New Explicit Solutions For Zakharov-Kuznetsov-Benjamin-Bona-Mahony Equation

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In this paper, we propose two techniques for constructing new explicit exact (EE) solutions of the the ZK-BBM equation. we utilize either the modified Kudryashov technique and a new Kudryashov technique in conjunction with symbolic computation to achieve this goal. In addition, 3D and 2D plots were generated to illustrate the features of the obtained solutions. This was achieved by meticulously choosing appropriate values for the relevant parameters. The results determine that these techniques are a mighty math instrument to solve nonlinear partial differential equations (NLPDEs) in terms of efficiency, validity and accuracy.

Keywords: ZK-BBM equation, new Kudryashov technique; modified Kudryashov technique; explicit exact solutions

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1. Introduction

Most phenomena that occur in the disciplines of engineering and mathematical physics can be effectively expressed with the help of NLPDEs. The NLPDEs have become very powerful tools for describing the engineering models and natural phenomena of science. Many issues involving protein chemistry, solid-state physics, geochemistry, quantum mechanics, chemical kinematics, optical fibers, wave propagation phenomena, heat flow, chemically reactive materials, electricity, etc., are frequently expressed using NLPDEs. With the help of EE solutions, researchers can better understand the phenomena modeled by these NLPDEs when they exist. Thus, studies of EE solutions for NLPDEs play a fundamental role in better understanding nonlinear physical phenomena. Many researchers have made remarkable endeavors to discover EE solutions for these NLEEs., and the number of efficient and powerful techniques have been expanded by these authors, such as the functional variable method [1], the (G′/G,1/G)-expansion method [2], the advanced exp (-$\phi(\xi)$)-expansion method [3, 4], the hyperbolic function and the methods of the sine-Gordon expansion [5], the extended direct algebraic method [6], the singular manifold method [7], the Bernoulli sub-ODE method [8, 9], the extended Fan sub-equation method [10], the modified Khater method [11], the method of He's the semi-inverse variational principle [12], modified simple equation the method [13], the symmetry approach [14], the generalized exponential rational function method [15], the first integral method [16], the modified exponential function method [17, 18], unified method [19–21], improved tan ($\phi/2$)- expansion approache [22, 23], the extended sinh-Gordon equation expansion method [24, 25] and so on [26–28].

Using the modified Kudryashov and new Kudryashov techniques, we'll get new EE answers to the ZK-BBM equation [29–31].

$$\bar{\omega}_{\varsigma} + \bar{\omega}_{\xi} - \alpha(\bar{\omega}^2)_{\xi} - (\beta\bar{\omega}_{\xi\varsigma} + k\bar{\omega}_{\zeta\varsigma})_{\xi} = 0, \tag{1}$$

where α, β, and k are constants.

Wazwaz specified the precise GZK-BBM equation solutions with various physical structures in 2005, utilizing tanh and sine-cosine techniques [30]. Also, the author solved the ZK-BBM and GZK-BBM equations using the extended tanh technique and successfully discovered several compacton solutions, solitons, solitary patterns, and periodic plane patterns. [31]. Bibi and Mohyud-Din [32] found soliton and periodic solutions for Eq. (1) using the sine-cosine approach. To obtain the ZK-BBM model's new wave solutions, the extended (G′/G)-expansion technique was investigated by Nahr [33]. Using the exp-function technique, Ali et al. [34] obtained generalized soliton solutions for the ZK-BBM equation. The determination of the exact wave solution for the ZK-BBM equation using the bifurcation structure has been studied by Zhao and Li [35]. In [36], Several periodic wave solutions of Eq. (1) are obtained using the bifurcation approach. Through the bifurcation approach of a dynamical system, Song and Yang studied the wave solutions of the ZK-BBM equation [37]. Güner et al. [38] investigated the ZK-BBM equation using the ansatz method to identify bright and dark solitary wave solutions. Some exact soliton solutions of Eq. (1) have been obtained using the tan-cot method by Kumar et al. [39]. Kayum et al. [40] used the sine-Gordon expansion method to identify the (2+1)-dimensional GZK-BBM equation's W-shaped, V-shaped, parabolic, compacton, bright, and dark solitons. The homotopy analysis method developed by Tasbozan and Kurt provides approximative analytical solutions to the GZK-BBM equation.[41]. Yang [39] used the solitary wave solutions for Eq. (1) using the discriminating system and direct integral method.

In the following, the paper is prepared as follows: In Sec. 2, the modified Kudryashov technique and a new Kudryashov technique are discussed; In Sec. 3, we apply the techniques presented in the previous section to the equation mentioned above and in Sec. 4 conclusions are given.

2. The kudryashov techniques

Consider the NLPDE in the form [42, 43]

$$F(\bar{\omega},\bar{\omega}_{\varsigma},\bar{\omega}_{\xi},\bar{\omega}_{\zeta},\bar{\omega}_{\varsigma\varsigma},...)=0 \tag{2}$$

where $\bar{\omega}(\varsigma\xi,\zeta)$ is a travelling wave solution of NLPDE Eq. (2) and F is a polynomial in v. We use the transformations

$$\bar{\omega}(\varsigma,\xi,\zeta)=X(\mu),\mu=\varsigma+\xi-\delta\zeta \tag{3}$$

where $\delta\neq 0$ is constants to be determined later.

With the help of Eq. (3), the NLPDE Eq. (2), an ordinary differential equation (ODE) is modified as:

$$G(X,X',X'',X''',...)=0 \tag{4}$$

where X',X'' etc., denote derivative of H with respect to μ.

With a polynomial in $\wedge(\mu)$, the solution of Eq. (4) can be expressed as:

$$X(\mu)=\Sigma_{j=0}^{N}b_j\wedge^j(\mu),\quad (b_N\neq 0) \tag{5}$$

where b_j, (j=0,1,...,N) are the coefficients and $\wedge(\mu)$ satisfies the following equations:

$$(\wedge'(\mu))^2=(\sigma(\ln V)a\wedge(\mu))^2\left(1-4ab\wedge^2(\mu)\right) \tag{6}$$

or

$$\wedge'(\mu)=\sigma(\ln V)\left(a\wedge^2(\mu)-\wedge(\mu)\right), \tag{7}$$

where $V>0$, $V\neq 1$, the solutions of ODEs Eqs. (6) and (7) are

$$\wedge(\mu)=\frac{1}{aV^{\sigma\mu}+bV^{-\sigma\mu}}$$
$$\wedge(\mu)=\frac{1}{a+bV^{\sigma\mu}}$$

The parameter can be determined by balancing the nonlinear sentence in Eq. (4) and the highest-order derivative sentence. We equate all coefficients $\wedge^j(\mu)$ with zero because the $\wedge(\mu)\neq 0$. It presents a system of algebraic equations, including $b_j s$ and a,b,δ parameters. Whenever b_j s and a,b,δ are determined, solutions are obtained with the help of parameters by using the explicit strategy. By combining Eq. (5) $\mu=\varsigma+\xi-\delta\zeta$, and the solutions of ODEs Eqs. (6) and (7), we can obtain the travelling wave solution to Eq. (2).

3. Solutions of the zk-mew equation

Here, we seek the EE solution for Eq. (1) in the form of:

$$\begin{aligned}\bar{\omega}(\varsigma,\xi,\zeta)&=X(\eta)\\ \mu&=\varsigma+\xi-\delta\zeta\end{aligned} \tag{8}$$

reduces the ZK-BBM equation to the following ODE:

$$-\delta X'+X'-\alpha(X^2)'-(-\beta\delta X''-k\delta X'')'=0 \tag{9}$$

By integrating Eq. (9) once and zeroing the integration constant, we get:

$$(1-\delta)X-\alpha X^2+\delta(\beta+k)X''=0 \tag{10}$$

Balancing the terms X'' and X^2 to get $N=2$. By substituting $N=2$ in to Eq. (10) to get:

$$X(\mu)=b_0+b_1\wedge(\mu)+b_2\wedge^2(\mu) \tag{11}$$

3.1. First method

Substituting Eq. (11) in Eq. (10), along with Eq. (6) and equating the coefficient of L yields

$$\begin{aligned}
&\wedge^0: \quad b_0(1-\delta-\alpha b_0)=0\\
&\wedge^1: \quad b_1(1-\delta-2\alpha b_0+\delta\beta\sigma^2(\ln^2 V)+\delta k\sigma^2(\ln^2 V))=0\\
&\wedge^2: \quad b_2-\delta b_2-2\alpha b_0 b_2-\alpha b_1^2+4\delta\beta b_2\sigma^2(\ln^2 V)\\
&\qquad\quad +4\delta k b_2\sigma^2(\ln^2 V)=0\\
&\wedge^3: \quad 2b_1(-\alpha b_2-4\delta\beta ab\sigma^2(\ln^2 V)-4\delta kab\sigma^2(\ln^2 V))=0\\
&\wedge^3: \quad b_2(-\alpha b_2-24\delta\beta ab\sigma^2)(\ln^2 V)-24\delta kab\sigma^2(\ln^2 V))=0
\end{aligned}$$

Solving the above system, we get:

Case I

$$\begin{aligned} b_0 &= 0 \\ b_1 &= 0 \\ b_2 &= \frac{24\sigma^2(\ln^2 V)(\beta+k)}{\alpha(4\sigma^2(\ln^2 V)(\beta+k)-1)} \\ \delta &= \frac{1}{4\sigma^2(\ln^2 V)(\beta+k)-1} \end{aligned} \tag{12}$$

Case II

$$\begin{aligned} b_0 &= \frac{4\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+4\sigma^2(\ln^2 V)(\beta+k))} \\ b_1 &= 0 \\ b_2 &= -\frac{24ab\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+4\sigma^2(\ln^2 V)(\beta+k))} \\ \delta &= \frac{1}{1+4\sigma^2(\ln^2 V)(\delta+k)} \end{aligned} \tag{13}$$

By substituting Eq. (12) in to Eq. (11) and along with a solution of ODE Eq. (6), we recover the following EE solution:

$$\bar{\omega}(\varsigma,\xi,\zeta) = \frac{24\sigma^2(\ln^2 V)(\beta+k)}{\alpha(4\sigma^2(\ln^2 V)(\beta+k)-1)}\left(\frac{1}{aV^{\sigma\mu}+bV^{-\sigma\mu}}\right)^2 \tag{14}$$

where $\mu = \varsigma+\xi+\frac{1}{4\sigma^2(\ln^2 V)(\beta+k)-1}\zeta$.

In Fig. 1, soliton Eq. (14) for $\alpha = 1.2, \beta = 1.5, \sigma = 1.5, k = 1, a = 2.5, b = 0.75, V = 2.8$ and $\zeta = 1$ within the interval $-10 \leq \varsigma, \xi \leq 10$, the left figure shows the 3D plot and the right figure shows the 2D plot for ξ=0,5,10.

By substituting Eq. (13) in to Eq. (11) and along with solution of ODE Eq. (6), we recover the following EE solution:

$$\bar{\omega}(\varsigma,\xi,\zeta) = \frac{4\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+4\sigma^2(\ln^2 V)(\beta+k))}\left(1-6ab\left(\frac{1}{aV^{\sigma\mu}+bV^{-\sigma\mu}}\right)^2\right) \tag{15}$$

where $\mu = \varsigma+\xi-\frac{1}{1+4\sigma^2(\ln^2 V)(\beta+k)}\zeta$.

In Fig. 2, soliton Eq. (15) for $\alpha = 0.5, \beta = 2, \sigma = 1.5, k = 0.25, a = 1, b = 1.5, V = 2.6$ and $\zeta = 1$ within the interval $-10 \leq \varsigma, \xi \leq 10$, the left figure shows the 3D plot and the right figure shows the 2D plot for $\xi = 0, 5, 10$.

3.2. Second method

Substituting Eq. (11) in Eq. (10), along with Eq. (7) and equating the coefficient of L yields

$$\begin{aligned} \wedge^0: &\quad b_0-\delta b_0-\alpha b_0^2 = 0 \\ \wedge^1: &\quad b_1(1-\delta-2\alpha b_0+\delta\beta\sigma^2(\ln^2 V)+\delta k\sigma^2(\ln^2 V)) = 0, \\ \wedge^2: &\quad \omega_2-\lambda\omega_2-2\alpha\omega_0\omega_2-\alpha\omega_1^2-3\delta\sigma^2(\ln^2 V)\beta a\omega_1 \\ &\quad +4\delta\sigma^2(\ln^2 V)\beta\omega_2-3\delta\sigma^2(\ln^2 V)ka\omega_1 \\ &\quad +4\delta\sigma^2(\ln^2 V)k\omega_2 = 0 \\ \wedge^3: &\quad -2\alpha b_1b_2+2\delta\beta a^2b_1\sigma^2(\ln^2 V)-10\delta\beta ab_2\sigma^2(\ln^2 V) \\ &\quad +2\delta ka^2b_1\sigma^2(\ln^2 V)-10\delta kab_2\sigma^2(\ln^2 V) = 0, \\ \wedge^3: &\quad b_2(-\alpha b_2+6\delta\beta a^2\sigma^2(\ln^2 V)+6\delta ka^2\sigma^2(\ln^2 V)) = 0. \end{aligned}$$

Solving the above system, we get:

Case I

$$\begin{aligned} b_0 &= 0 \\ b_1 &= -\frac{6a\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1-\sigma^2(\ln^2 V)(\beta+k))} \\ b_2 &= \frac{6a^2\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1-\sigma^2(\ln^2 V)(\beta+k))} \\ \delta &= \frac{1}{1-\sigma^2(\ln^2 V)(\beta+k)} \end{aligned} \tag{16}$$

Case II

$$\begin{aligned} b_0 &= \frac{\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+\sigma^2(\ln^2 V)(\beta+k))} \\ b_1 &= -\frac{6a\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+\sigma^2(\ln^2 V)(\beta+k))} \\ b_2 &= \frac{6a^2\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+\sigma^2(\ln^2 V)(\beta+k))} \\ \delta &= \frac{1}{1+\sigma^2(\ln^2 V)(\beta+k)} \end{aligned} \tag{17}$$

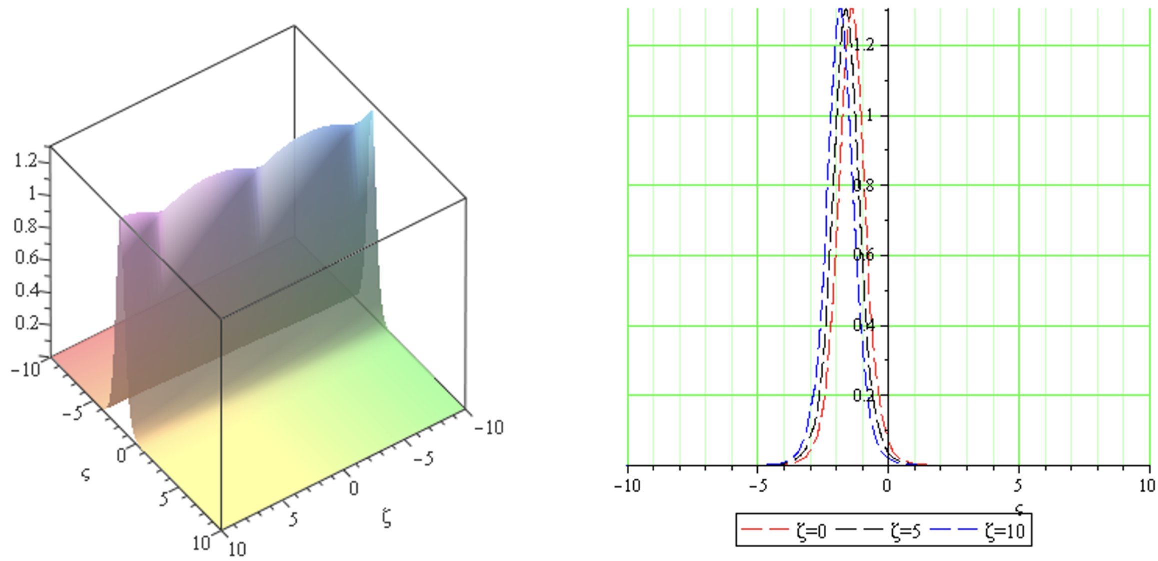
By substituting Eq. (16) in to Eq. (11) and along with a solution of ODE Eq. (7), we recover the following EE solution:

$$\bar{\omega}(\varsigma,\xi,\zeta) = \frac{6a\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1-\sigma^2(\ln^2 V)(\beta+k))}\left(-\frac{1}{a+bV^{\sigma\mu}}+a\left(\frac{1}{a+bV^{\sigma\mu}}\right)^2\right) \tag{18}$$

where $\mu = \varsigma+\xi-\frac{1}{1-\sigma^2(\ln^2 V)(\beta+k)}\zeta$.

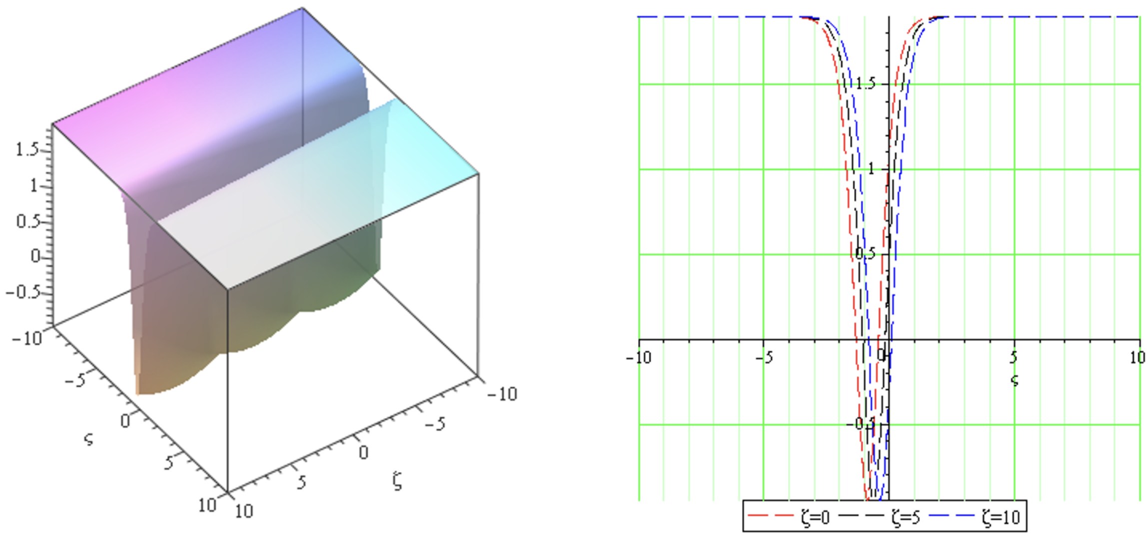
In Fig. 3, soliton Eq. (18) for $\alpha = 0.25, \beta = 1.2, \sigma = 0.75, k = 1, a = 2, d = 0.5, V = 2.7$ and $\zeta = 1$ within the interval $-10 \leq \varsigma, \xi \leq 10$, the left figure shows the 3D plot and the right figure shows the 2D plot for $\xi = 0, 5, 10$.

By substituting Eq. (17) in to Eq. (11) and along with a solution of ODE Eq. (7), we recover the following EE solution:



(a) Three-dimensional plot (b) two-dimensional plot

Fig. 1. Three-dimensional and two-dimensional plots for Eq. (14) with $\alpha = 1.2, \beta = 1.5, \sigma = 1.5, k = 1, a = 2.5, b = 0.75, V = 2.8$ and $\zeta = 1$ when $-10 \le \varsigma, \xi \le 10$.



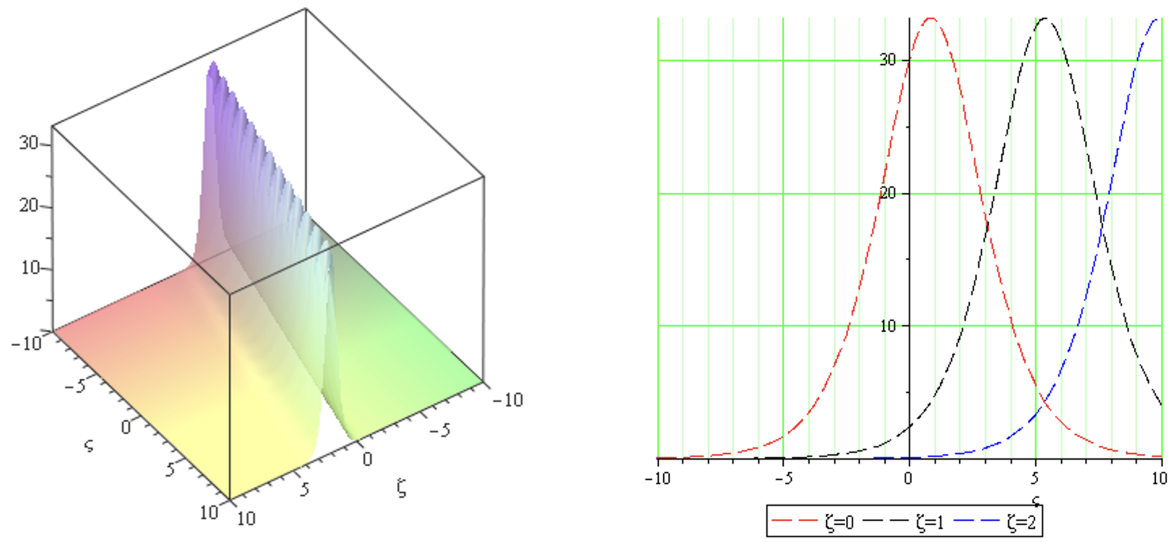
(a) Three-dimensional plot (b) two-dimensional plot

Fig. 2. Three-dimensional and two-dimensional plots for Eq. (14) with $\alpha = 0.5, \beta = 2, \sigma = 1.5, k = 0.25, a = 1, b = 1.5, V = 2.6$ and $\zeta = 1$ when $-10 \le \varsigma, \xi \le 10$.

$$\bar{\omega}(\varsigma,\xi,\zeta) = \frac{\sigma^2(\ln^2 V)(\beta+k)}{\alpha(1+\sigma^2(\ln^2 V)(\beta+k))} \left(1 - 6a\frac{1}{a+bV^{\sigma\mu}} + 6a^2\left(\frac{1}{a+bV^{\sigma\mu}}\right)^2\right) \tag{19}$$

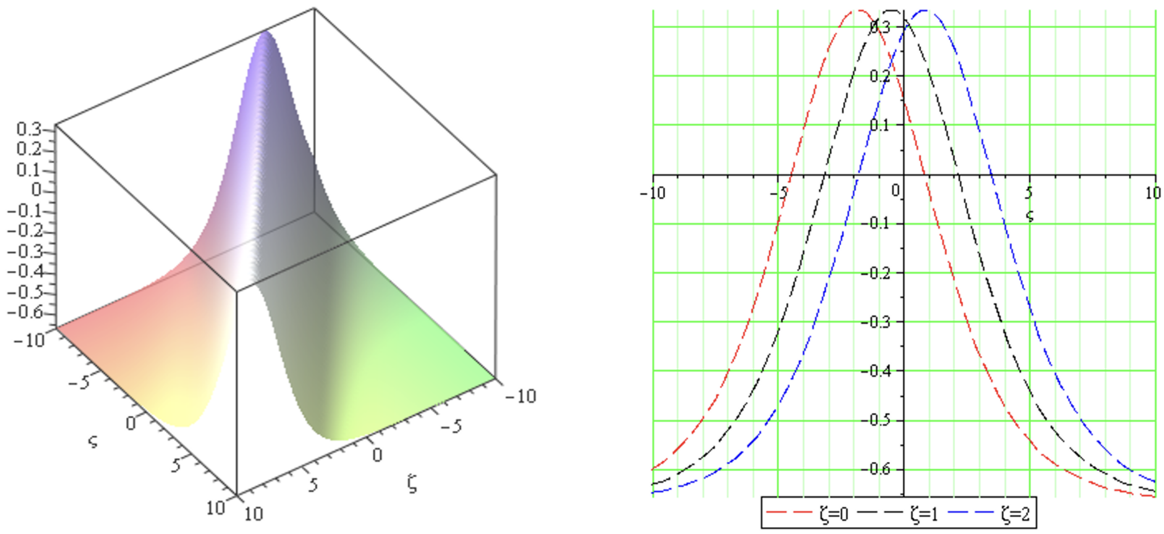
where $\mu = \varsigma + \xi - \frac{1}{1+\sigma^2(\ln^2 V)(\beta+k)}\zeta$.

In Fig. 4, soliton Eq. (19) for $\alpha = 0.5, \beta = 1, \sigma = 0.5, k = -2, a = 1, d = 1.5, V = e$ and $\zeta = 1$ within the interval $-10 \le \varsigma, \xi \le 10$, the left figure shows the 3D plot and the right figure shows the 2D plot for $\xi = 0, 5, 10$.



(a) Three-dimensional plot (b) two-dimensional plot

Fig. 3. Three-dimensional and two-dimensional plots for Eq. (14) with $\alpha = 0.25, \beta = 1.2, \sigma = 0.75, k = 1, a = 2, d = 0.5, V = 2.7$ and $\zeta = 1$ when $-10 \leq \varsigma, \xi \leq 10$



(a) Three-dimensional plot (b) two-dimensional plot

Fig. 4. Three-dimensional and two-dimensional plots for Eq. (14) with $\alpha = 0.5, \beta = 1, \sigma = 0.5, k = -2, a = 1, d = 1.5, V = e$ and $\zeta = 1$ when $-10 \leq \varsigma, \xi \leq 10$.

4. Conclusion

In this work, we developed novel EE solutions of travelling wave solutions to the ZK-BBM equation using the modified Kudryashov and new Kudryashov techniques. We show that the keys previously provided by other authors differ from those we found in this article. The solutions obtained here may be potentially useful for applications to engineers and physicists. These techniques are adequate, concise and direct. The availability of math software such as Maple has made complex algebraic calculations easier. Therefore, as a result, the present technique can be expanded to solve nonlinear problems that rise in other fields.

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