On Optical Solutions To The Kadomtsev–Petviashviliequation With A Local Conformable Derivativ

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In fact, due to the existence of this category of equations, our understanding of many phenomena around us becomes more complete. In this paper, we study an integrable partial differential equation called the Kadomtsev–Petviashvili equation with a local conformable derivative. This equation is used to describe nonlinear motion. In order to solve the equation, it is first necessary to convert the form of the equation from a partial derivative to an equation with ordinary derivatives using a suitable variable change. The resulting form will then be the basis of our work to determine the main solutions. All the solutions reported in the paper for the present equation are quite different from the previous findings in other papers. All necessary calculations are provided using symbolic computing software in Maple.

Keywords: Conformable potential Kadomtsev–Petviashvili equation; new extended direct algebraic method; exact wave solutions

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1. Introduction

Differential equations with partial derivatives play an important role in describing many phenomena in the world around us [1–5]. In fact, due to the existence of this category of equations, our understanding of many phenomena around us becomes more complete [5–59].

One of the most important examples of partial differential equations is the Kadomtsev–Petviashvili, which has many applications to describe nonlinear wave motion [60]. In recent years, based on the applications that exist for this equation, various methods have been proposed to study this equation. For example, a class of lump solutions for the (2 + 1)-dimensional Kadomtsev–Petviashvili equation is studied in [61]. In [62], an efficient generalized Jacobi elliptic function expansion method has been employed to solve a (3+1)-dimensional form of the Kadomtsev-Petviashvili equation. Moreover, the authors of [63] have applied two efficient techniques namely the Hirotabilinear method and an ansatz technique to consider the Kadomtsev–Petviashvili equation with a self-consistent source. The authors of [64] have analysis soliton solutions and then verified the Hirota-soliton condition for the B-type Kadomtsev–Petviashvili equation in the framework of the Hirota bilinear formulation. In [65], various exact analytical solutions to the cKP equation are determined based on its bilinear representation through taking four types of *antanz* forms. Moreover, they studied the interaction phenomena between lump wave and a stipe, and lump wave and soliton solution. On the basis of the Grammian determinant solution, a class of exponentially localized wave solutions to the Bogoyavlenskii–Kadomtsev–Petviashvili equation has been proposed in [66]. Further, the dynamics of soliton waves, lump solutions and interaction solutions to a (2+1)-dimensional generalized Bogoyavlensky-Konopelchenko equation has been studied in [67]. The authors of [68] constructed Wronskian determinant solutions, based on its associated trilinear equation, instead of a Hirotabilinear form to the Kadomtsev–Petviashvili equation. Further, a (2+1)-dimensional variable-coefficient Bogoyavlensky-Konopelchenko equation is investigated in [69].

In this paper, we will construct the new exact solitons solutions of the following conformable potential Kadomtsev–Petviashvili (CpKP) equation [70]

$$\frac{\partial^{2\mu} x}{\partial\tau^{\mu}\partial\eta^{\mu}} + \frac{3}{2}\frac{\partial^{\mu} x}{\partial\eta^{\mu}}\frac{\partial^{2\mu} x}{\partial\eta^{2\mu}} + \frac{1}{4}\frac{\partial^{4\mu} x}{\partial\omega^{4\mu}} + \frac{3}{4}\frac{\partial^{2\mu} x}{\partial\omega^{2\mu}} = 0, \quad \tau,\eta,\omega > 0, \quad 0 < \mu \le 1, \tag{1}$$

where $\frac{\partial^{\mu}}{\partial\tau^{\mu}}$ is the conformable derivative operator of order $\mu \in (0,1)$ is defined by [71]

$$\frac{d^{\mu} f(\tau)}{d\tau^{\mu}} = \lim_{\varepsilon\to 0}\frac{f(\tau+\varepsilon\tau^{1-\mu}) - f(\tau)}{\varepsilon}, \quad f:(0,\infty)\to R, \quad \mu\in(0,1), \quad \tau>0.$$

In this paper, based on the new extended direct algebraic method [72], we obtain different classes of analytical solutions for Eq. (1). To the best of the authors' knowledge, this form of the equation has not yet been explored using the new extended direct algebraic method. For this purpose, the following article has been compiled as follows. The main results of the paper are presented in Section 2. Some graphical representation for the obtained solutions with different parameters is illustrated in Section 3. Conclusions are also provided at the end of the article.

2. The main analysis

Let's assume the transformations:

$$\mathcal{X}(\tau,\eta,\omega) = \mathcal{X}(\varpi), \varpi = \frac{1}{\mu}(\rho\eta^{\mu} + \sigma\omega^{\mu} - \delta\tau^{\mu}), \tag{2}$$

where ρ, σ and δ are constants. By using Eq. (2) into Eq. (1), we get the following ODE

$$\left(\frac{3}{4}\sigma^2 - \rho\delta\right)\mathcal{X}'' + \frac{3}{2}\rho^3\mathcal{X}'\mathcal{X}'' + \frac{1}{4}\rho^4\mathcal{X}'''' \tag{3}$$

By integrating the Eq. (3) and ignoring the constant of integration, we have

$$\left(\frac{3}{4}\sigma^2 - \rho\delta\right)\mathcal{X}' + \frac{3}{2}\rho^3(\mathcal{X}')^2 + \frac{1}{4}\rho^4\mathcal{X}''' = 0 \tag{4}$$

By applying the homogenous balance technique between the terms $\mathcal{X}'''$ and $(\mathcal{X}')^3$, we get

$$\mathcal{X}(\varpi) = \alpha_0 + \alpha_1\Theta(\varpi), \tag{5}$$

where $\mathcal{X}(\varpi)$ satisfies

$$\Theta'(\varpi) = \ln(\Omega)(a + b\Theta(\varpi) + c\Theta^2(\varpi)), \Omega \ne 0, 1. \tag{6}$$

where a, b along with c are the real constants.

After plugging Eq. (5) in Eq. (4), we get a structure of the respective algebraic equations and coefficients of different powers of $\Theta(\varpi)$ are equalized.

$$\begin{aligned}
\Theta^0(\varpi) :& \left(\frac{3}{4}\sigma^2 - \rho\delta\right)\alpha_1 a\ln(\Omega) + \frac{3}{4}\rho^3\alpha_1^2a^2\ln^2(\Omega) \\
&+ \frac{1}{4}\rho^4\alpha_1 a(2ca+b^2)\ln^3(\Omega) = 0 \\
\Theta^1(\varpi) :& \left(\frac{3}{4}\sigma^2 - \rho\delta\right)\alpha_1 b\ln(\Omega) + \frac{3}{2}\rho^3\alpha_1^2ab\ln^2(\Omega) \\
&+ \frac{3}{2}\rho^4\alpha_1 abc\ln^3(\Omega) + \frac{1}{4}\rho^4\alpha_1 b(2ca+b^2)\ln^3(\Omega) \\
&= 0 \\
\Theta^2(\varpi) :& \left(\frac{3}{4}\sigma^2 - \rho\delta\right)\alpha_1 c\ln(\Omega) + \frac{3}{4}\rho^3\alpha_1^2(2ca+b^2)\ln^2(\Omega) \\
&+ \frac{3}{2}\rho^4\alpha_1 ac^2\ln^3(\Omega) + \frac{3}{2}\rho^4\alpha_1 b^2c\ln^3(\Omega) \\
&+ \frac{1}{4}\rho^4\alpha_1 c(2ca+b^2)\ln^3(\Omega) = 0, \\
\Theta^3(\varpi) :& \frac{3}{2}\rho^3\alpha_1^2 ba\ln^2(\Omega) + 3\rho^4\alpha_1 bc^2\ln^3(\Omega) = 0, \\
\Theta^4(\varpi) :& \frac{3}{4}\rho^3\alpha_1^2c^2\ln^2(\Omega) + \frac{3}{2}\rho^4\alpha_1 c^3\ln^2(\Omega) = 0.
\end{aligned} \tag{7}$$

Employing computational program to solve the above algebraic equations, the following set of solution is obtained:

$$\begin{aligned}
\alpha_0 &= \alpha_0, \\
\alpha_1 &= -2\rho c\ln(\Omega), \\
\delta &= -\frac{1}{4}\frac{-3\sigma^2 - 4\rho^4(b^2-4ca)\ln^2(\Omega)}{\rho}.
\end{aligned} \tag{8}$$

After plugging the values of α_0 and α_1 via Eq. (8) into Eq. (5), which represents the regarding solutions of Eq. (4). General solutions of Eq. (1) as regards with parameters a, b and c are as follows [72].

(I) If $\Pi = b^2 - 4ca < 0$ and $c \ne 0$

$$\mathcal{X}_1(\tau,\eta,\omega) = \alpha_0 + \rho\ln(\Omega)\left(b - \sqrt{-\Pi}\tan_\Omega\left(\frac{\sqrt{-\Pi}}{2}\bar{\omega}\right)\right),$$

$$\mathcal{X}_2(\tau,\eta,\omega) = \alpha_0 + \rho\ln(\Omega)\left(-b + \sqrt{-\Pi}\cot_\Omega\left(\frac{\sqrt{-\Pi}}{2}\bar{\omega}\right)\right),$$

$$\mathcal{X}_3(\tau,\eta,\omega) = \alpha_0 + \rho\ln(\Omega)$$
$$\left(b - \sqrt{-\Pi}\left(\tan_\Omega(\sqrt{-\Pi}\bar{\omega}) \pm \sqrt{mn}\sec_\Omega(\sqrt{-\Pi}\bar{\omega})\right)\right),$$

$$\mathcal{X}_4(\tau,\eta,\omega) = \alpha_0 - \rho\ln(\Omega)$$
$$\left(-b + \sqrt{-\Pi}(\cot_\Omega(\sqrt{-\Pi}\eta) \pm \sqrt{mn}\csc_\Omega(\sqrt{-\Pi}\eta))\right),$$

$$\mathcal{X}_5(\tau,\eta,\omega) = \alpha_0 - \rho\ln(\Omega)$$
$$\left(-b + \frac{\sqrt{-\Pi}}{2}\left(\tan_\Omega\left(\frac{\sqrt{-\Pi}}{4}\bar{\omega}\right) - \cot_\Omega\left(\frac{\sqrt{-\Pi}}{4}\bar{\omega}\right)\right)\right),$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu + \frac{1}{4}\frac{-3\sigma^2 - 4\rho^4\Pi\ln^2(\Omega)}{\rho}\tau^\mu\right)$.

(II) If $\Pi = b^2 - 4ca > 0$ and $c \neq 0$

$$\mathcal{X}_6(\tau,\eta,\omega) = \alpha_0 + \rho\ln(\Omega)\left(b + \sqrt{\Pi}\tan h_\Omega\left(\frac{\sqrt{\Pi}}{2}\bar{\omega}\right)\right),$$

$$\mathcal{X}_7(\tau,\eta,\omega) = \alpha_0 + \rho\ln(\Omega)\left(b + \sqrt{\Pi}\cot h_\Omega\left(\frac{\sqrt{\Pi}}{2}\bar{\omega}\right)\right),$$

$$\mathcal{X}_8(\tau,\eta,\omega) = \alpha_0 - \rho\ln(\Omega)$$
$$\left(-b + \sqrt{\Pi}(-\tanh_\Omega(\sqrt{\Pi}\bar{\omega}) \pm i\sqrt{mn}\sec h_\Omega(\sqrt{\Pi}\bar{\omega}))\right),$$

$$\mathcal{X}_9(\tau,\eta,\omega) = \alpha_0 - \rho\ln(\Omega)$$
$$\left(-b + \sqrt{\Pi}(-\coth_\Omega(\sqrt{\Pi}\bar{\omega}) \pm \sqrt{mn}\csc h_\Omega(\sqrt{\Pi}\bar{\omega}))\right),$$

$$\mathcal{X}_{10}(\tau,\eta,\omega) = \alpha_0 + \rho\ln(\Omega),$$
$$\left(b + \frac{\sqrt{\Pi}}{2}\left(\tanh_\Omega\left(\frac{\sqrt{\Pi}}{4}\bar{\omega}\right) + \coth_\Omega\left(\frac{\sqrt{\Pi}}{4}\bar{\omega}\right)\right)\right),$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu + \frac{1}{4}\frac{-3\sigma^2 - 4\rho^4\Pi\ln^2(\Omega)}{\rho}\tau^\mu\right)$.

(III) If $ac > 0$ and b=0

$$\mathcal{X}_{11}(\tau,\eta,\omega) = \alpha_0 - 2\sqrt{ac}\rho\ln(\Omega)\tan_\Omega(\sqrt{ac}\bar{\omega}),$$
$$\mathcal{X}_{12}(\tau,\eta,\omega) = \alpha_0 + 2\sqrt{ac}\rho\ln(\Omega)\cot_\Omega(\sqrt{ac}\bar{\omega}),$$
$$\mathcal{X}_{13}(\tau,\eta,\omega) = \alpha_0 - 2\sqrt{ac}\rho\ln(\Omega)$$
$$(\tan_\Omega(2\sqrt{ac}\bar{\omega}) \pm \sqrt{mn}\sec_\Omega(2\sqrt{ac}\bar{\omega})),$$
$$\mathcal{X}_{14}(\tau,\eta,\omega) = \alpha_0 - 2\sqrt{ac}\rho\ln(\Omega)$$
$$(-\cot_\Omega(2\sqrt{ac}\bar{\omega}) \pm \sqrt{mn}\csc_\Omega(2\sqrt{ac}\bar{\omega})),$$
$$\mathcal{X}_{15}(\tau,\eta,\omega) = \alpha_0 - \sqrt{ac}\rho\ln(\Omega)$$
$$\left(\tan_\Omega\left(\frac{\sqrt{ac}}{2}\bar{\omega}\right) - \cot_\Omega\left(\frac{\sqrt{ac}}{2}\bar{\omega}\right)\right),$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu + \frac{1}{4}\frac{-3\sigma^2 + 16\rho^4 ac\ln^2(\Omega)}{\rho}\tau^\mu\right)$.

(IV) If $ac < 0$ and $b = 0$

$$\mathcal{X}_{16}(\tau,\eta,\omega) = \alpha_0 + 2\rho\ln(\Omega)\sqrt{-ac}\tanh_\Omega(\sqrt{-ac}\bar{\omega}),$$
$$\mathcal{X}_{17}(\tau,\eta,\omega) = \alpha_0 + 2\rho\ln(\Omega)\sqrt{-ac}\coth_\Omega(\sqrt{-ac}\bar{\omega}),$$
$$\mathcal{X}_{18}(\tau,\eta,\omega) = \alpha_0 - 2\sqrt{-ac}\rho\ln(\Omega)$$
$$(-\tanh_\Omega(2\sqrt{-ac}\bar{\omega}) \pm i\sqrt{mn}\sec h_\Omega(2\sqrt{-ac}\bar{\omega})),$$
$$\mathcal{X}_{19}(\tau,\eta,\omega) = \alpha_0 - 2\sqrt{-ac}\rho\ln(\Omega)$$
$$(-\coth_\Omega(2\sqrt{-ac}\bar{\omega}) \pm \sqrt{mn}\csc h_\Omega(2\sqrt{-ac}\bar{\omega})),$$
$$\mathcal{X}_{20}(\tau,\eta,\omega) = \alpha_0 + \sqrt{-ac}\rho\ln(\Omega)$$
$$\left(\tanh_\Omega\left(\frac{\sqrt{-ac}}{2}\bar{\omega}\right) + \coth_\Omega\left(\frac{\sqrt{-ac}}{2}\bar{\omega}\right)\right),$$

where $\bar{\omega} = \frac{1}{\mu}(\rho\eta^\mu + \sigma\omega^\mu + \frac{-3\sigma^2 - 4\rho^4 b^2\ln^2(\Omega)}{4\rho}\tau^\mu)$.

(V) If $a = c$ and $b = 0$

$$\mathcal{X}_{21}(\tau,\eta,\omega) = \alpha_0 - \rho a\ln(\Omega)\tan_\Omega(a\bar{\omega}),$$
$$\mathcal{X}_{22}(\tau,\eta,\omega) = \alpha_0 + 2\rho a\ln(\Omega)\cot_\Omega(a\bar{\omega},$$
$$\mathcal{X}_{23}(\tau,\eta,\omega) = \alpha_0 - 2\rho\ln(\Omega)$$
$$(\tan_\Omega(2a\bar{\omega}) \pm \sqrt{mn}\sec_\Omega(2a\bar{\omega})),$$
$$\mathcal{X}_{24}(\tau,\eta,\omega) = \alpha_0 - 2\rho a\ln(\Omega)$$
$$(-\cot_\Omega(2a\bar{\omega}) \pm \sqrt{mn}\csc_\Omega(2a\bar{\omega})),$$
$$\mathcal{X}_{25}(\tau,\eta,\omega) = \alpha_0 - \rho a\ln(\Omega)$$
$$\left(\tan_\Omega\left(\frac{a}{2}\bar{\omega}\right) - \cot_\Omega\left(\frac{a}{2}\bar{\omega}\right)\right),$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu + \frac{-3\sigma^2 + 16\rho^4 a^2\ln^2(\Omega)}{4\rho}\tau^\mu\right)$.

(VI) If $a = -c$ and $b = 0$

$$\mathcal{X}_{26}(\tau,\eta,\omega) = \alpha_0 - 2\rho a\ln(\Omega)\tanh_\Omega(a\bar{\omega}),$$
$$\mathcal{X}_{27}(\tau,\eta,\omega) = \alpha_0 - 2\rho a\ln(\Omega)\coth_\Omega(a\bar{\omega}),$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu + \frac{-3\sigma^2 - 16\rho^4 a^2\ln^2(\Omega)}{4\rho}\tau^\mu\right)$.

(VII) If $a = b = 0$

$$\mathcal{X}_{31}(\tau,\eta,\omega) = \alpha_0 + \frac{2\rho}{\bar{\omega}},$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu - \frac{3}{4}\frac{\sigma^2}{\rho}\tau^\mu\right)$.

(VIII) If $a = 0$ and $b \neq 0$

$$\mathcal{X}_{32}(\tau,\eta,\omega) = \alpha_0 + \ln(\Omega)\frac{2\rho bm}{(\cosh_\Omega(b\bar{\omega}) - \sinh_\Omega(b\bar{\omega}) + m)'}$$
$$\mathcal{X}_{33}(\tau,\eta,\omega) = \alpha_0 + \ln(\Omega)\frac{2\rho cb(\sinh_\Omega(b\bar{\omega}) + \cosh_\Omega(b\bar{\omega}))}{(\sinh_\Omega(b\bar{\omega}) + \cosh_\Omega(b\bar{\omega}) + n)'}$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^\mu + \sigma\omega^\mu + \frac{-3\sigma^2 - 4\rho^4 b^2\ln^2(\Omega)}{4\rho}\tau^\mu\right)$.

(IX) If $b = h$, $c = kh$ and $a = 0$

$$\mathcal{X}_{34}(\tau, \eta, \omega) = \alpha_0 + \ln(\Omega)\frac{2\rho khm\Omega^{h\bar{\omega}}}{m - kn\Omega^{h\bar{\omega}'}}$$

where $\bar{\omega} = \frac{1}{\mu}\left(\rho\eta^{\mu} + \sigma\omega^{\mu} + \frac{-3\sigma^2 - 4\rho^4 h^2 \ln^2(\Omega)}{4\rho}\tau^{\mu}\right)$.

We have checked all the obtained results in the article with Maple software and concluded that they all satisfy the main equation.

3. Comparison and physical explanation

In this section, the3D, contour and 2D plots of the results obtained from the modulus of the solutions of Conformable potential Kadomtsev–Petviashvili equation are shown in Figs. 1 to 3. In Figs. 1 and 2, we plot the modulus of the solutions $\mathcal{X}_1$ and $\mathcal{X}_6$, by taking different values of parameters of, a, b, c, Ω, m, n, μ, α_0, σ, ω respectively. We have shown the modulus of the solution of $\mathcal{X}_{33}$ with parameters $h = k = 0.5$, $\Omega = e$, $n = \rho = \omega = 1$, $m = \mu = 0.9$, in Fig. 3.

4. Conclusions

In this research, through symbolic calculations with Maple, a class of solutions for the Kadomtsev-Petviashvili equation is presented. The difference between the equation considered in this paper and other research activities is the use of local Conformable derivative in its structure. The solution structures introduced in this paper include different forms for the equation that can be effective in practical applications. Another advantage of the results is the expression of solutions based on known basic functions that are easily directly applicable. It should be noted that all the solutions obtained for the equation in this paper are completely different and different from other previous findings in other studies in terms of form and structure. The method used in this article to determine the solutions is the new extended direct algebraic method, which has a very convenient and direct use. Moreover, it can be further applied to some of the other nonlinear equations that arise in mathematics, physics, and engineering applications that provide scope for future work.

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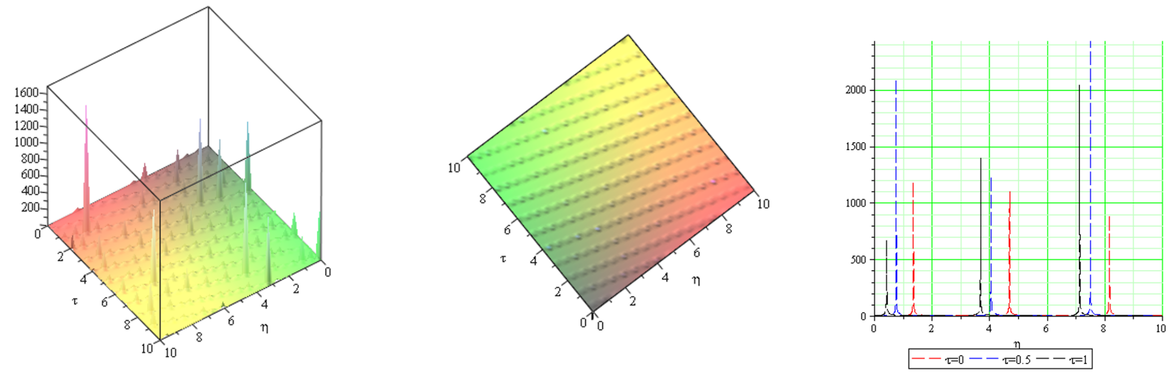
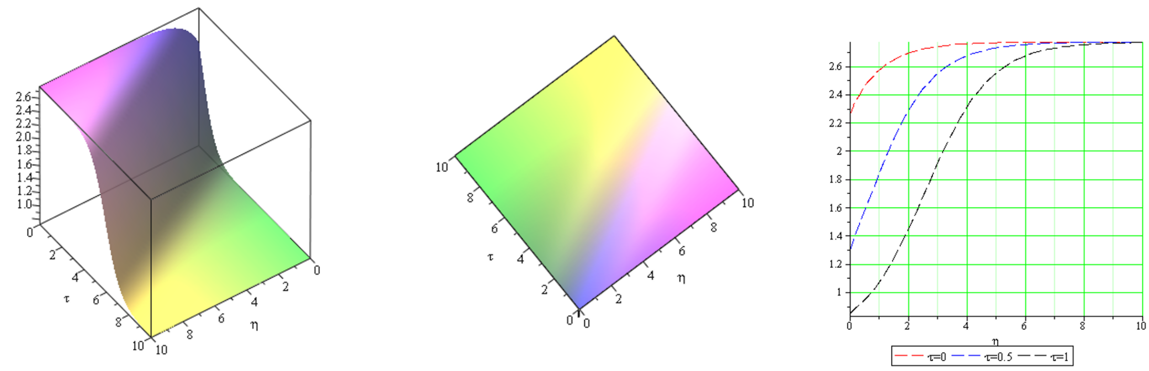


(a) **(b)** **(c)**

Fig. 1. (a) 3D-plot (b) the contour plot (c) 2D plot of the travelling wave solution of $|\mathcal{X}_1|$ at τ=0, τ=0.5, $\tau = 1$, respectively, when a=-2, b=1.5, c=-0.5, ρ=1.5, Ω=2.6, α_0=0.5, σ=0.25, m=1, n=1, ω=1, and μ=0.95.



(a) **(b)** **(c)**

Fig. 2. (a) 3D-plot (b) the contour plot (c) 2D plot of the travelling wave solution of $|\mathcal{X}_6|$ at τ=0, τ=0.5, $\tau = 1$, respectively, when a=-2, b=-0.5, c=0.5, ρ=0.5, Ω=2.7, α_0=1.5, σ=0.5, m=0.95, n=0.9, ω=1, and μ=0.95.

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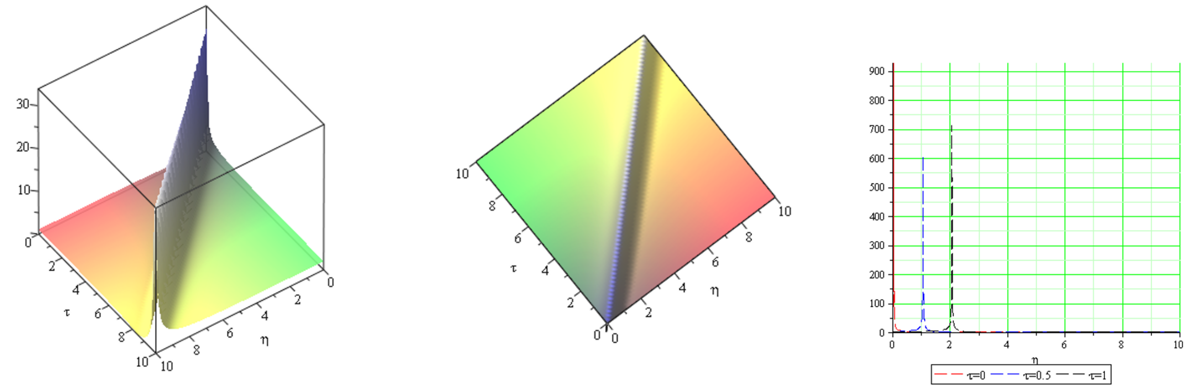

(a) **(b)** **(c)**

Fig. 3. (a) 3D-plot (b) the contour plot (c) 2D plot of the travelling wave solution of $|\mathcal{X}_{34}|$ at τ=0, τ=0.5, $\tau = 1$, respectively, when h=0.5, k=0.5, ρ=1, Ω=1.3, α_0=2, σ=1, m=0.9, n=1, ω=1, and μ=0.9.

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