.

The set with the highest score for a specific IVITrFS is ranked highly. Use ambiguity indices to compare if value indices are equal. The most liked set has a high ambiguity index. They are considered to be equal if the value index and the ambiguity index are both the same.

3.2.4. *Jianqiang and Zhong [19]*

Jianqiang and Zhong [19] defined a new score and accuracy degree of IVITrFSs and developed a ranking procedure.

The score function $S(\widetilde{A})$ is defined as,

$$S(\widetilde{A}) = \left[\frac{a+2b+2c+d}{6}\right]\left[\frac{m_{\tilde{A}}^L + m_{\tilde{A}}^U - n_{\tilde{A}}^L - n_{\tilde{A}}^U}{2}\right]$$

and the accuracy function $H(\tilde{A})$ is defined as,

$$H(\tilde{A}) = \left[\frac{a+2b+2c+d}{6}\right]\left[\frac{m_{\tilde{A}}^L + m_{\tilde{A}}U^U + n_{\tilde{A}}^L + n_{\tilde{A}}^U}{2}\right]$$

For given IVITrFSs, the set with more score is ranked high. If scores are equal, then compare with accuracy. The set with high accuracy is preferred the most.

4. Proposed ranking method

4.1. Methodology

The geometric representation of an IVITrFS is represented in Fig. 1Here the region is divided into three parts S_1, S_2, and S_3 as follows.

First, the centroids of the regions $S_1(x_1, y_1), S_2(x_2, y_2), S_3(x_3, y_3)$ are calculated. By using the centroid coordinates, the Improved Score function of IVITrFS is defined.

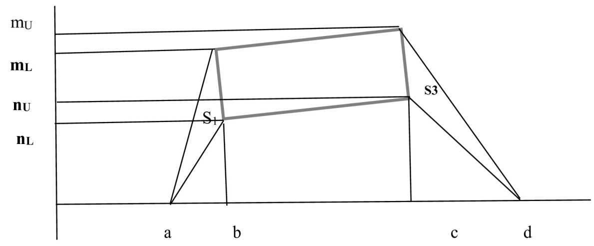

Fig. 1. The geometric representation of an IVITrFS

Let $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) represents the centroids of S_1, S_2, and S_3. Where

$$(x_1, y_1) = \left[\frac{a+2b}{3}, \frac{n_L+m_L}{3}\right]$$

$$(x_2, y_2) = \left[\frac{b+c}{2}, \frac{n_L+n_U+m_L+m_U}{4}\right]$$

$$(x_3, y_3) = \left[\frac{2c+d}{3}, \frac{n^U+m^U}{3}\right]$$

Then the improved score function is defined as

$$I(S(A)) = x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3$$

For any two IVITFSs A_1, A_2 the ranking is defined as

i) If $I(S(A_1)) > I(S(A_2))$ then $A_1 > A_2$

ii) If $I(S(A_1)) < I(S(A_2))$ then $A_1 < A_2$

iii) If $I(S(A_1)) = I(S(A_2))$ then $A_1 = A_2$.

4.2. Numerical Example

Let $A_1 = ([0.1, 0.4, 0.5, 0.6]; [1, 1]; [0, 0])$, and $A_2 = ([0.2, 0.3, 0.5, 0.6]; [1, 1]; [0, 0])$, then

For A_1 we have

$(x_1, y_1) = (0.3, 0.33), (x_2, y_2) = (0.45, 0.5)$ and $(x_3, y_3) = (0.53, 0.33)$

And $I(S(A_1) = 0.5207$

For A_2 we have

$(x_1, y_1) = (0.26, 0.33), (x_2, y_2) = (0.4, 0.5)$ and $(x_3, y_3) = (0.53, 0.33)$

And $I(S(A_2)) = 0.4667$,

Which implies that $A_1 > A_2$

4.3. Comparative Study:

In this section, the proposed method is compared with the existing methods from literature namely Wan [11], Wu and Liu [12], Jianqiang and Zhong [19] and Veeramachaneni and Kandikonda [18] and the results are given in Table 1.

In example 2, the ranking methods proposed by Wan [11] and Wu and Liu [12] are not able to rank effectively, but the proposed method works effectively to rank the given sets.

Table 1. Comparison of time performance and performance with different superpixel methods

Example	Wan [11]	Wu and Liu [12]	Jianqiang and Zhong[19]	Veeramachaneni and Kandikonda [18]	Proposed method
1. A1=([0.2,0.3,0.4,0.5]; [0.4,0.5];[0.2,0.3]) A2=([0.4,0.5,0.6,0.7]; [0.4,0.5];[0.2,0.3])	$S_E(A_1) = 0.07$ $S_E(A_2) = 0.0825$ $A_1 < A_2$	$I(S_x(A_1)) = 0.875$ $I(S_x(A_2)) = 0.0825$ $A_1 > A_2$	$S_E(A_1) = 0.07$ $S_E(A_2) = 0.0825$ $A_1 < A_2$	$V(A_1) = 0.04375$, $V(A_2) = 0.04125$ $A_1 > A_2$	$I(S(A_1)) = 0.3058$ $I(S(A_2)) = 0.4148$ $A_1 < A_2$
2. A_1=([0.5,0.6,0.7, 0.75];[1,1];[0,0]) A_2=([0.45,0.65,0.7, 0.75];[1,1];[0,0])	$S_E(A_1) = 0.6375$ $S_E(A_2) = 0.6375$ $H_E(A_1) = 0.6375$ $H_E(A_2) = 0.6375$ $A_1 = A_2$	$I(S_x(A_1)) = 0.6375$ $I(S_x(A_2)) = 0.6375$ $I(H_x(A_1)) = 0.6375$ $I(H_x(A_2)) = 0.6375$ $A_1 = A_2$	$A_1 = 0.6416$ $A_2 = 0.65$ $A_2 > A_1$	$V(A_1) = 0.3208$, $V(A_2) = 0.325$ $A_2 > A_1$	$I(S(A_1)) = 0.8722$ $I(S(A_2)) = 0.8902$ $A_2 > A_1$
3. A_1=([0.3,0.4,0.5,0.6]; [1,1];[0,0]) A_2=([0.2,0.3,0.6,0.7]; [1,1];[0,0]) A_3=([0.1,0.4,0.5,0.8]; [1,1];[0,0]);	$S_E(A_1) = 0.45$ $S_E(A_2) = 0.45$ $S_E(C) = 0.45$ $H_E(A_1) = 0.45$ $H_E(A_2) = 0.45$ $H_E(A_3) = 0.45$ $A_1=A_2=A_3$	$I(S_x(A_1)) = 0.45$ $I(S_x(A_2)) = 0.45$ $I(S_x(C)) = 0.45$ $I(H_x(A_1)) = 0.45$ $I(H_x(A_2)) = 0.45$ $I(H_x(A_3)) = 0.45$ $A_1=A_2=A_3$	$S_E(A_1) = 0.45$ $S_E(A_2) = 0.45$ $S_E(C) = 0.45$ $H_E(A_1) = 0.45$ $H_E(A_2) = 0.45$ $H_E(A_3) = 0.45$ $A_1=A_2=A_3$	$V(A_1) = 0.225$, $V(A_2) = 0.225$, $V(C) = 0.225$ $A(A_1) = 0.0833$, $A(A_2) = 0.1833$, $A(A_3) = 0.15$, $A_2>A_3>A_1$	$I(S(A_1)) = 0.6138$ $I(S(A_2)) = 0.6305$ $I(S(A_3)) = 0.625$ $A_2>A_3>A_1$

In example 3, the ranking methods proposed by Wan [11], Wu and Liu [12] and Jianqiang and Zhong [19] are not able to rank the sets effectively, but the proposed method works effectively to rank the given sets.

The existing ranking methods are good at ranking in several cases, but if the membership and non-membership values are ideal sets (i.e $[1,1];[0,0]$) the existing methods fails in ranking. The proposed ranking method is effective in ranking in such cases also.

The comparative analysis with existing rakings defined using score and accuracy illustrates that the proposed method is strictly ranking when compared to Wan [11], Wu and Liu [12], and Jianqiang and Zhong [19] and is almost coinciding with Veeramachaneni and Kandikonda [18].

5. Conclusion

Ranking is a difficult endeavour, and each ranking method has its own significance and application. Numerous researchers have proposed a variety of ranking methods, and there is no single method that is applicable to all applications. The existing ranking methods are good at ranking in several cases, but if the membership and non-membership values are ideal sets (i.e $[1,1];[0,0]$) then the existing methods fails in ranking. In this study, the proposed ranking method is derived with this objective in mind and the proposed ranking method is effective in ranking such cases also.

Additional numerical examples and a comparative analysis were provided to demonstrate the superiority of the proposed method over other existing ranking techniques. The present work is confined to the ranking of IVITrFS.

Some of the examples in this paper either produce similar result with existing ranking methods or they are in the same direction as the related works in this area, but in some cases the results follow a different approach. We believe that this paper will leave enough scope for future work in ranking. An immediate query in this direction would be whether this proposed ranking fit for all situations or not. In future work it is proposed to discuss the applications of ranking method in decision making problems and in optimization problems.

6. Acknowledgments

The author expresses his heartfelt gratitude to the anonymous reviewer for such excellent comments and suggestions which have enormously enhanced the quality and presentation of this paper. And thanks to the Editor in Chief for his support and valuable suggestions.

References

[1] L. A. Zadeh, (1965) *"Information and control"* **Fuzzy sets** *8*(3): 338–353.

[2] K. T. Atanassov and K. T. Atanassov. *Intuitionistic fuzzy sets*. Springer, 1999. DOI: 10.1007/978-3-7908-1870-3_1.

[3] L. A. Zadeh, (1965) *"Fuzzy sets"* **Information and control** *8*(3): 338–353. DOI: 10.1016/S0019-9958(65)90241-X.

[4] H. Bustince and P. Burillo, (1996) *"Vague sets are intuitionistic fuzzy sets"* **Fuzzy sets and systems** *79*(3): 403–405. DOI: 10.1016/0165-0114(95)00154-9.

[5] W.-L. Gau and D. J. Buehrer, (1993) *"Vague sets"* **IEEE transactions on systems, man, and cybernetics** *23*(2): 610–614. DOI: 10.1109/21.229476.

[6] K. T. Atanassov and K. T. Atanassov. *Interval valued intuitionistic fuzzy sets*. Springer, 1999.

[7] D. J. Dubois. *Fuzzy sets and systems: theory and applications*. ***144***. Academic press, 1980.

[8] M.-H. Shu, C.-H. Cheng, and J.-R. Chang, (2006) *"Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly"* **Microelectronics Reliability** *46*(12): 2139–2148. DOI: 10.1016/j.microrel.2006.01.007.

[9] D.-F. Li, (2008) *"A note on "using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly""* **Microelectronics Reliability** *48*(10): 1741. DOI: 10.1016/j.microrel.2008.07.059.

[10] J. Wang, (2008) *"Overview on fuzzy multi-criteria decision-making approach"* **Control and decision** *23*(6): 601–606.

[11] S.-P. Wan, (2011) *"Multi-attribute decision making method based on interval-valued intuitionistic trapezoidal fuzzy number"* **Control and Decision** *26*(6):

[12] J. Wu and Y. Liu, (2013) *"An approach for multiple attribute group decision making problems with interval-valued intuitionistic trapezoidal fuzzy numbers"* **Computers & Industrial Engineering** *66*(2): 311–324. DOI: 10.1016/j.cie.2013.07.001.

[13] N. Konwar, A. Esi, and P. Debnath, (2019) *"New fixed point theorems via contraction mappings in complete intuitionistic fuzzy normed linear space"* **New Mathematics and Natural Computation** *15*(01): 65–83. DOI: 10.1142/S1793005719500042.

[14] S. MurtyKodukulla, V. Sireesha, and V. Anusha, (2022) *"Distance Measure Approaches to Rank Interval-Valued Trapezoidal Intuitionistic Fuzzy Sets"* **Mathematical Statistician and Engineering Applications** *71*(4): 4335–4353. DOI: 10.17762/msea.v71i4.1020.

[15] N. Konwar and P. Debnath, (2017) *"Continuity and Banach contraction principle in intuitionistic fuzzy n-normed linear spaces"* **Journal of Intelligent & Fuzzy Systems** *33*(4): 2363–2373. DOI: 10.3233/JIFS-17500.

[16] P. Debnath, (2012) *"Lacunary ideal convergence in intuitionistic fuzzy normed linear spaces"* **Computers & Mathematics with Applications** *63*(3): 708–715. DOI: 10.1016/j.camwa.2011.11.034.

[17] C. Mallika and V. Sireesha, (2023) *"An interval-valued trapezoidal intuitionistic fuzzy TOPSIS approach for decision-making problems"* **SN Computer Science** *4*(4): 327. DOI: 10.1007/s42979-023-02488-4.

[18] S. Veeramachaneni and H. Kandikonda, (2016) *"An ELECTRE Approach for Multicriteria Interval-Valued Intuitionistic Trapezoidal Fuzzy Group Decision Making Problems"* **Advances in Fuzzy systems** *2016*(1): 1956303. DOI: 10.1155/2016/1956303.

[19] W. Jianqiang and Z. Zhong, (2009) *"Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems"* **Journal of Systems Engineering and Electronics** *20*(2): 321–326.