Some Characteristics Of Completeness Property In Fuzzy Soft b-Metric Space

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In this study, we developed a novel idea known as the fuzzy soft b-metric, and investigated some fundamental aspects of fuzzy soft b-metric. Moreover, various topological features of this new space, such as fuzzy soft open ball, and fuzzy soft Hausdorff b- metric space are defined, also, some fundamental theorems about the ideas are developed.

Keywords: $\mathcal{F}_{SS}$-sets; $\mathcal{F}_{SS}-$ b-metric space; $\mathcal{F}_{SS}-$ Hausdorff-BMS; $\mathcal{F}_{SS}-$ limit point; $\mathcal{F}_{SS}-$ Cauchy sequence

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1. Introduction

Numerous real-world issues deal with ambiguous data and cannot be adequately modeled by classical mathematics. set in fuzzy theory, developed by Zadeh in 1965 [1], also the soft sets developed by Molodtsov in 1999 [2], these two kinds of mathematical tools that can be employed to deal with uncertainties and help with difficulties in a variety of fields. Numerous operations in soft sets have been developed by Maji et al.in 2002 [3] and They conducted a theoretical analysis of soft set theory and offered a soft set application to a decision-making issue, also the idea of b-metric space was first suggested by Backhtin [4] in 1989 such that explored the features of fuzzy soft metric space and defined it in terms of fuzzy soft point. Czerwik [5] expanded the b-metric spaces results in 1993. Following that, several authors examined the use of fuzzy and soft set theory in various fields [6–11] and generalized some important theorems in the b-metric space by using this idea. Cetkin et al. in [12] they introduced 2-metric spaces in terms of soft points, called 2s-metric spaces, and studied some of its topological structures, such as open balls, open (closed) sets, completeness, etc. In [13], they induced a topology from a given b2-metric and studied the properties of the topology induced this way. Also, they define the notion of e-ball in b2-metric spaces.

In this work, we present a novel definition for fuzzy soft b-metric spaces utilizing fuzzy soft points of fuzzy soft sets and fuzzy soft real numbers of fuzzy soft real sets, the definitions of fuzzy soft b- metric are completely unrelated to this new idea. We provide the convergent fuzzy soft sequence has unique limit point and Hausdorff property by using triangle inequality in fuzzy soft b-metric space and provide some characteristics of the fuzzy soft metric.

2. Preliminaries

This part discusses some fundamental definitions of fuzzy soft sets and its properties. Throughout this article, U denotes an initial universe, E is the set of all parameters, $(\mathcal{G}, A)$ denotes a fuzzy soft set, and $\left\langle u^n \mu_{\mathcal{G}(e_n)} \right\rangle$ denotes $\mathcal{F}_{sS}-$ sequence of element in this set.

Definition 2.1. [14] Let X be a non-empty set and $S \geq 1$ be a given real number, bmetric space is a triplet $(X, \mathcal{B}, S)$, where $\mathcal{B} : X \times X \to [0, \infty)$ be a function satisfy the following statements for all $x, y, z \in X$.

1. $\mathcal{B}(x, y) = 0$ if and only if $x = y$,
2. $\mathcal{B}(x, y) = \mathcal{B}(y, x)$,

3. $\mathcal{B}(x,y) \leq S[\mathcal{B}(x,z) + \mathcal{B}(z,y)]$,

Definition 2.2. [15] the fuzzy set $\tilde{X}$ under a universal set U is a set describe by function of membership $\mu_{\tilde{X}} : U \to I$, where $I = [0,1]$ and $\tilde{X}$ an ordered pair collection defined by $\widetilde{X} = \{(u, \mu_{\tilde{X}}(u)) : u \in U, \mu_{\tilde{X}}(u) \in I\}$, where $\mu_{\tilde{X}}(u)$ is namely degree membership of u in $\tilde{X}$, and I^U be the family of all fuzzy subsets of U.

Definition 2.3. [16] Let F_n is the family of all fuzzy sets in general fuzzy numbers, $b \geq 1$ and $\bar{R}$ fuzzy number non negative, fuzzy b- metric space is a triplet $(F_n, \mathcal{B}, \mathrm{b})$, where $\mathcal{B} : F_n \times F_n \to \bar{R}$ be a function satisfy the following statements for all $\tilde{\mathrm{x}}, \tilde{\mathrm{y}}, \tilde{\mathrm{z}} \in F_n$.

1. $\mathcal{B}(\tilde{\mathrm{x}}, \tilde{\mathrm{y}}) = 0$ if and only if $\tilde{\mathrm{x}} = \tilde{\mathrm{y}}$,
2. $\mathcal{B}(\tilde{\mathrm{x}}, \tilde{\mathrm{y}}) = \mathcal{B}(\tilde{\mathrm{y}}, \tilde{\mathrm{x}})$,
3. $\mathcal{B}(\tilde{\mathrm{x}}, \tilde{\mathrm{y}}) \leq \mathrm{b}[\mathcal{B}(\tilde{\mathrm{x}}, \tilde{\mathrm{z}}) + \mathcal{B}(\tilde{\mathrm{z}}, \tilde{\mathrm{y}})]$,

Definition 2.4. [15] Assume that U is a universal set, E is a set of parameters, and $A \subseteq E$. So the pair $(\mathcal{G}, A)$ which tall a soft set under U and defined as a set $\mathcal{G}_A = \{(e, \mathcal{G}_A(e)) : e \in E, \mathcal{G}_A(e) \in P(U)\}$, such that $\mathcal{G}$ a mapping provided as $\mathcal{G} : A \to P(U)$ and $P(U)$ is a power set of U.

Definition 2.5. [17] Let $(\mathcal{G}, A)$ be non-empty soft set, $k \geq 1$ and $R_S(E)^*$ soft real number non-negative, soft b-metric space is a triplet $((\mathcal{G}, A), D, k)$, where $D : (\mathcal{G}, A) \times (\mathcal{G}, A) \to R_S(E)^*$ be a function satisfy the following statements for all $x_\lambda, y_\lambda, z_\lambda \in (\mathcal{G}, A)$.

1. $D(x_\lambda, y_\lambda) = \bar{0}$ if and only if $x_\lambda = y_\lambda$,
2. $D(x_\lambda, y_\lambda) = D(y_\lambda, x_\lambda)$,
3. $D(x_\lambda, y_\lambda) \leq k[D(x_\lambda, z_\lambda) + D(z_\lambda, y_\lambda)]$,

Definition 2.6. [15] A set $(\mathcal{G}, A)$ is refer to be fuzzy soft set over U, whenever $\mathcal{G}$ is mapping $\mathcal{G} : A \to I^U$, and $\{\mathcal{G}(e) \in I^U : e \in A\}$. The collection of all fuzzy soft set, is symbolized by $\mathcal{F}_{SS}(U)$ and fuzzy soft in short denoted by $\mathcal{F}_{SS}$.

Definition 2.7. [18] Assume that $(\mathcal{G}_1, A)$ and $(\mathcal{G}_2, B)$ be two $\mathcal{F}_{SS}-$ sets over the same one set U

1. Then $(\mathcal{G}_1, A)$ namely a $\mathcal{F}_{SS}-$ subset of $(\mathcal{G}_2, B)$ if $A \subseteq B$, and $\mathcal{G}_1(e) \subseteq \mathcal{G}_2(e)$ that is $\mu\mathcal{G}_1(e) \leq \mu\mathcal{G}_2(e)$ for all $e \in A$. and written as $(\mathcal{G}_1, A) \widetilde{\subseteq} (\mathcal{G}_2, B)$.
2. The two $\mathcal{F}_{ss}-$ sets $(\mathcal{G}_1, A)$ and $(\mathcal{G}_2, B)$ are said to be equal $\mathcal{F}_{ss}-$ set, with denoted by $(\mathcal{G}_1, A) \cong (\mathcal{G}_2, B)$, if $(\mathcal{G}_1, A) \widetilde{\subseteq} (\mathcal{G}_2, B)$ and $(\mathcal{G}_2, B) \widetilde{\subseteq} (\mathcal{G}_1, A)$.

Next, we recall the definitions of the intersection and union of $\mathcal{F}_{SS}-$ sets.

Definition 2.8. [19] Consider the $\mathcal{F}_{SS}-$ sets $(\mathcal{G}_2, B)$ and $(\mathcal{G}_1, A)$ over the same universal set U then

1. $(\mathcal{G}_1, A) \tilde{\cup} (\mathcal{G}_2, B) \cong (\mathcal{G}_3, C)$, where $B \cup A = C$ and for all $e \in C, u \in U$

$$\mu\mathcal{G}_3(\mathrm{e})(\mathrm{u}) = \begin{cases} \mu\mathcal{G}_1(\mathrm{e})(u), \text{ ife } \in A - B, u \in \mathrm{U} \\ \mu\mathcal{G}_2(\mathrm{e})(u), \text{ ife } \in B - A, u \in \mathrm{U} \\ \max[\mu\mathcal{G}_1(\mathrm{e})(u), \mu\mathcal{G}_2(\mathrm{e})(u)], \\ \text{ife } \in A \cap B, u \in \mathrm{U} \end{cases}$$

2. $(\mathcal{G}_1, A) \tilde{\cap} (\mathcal{G}_2, B) \cong (\mathcal{G}_3, C)$, where $C = A \cup B$ and for all $\in C$,

$$\mu\mathcal{G}_3(\mathrm{e})(\mathrm{u}) = \begin{cases} \mu\mathcal{G}_1(\mathrm{e})(u), \text{ if } e \in A - B, u \in \mathrm{U} \\ \mu\mathcal{G}_2(\mathrm{e})(u), \text{ if } e \in B - A, u \in \mathrm{U} \\ \min[\mu\mathcal{G}_1(\mathrm{e})(u), \mu\mathcal{G}_2(\mathrm{e})(u)], \\ \text{if } e \in A \cap B, u \in \mathrm{U} \end{cases}$$

Definition 2.9. [20] A $\mathcal{F}_{SS} - \text{set}(\mathcal{G}, A)$ over a universal set U is namely.

1. A set of absolute $\mathcal{F}_{SS}-$ set, represented by C_A, if $\mu\mathcal{G}(e) = 1$ for each $e \in A$.
2. A null $\mathcal{F}_{SS}$ - set, symbolized by $\widetilde{\Phi}$, if for all $e \in A$, we have $\mu\mathcal{G}(e) = 0$.

Now, it important that, recalling definition of $\mathcal{F}_{SS}-$ point with symbolized by $u_{\mu_{\mathcal{G}(e)}}$.

Definition 2.10. [21] The $\mathcal{F}_{SS} - \text{set}(\mathcal{G}, A)$ over U is called $\mathcal{F}_{SS}-$ point and symbolized by $u_{\mu_{\mathcal{G}(e)}}$, if $e \in A$ and $\in U$,

Definition 2.11. [21] Consider the set of all real integers $\mathbb{R}$, where E is a parameter set, $A \subseteq E$ and $\mathcal{F}^{\mathbb{B}(\mathbb{R})}$ be the set of all non-empty bounded fuzzy subsets of $\mathbb{R}$, then (R, A) namely $\mathcal{F}_{SS}-$ real set over $\mathbb{R}$ and is defined as a set of $R_A = \{(e, R_A(e)) : e \in A, R_A(e) \in \mathcal{F}^{\mathbb{B}(\mathbb{R})}\}$, where R is a mapping provide as $R : A \to \mathcal{F}^{\mathbb{B}(\mathbb{R})}$. A is referred to as the support of R_A.

Definition 2.12. [21] (R, A) is namely a $\mathcal{F}_{SS}-$ real number in $\mathbb{R}$, with describe as (r, A) (shortly $\bar{r}$), whenever is a singleton $\mathcal{F}_{ss}-$ real set, such as $\mathbb{R}(A)$ represent the set of each $\mathcal{F}_{SS}-$ real values and $\mathbb{R}^+(A)$ represents the collection of all $\mathcal{F}_{SS}-$ real values that are not negative.

3. Fuzzy soft b- metric

Now, we generalize the definition of b-metric space on $\mathcal{F}_{SS}-$ sets, with introduce some characterization of this concept.

Definition 3.1. Let $\mathcal{F}_{SS}(U)$ be the family of all nonempty $\mathcal{F}_{SS}-$ sets on universal U, a $\mathcal{F}_{SS}-$ b-metric space is $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$, where $\widetilde{\mathcal{B}} : \mathcal{F}_{SS}(U) \times \mathcal{F}_{SS}(U) \to \mathbb{R}^+(\mathrm{A})$ be a function with $S \geq 1$ satisfy the following statements for all $u_{\mu^1_{\mathcal{G}(e_1)}}, u^2_{\mu_{\mathcal{G}(e_2)}}, u^3_{\mu_{\mathcal{G}(e_3)}} \widetilde{\in} \mathcal{F}_{SS}(U)$

1. $\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, u^2_{\mu_{\mathcal{G}(e_2)}}\right) \geq 0$

2. $\widetilde{\mathcal{B}}\left(u^1_{\mu^1_{\mathcal{G}(e_1)}}, u^1_{\mu_{\mathcal{G}(e_1))}}\right) = 0$

3. $\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}},\quad u^2_{\mu_{\mathcal{G}(e_2)}}\right) = \widetilde{\mathcal{B}}\left(u^2_{\mu_{\mathcal{G}(e_2)}}, u^1_{\mu_{\mathcal{G}(e_1)}}\right)$

4. $\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, u^3_{\mu_{\mathcal{G}(e_3)}}\right) \leq$ S $\left[\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, u^2_{\mu_{\mathcal{G}(e_2)}}\right) + \widetilde{\mathcal{B}}\left(u^2_{\mu_{\mathcal{G}(e_2)}}, u^3_{\mu_{\mathcal{G}}(e_3)}\right)\right]$, $\mathcal{F}_{SS}-$ b-metric space in short written as $(\mathcal{F}_{SS} - BMS)$.

The following remakes explain the relation among b-metric space, fuzzy b-metric space, and fuzzy soft b-metric space

Remakes 3.2.

1. b- metric space is special case of fuzzy b- metric space.
2. fuzzy b- metric space is special case of soft b- metric space .so the aide of soft b- metric space is more general than the aide fuzzy b- metric.
3. soft b- metric space is special case of fuzzy soft b- metric space so the aide of fuzzy soft b- metric space is more general than the aide soft b- metric.

The following example explain $\mathcal{F}\_SS - BMS$.

Example 3.3. Let $U = \{u_1, u_2, u_3\}$ and $E = \{e\}$ a set of parameters. Then $\mathcal{F}_{SS}(U) = \widetilde{\widetilde{\mathcal{B}}}\left(u_{i_{\mathcal{G}(e)}}, u_{i_{\mu_{\mathcal{G}}(e)}}\right) = \bar{0}$ for all $u_{i_{\mu_{\mathcal{G}}(e)}} \widetilde{\widetilde{E}} \mathcal{F}_{SS}(U), i, j = \{1, 2, 3\}$

$$\widetilde{\widetilde{\mathcal{B}}}\left(u_{i_{\mu_{\mathcal{G}}(e)}}, u_{j_{\mu_{\mathcal{G}}(e)}}\right) = \widetilde{\widetilde{\mathcal{B}}}\left(u_{j_{\mu_{\mathcal{G}}(e)}}, u_{i_{\mu_{\mathcal{G}}(e)}}\right)$$

$$\widetilde{\widetilde{\mathcal{B}}}\left(u^1_{\mu_{\mathcal{G}}(e),}, u^2_{\mu_{\mathcal{G}}(e)}\right) = \overline{43}, \widetilde{\widetilde{\mathcal{B}}}\left(u^1_{\mu_{\mathcal{G}}(e)}, u^3_{\mu_{\mathcal{G}(e)}}\right) = \overline{1590}, \widetilde{\widetilde{\mathcal{B}}}\left(u^2_{\mu_{\mathcal{G}(e)}}, u^3_{\mu_{\mathcal{G}(e)}}\right) = \overline{63}$$

Then $\left(\mathcal{F}_{SS}(U), \widetilde{\widetilde{\mathcal{B}}}\right)$ is $\mathcal{F}_{SS} - BMS$ with $S = 15$.

Remakes 3.4. Let Let $\left(\mathcal{F}\_SS(U), \widetilde{\mathcal{B}}\right)$ be $\mathcal{F}\_SS - BMS$ and $(\mathcal{G}, A), (\mathcal{G}, B) \widetilde{\approx} \mathcal{F}\_SS(U)$.

1. The distance between $(\mathcal{G}, A)$ and $(\mathcal{G}, B)$ denoted by $\widetilde{\mathcal{B}}((\mathcal{G}, A), (\mathcal{G}, B))$ is defined as $\widetilde{\mathcal{B}}((\mathcal{G}, A), (\mathcal{G}, B)) = \inf\left\{\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(a)}}, u_{\mu_{\mathcal{G}(b)}}\right) : u_{\mu_{\mathcal{G}(a)}} \widetilde{\in} (\mathcal{G}, A), u_{\mu_{\mathcal{G}(b)}} \widetilde{\in} (\mathcal{G}, B)\right\}$.
2. If $\left\{u_{\mu_{\mathcal{G}(a)}}\right\} \widetilde{\widetilde{\notin}} (\mathcal{G}, B)$, then $\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(a)}}, (\mathcal{G}, B)\right) = \inf\left\{\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(a)}}, u_{\mu_{\mathcal{G}(b)}}\right) : u_{\mu_{\mathcal{G}(b)}} \widetilde{\in} (\mathcal{G}, B)\right\}$ is called distance of point $u_{\mu_{\mathcal{G}(a)}}$ from the set $(\mathcal{G}, B)$. In case $\left\{u_{\mu_{\mathcal{G}(a)}}\right\} \widetilde{\in} (\mathcal{G}, B)$ then $\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(a)}}, (\mathcal{G}, B)\right) = \bar{0}$
3. For any distance poin $u^1_{\mu_{\mathcal{G}(e_1)}}, u^2_{\mu_{\mathcal{G}(e_2)}} \widetilde{\in} \mathcal{F}_{SS}(U)$, one can have $\left|\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, (\mathcal{G}, A)\right) - \widetilde{\mathcal{B}}\left(u^2_{\mu_{\mathcal{G}(e_2)}}, (\mathcal{G}, A)\right)\right| \leq \widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, u^2_{\mu_{\mathcal{G}(e_2)}}\right)$
4. The diameter of $(\mathcal{G}, A)$ denoted by $\widetilde{\widetilde{\mathcal{B}}}((\mathcal{G}, A))$ is define as $\widetilde{\mathcal{B}}((\mathcal{G}, A)) = \sup\left\{\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(a_1)}}, u^2_{\mu_{\mathcal{G}(a_2)}}\right) : u^1_{\mu_{\mathcal{G}(a_1)}}, u^2_{\mu_{\mathcal{G}(a_2)}} \widetilde{\in} (\mathcal{G}, A)\right\}$
   In case $(\mathcal{G}, A)$ an empty set, following convention are adopted
   a) $\widetilde{\mathcal{B}}(\widetilde{\Phi}) = -\infty$, some authors take $\widetilde{\mathcal{B}}(\widetilde{\Phi}) = 0$.
   b) $\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, \widetilde{\Phi}\right) = \infty$, that means distance of a point $u_{\mu_{\mathcal{G}(e)}}$ from empty set is ∞.
   c) $\widetilde{\mathcal{B}}((\mathcal{G}, B), \widetilde{\Phi}) = \infty$, where $(\mathcal{G}, B)$ is any $\mathcal{F}_{SS}-$ set which isn't empty.
5. $(\mathcal{G}, A)$ namely bounded if a diameter of $(\mathcal{G}, A)$ is finite that means $\widetilde{\mathcal{B}}((\mathcal{G}, A)) < \infty$. Now, we will discussion the $\mathcal{F}_{SS}-$ sequence in $\mathcal{F}_{SS} - BMS$.

Definition 3.5. Let $\left\langle u^n_{\mu_{\mathcal{G}(en)}}\right\rangle$ be a $\mathcal{F}_{SS}-$ sequence in a $\mathcal{F}_{SS} - BMS\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$, we say

1. $\left\langle u^n_{\mu_{\mathcal{G}(en)}}\right\rangle$ converges to $u_{\mu_{\mathcal{G}(e_0)}} \tilde{\in} \mathcal{F}_{SS}(U)$, when for each $\tilde{\epsilon} > \tilde{0}$ there is $\bar{\delta} > \bar{0}$ with positive integer $N = N(\tilde{\epsilon})$ such as $\widetilde{\mathcal{B}}\left(u^n_{\mu_{\mathcal{G}(en)}}, u_{\mu_{\mathcal{G}(e_0)}}\right) < \bar{\delta}$ implies $\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}(e_0)}(s)\right| < \tilde{\varepsilon}$ whenever $n \geq N, \bar{\delta} \in \mathbb{R}^+(A)$ and $\tilde{\varepsilon} \tilde{\in} [0, 1]$. It is usually denoted as $\lim_{n\to\infty} u^n_{\mu_{\mathcal{G}(en)}} = u_{\mu_{\mathcal{G}(e_0)}}$.
2. $\left\langle u^n_{\mu_{\mathcal{G}(en)}}\right\rangle \mathcal{F}_{ss}-$ Cauchy sequence when for every $\tilde{\epsilon} > \tilde{0}$ there is $\bar{\delta} > \bar{0}$ with a positive integer $N = N(\tilde{\epsilon})$ such as $\widetilde{\mathcal{B}}\left(u^n_{\mu_{\mathcal{G}(en)}}, u^m_{\mu_{\mathcal{G}(em)}}\right) \leq \tilde{\delta}$ implies $\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}(e_m)}(s)\right| < \tilde{\varepsilon}$

   whenever $n, m \geq N$ that is $\widetilde{\mathcal{B}}\left(u^n_{\mu_{\mathcal{G}(en)}}, u^m_{\mu_{\mathcal{G}}(e_m)}\right) \to \tilde{0}$ as $n, m \to \infty$.
3. if every $\mathcal{F}_{SS}-$ Cauchy sequence in $\mathcal{F}_{SS} - BMS\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ to be $\mathcal{F}_{SS}-$ convergent sequence, then $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ is namely complete $\mathcal{F}_{SS} - BMS$.

Definition 3.6. Let $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ be $\mathcal{F}_{SS} - BMS$, then for all $u_{\mu_{\mathcal{G}(a)}} \widetilde{\in} \mathcal{F}_{SS}(U), \bar{r} \in \mathbb{R}^+(A), \tilde{\varepsilon} \widetilde{\in} (0, 1)$ the $\mathcal{F}_{SS}-$ open ball define by $\widetilde{\mathcal{B}}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(a)}}\right) = \left\{u_{\mu_{\mathcal{G}(e)}} \widetilde{\mathcal{E}} \mathcal{F}_{SS}(U) : \widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, u_{\mu_{\mathcal{G}(a)}}\right) < \bar{r}\right\}$,

with $\mid u_{\mu_{\mathcal{G}(e)}}(s) - u_{\mu_{\mathcal{G}(a)}}(s) \mid < \tilde{\varepsilon}$ and $\mathcal{F}_{SS} -$ closed ball by $\widetilde{\mathcal{B}}_{\bar{r}}\left[u_{\mu_{\mathcal{G}(a)}}\right] = \left\{u_{\mu_{\mathcal{G}(e)}} \widetilde{\in} \mathcal{F}_{SS}(U) : \widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, u_{\mu_{\mathcal{G}(a)}}\right) \leq \bar{r}\right\}$,

with $\left|u_{\mu_{\mathcal{G}(e)}}(s) - u_{\mu_{\mathcal{G}(a)}}(s)\right| \leq \tilde{\varepsilon}$

Definition 3.7. Let $\left(\mathcal{F}_{sS}(U), \widetilde{\mathcal{B}}\right)$ be $\mathcal{F}_{sS} - BMS$ and $(\mathcal{G}, A) \widetilde{\subseteq} \mathcal{F}_{sS}(U)$, the $\mathcal{F}_{ss}$ - point $u_{\mu_{\mathcal{G}(e_0)}}$, is $\mathcal{F}_{SS}-$ limit point of $(\mathcal{G}, A)$ if and only if for every $\overline{0} < \bar{r} \in \mathbb{R}^{+}(A)$ we have $\widetilde{\mathcal{B}}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e_0)}}\right) \widetilde{\cap} (\mathcal{G}, A) \neq \Phi$

Theorem 3.8. Let $\left\langle u^{n}_{\mu_{\mathcal{G}(e_n)}} \right\rangle$ be a $\mathcal{F}_{sS}-$ sequence in a $\mathcal{F}_{sS} - BMS\left(\mathcal{F}_{sS}(U), \widetilde{\mathcal{B}}\right)$, then $\mathcal{F}_{ss}$ -limit point of $\left\langle u_{\mu^{n}_{\mathcal{G}(e_n)}} \right\rangle$ is unique.

Proof. Let $\left\langle u_{\mu^{n}_{\mathcal{G}(e_n)}} \right\rangle$ be a $\mathcal{F}_{ss}-$ sequence in $\left(\mathcal{F}_{sS}(U), \widetilde{\mathcal{B}}\right)$ such that $u^{n}_{\mu_{\mathcal{G}}(e_n)} \to u^{1}_{\mu_{\mathcal{G}(e_1)}}$ and $u^{n}_{\mu_{\mathcal{G}(e_n)}} \to u^{2}_{\mu_{\mathcal{G}(e_2)}}$ we will prove that $u^{1}_{\mu_{\mathcal{G}(e_1)}} = u^{2}_{\mu_{\mathcal{G}(e_2)}}$

Assume that $u^{1}_{\mu_{\mathcal{G}(e_1)}} \neq u^{2}_{\mu_{\mathcal{G}(e_2)}}$, then $\widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) > \tilde{0}$, choose $\bar{\delta} = \widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)$

Now, Since $u^{n}_{\mu}G_{(en)} \to u^{1}{}_{\mu}G(e1)$, for each $\varepsilon^{\sim}_{1} > 0^{\sim}$, there is $\sigma^{-}_{1} > 0^{-}$ and $\mathrm{N}_1 = \mathrm{N}_1(\varepsilon_1)$ such that $\widetilde{\mathcal{B}}\left(u^{n}_{\mu_{\mathcal{G}}(e_n)}, u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) < \bar{\sigma}_1$ implies $\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}(e_1)}(s)\right| < \widetilde{\varepsilon_1}$ Similarly, there is $\bar{\sigma}_2 > \overline{0}$ for every $\widetilde{\varepsilon_2} > \tilde{0}$ and $N_2 = N_2(\tilde{\varepsilon}_2)$ such that $\widetilde{\mathcal{B}}\left(u^{n}_{\mu_{\mathcal{G}(e_n)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) < \bar{\sigma}_2$ implies $\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}(e_2)}(s)\right| < \widetilde{\varepsilon_2}$

Construct fuzzy soft open balls $\widetilde{\mathcal{B}}_{\bar{r}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}\right)$ and $\widetilde{B}_{\bar{r}}\left(u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)$ with radius $\bar{r} \leq \frac{\bar{\delta}}{2S}$ and centers $u^{1}_{\mu_{\mathcal{G}(e_1)}}$ and $u^{2}_{\mu_{\mathcal{G}(e_2)}}$ where $\tilde{\varepsilon} = \min\{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\}$ such that they are disjoint

IF $u_{\mu_{\mathcal{G}(a)}} \widetilde{\in} \widetilde{B}_{\bar{r}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) \widetilde{\cap} \widetilde{B}_{\bar{r}}\left(u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)$

With $\left|\mu_{\mathcal{G}(e_1)}(s) - \mu_{\mathcal{G}(a)}(s)\right| \leq \frac{\tilde{\varepsilon}}{2S}$ and $\left|\mu_{\mathcal{G}(e_2)}(s) - \mu_{\mathcal{G}(a)}(s)\right| \leq \frac{\tilde{\varepsilon}}{2S}$

Then $\bar{\delta} = \widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) \leq \mathrm{S}\left[\widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u_{\mu_{\mathcal{G}(a)}}\right) + \widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(a)}},\right.\right.$

$\left.\left. u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)\right] \leq \mathrm{S}[\bar{r} + \bar{r}] \leq \mathrm{S}[\bar{\delta}/2\,\mathrm{S} + \bar{\delta}/2\,\mathrm{S}] =$

$\bar{\delta}, \left|\mu_{\mathcal{G}(e_1)}(s) - \mu_{\mathcal{G}(e_2)}(s)\right| \leq S\left[\left|\mu_{\mathcal{G}(e_1)}(s) - \mu_{\mathcal{G}(a)}(s)\right| + \right.$

$\left.\left|\mu_{\mathcal{G}(a)}(s) - \mu_{\mathcal{G}(e_2)}(s)\right|\right. \leq S\left[\frac{\tilde{\varepsilon}}{2S} + \frac{\tilde{\varepsilon}}{2S}\right] < \tilde{\varepsilon}$

thus, $u^{n}_{\mu_{\mathcal{G}(e_n)}} \widetilde{\in} \widetilde{\mathcal{B}}_{\bar{r}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) \widetilde{\cap} \widetilde{\mathcal{B}}_{\bar{r}}\left(u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)$

This contradicts the fact that the fuzzy soft balls are disjoint. So, we conclude that $u^{1}_{\mu_{\mathcal{G}(e_1)}} \cong u^{2}_{\mu_{\mathcal{G}(e_2)}}$

Definition 3.9. A $\mathcal{F}_{SS} - \mathrm{set}(\mathcal{G}, A)$ in a $\mathcal{F}_{SS} - BMS\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ is said to be $\mathcal{F}_{SS}$-open if for each $\mathcal{F}_{SS}$ - point $u_{\mu_{\mathcal{G}(e)}}$ of $(\mathcal{G}, \mathrm{A})$ there exist a $\mathcal{F}_{SS}$-open ball $\widetilde{\mathcal{B}}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e)}}\right) \widetilde{\subseteq} (\mathcal{G}, A)$.

Theorem 3.10. If $\left(\mathcal{F}_{\mathrm{sS}}(\mathrm{U}), \widetilde{\mathcal{B}}\right)$ be $\mathcal{F}_{\mathrm{sS}} - \mathrm{BMS}$ then the sphere $\widetilde{\mathcal{B}}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e)}}\right)$ is $\mathcal{F}_{\mathrm{ss}}$-open set.

Proof: Let $u_{\mu_{\mathcal{G}(e_k)}} \widetilde{\in} \widetilde{\mathcal{B}}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e)}}\right)$ then $\widetilde{\mathcal{B}}\left(\mathrm{u}_{\mu_{\mathcal{G}}(\mathrm{e})}, \mathrm{u}_{\mu_{\mathcal{G}(\mathrm{e_k})}}\right) < \bar{r}$. With $\left|\mu_{\mathcal{G}(e)}(s) - \mu_{\mathcal{G}(e_k)}(s)\right| < \tilde{\varepsilon}$ choos $\left|\mu_{\mathcal{G}(e)}(s) - \mu_{\mathcal{G}(e_k)}(s)\right| = \frac{\tilde{\epsilon}_1}{S}$ Then we can consider a fuzzy soft real number $\frac{\overline{r_1}}{S} < -\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, u_{\mu_{\mathcal{G}(e_k)}}\right) + \frac{\bar{r}}{S}$ where $S \geq 1$ and a real number $\tilde{\epsilon}_2$ in $(0, 1)$ such that $\tilde{\epsilon}_2 < \tilde{\epsilon} - \tilde{\epsilon}_1$, also $u_{\mu_{\mathcal{G}(e_Z)}} \tilde{\in} \tilde{B}_{\frac{\bar{r}_1}{S}}\left(u_{\mu_{\mathcal{G}(e_k)}}\right)$ that is, $\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e_z)}}, u_{\mu_{\mathcal{G}(e_k)}}\right) < \frac{\overline{r_1}}{S}$ with $\left|\mu_{\mathcal{G}(e_z)}(s) - \mu_{\mathcal{G}(e_k)}(s)\right| < \frac{\tilde{\epsilon}_2}{S}$ $\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, u_{\mu_{\mathcal{G}(e_Z)}}\right) \leq S\left[\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, u_{\mu_{\mathcal{G}(e_k)}}\right) + \widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}}(e_k)}, u_{\mu_{\mathcal{G}(e_Z)}}\right)\right] < S\left[\widetilde{\mathcal{B}}\left(u_{\mu_{\mathcal{G}(e)}}, u_{\mu_{\mathcal{G}(e_k)}}\right) + \frac{\overline{r_1}}{S}\right] < S\left[\frac{\bar{r}}{S}\right] = \bar{r}, \left|\mu_{\mathcal{G}(e)}(s) - \mu_{\mathcal{G}(e_k)}(s)\right| \leq S\left[\left|\mu_{\mathcal{G}(e)}(s) - \mu_{\mathcal{G}(e_k)}(s)\right| + \left|\mu_{\mathcal{G}(e_k)}(s) - \mu_{\mathcal{G}(e_Z)}(s)\right|\right] < S\left[\frac{\tilde{\epsilon}_1}{S} + \frac{\tilde{\epsilon}_2}{S}\right] =< \tilde{\epsilon}$

Thus $\widetilde{B}\frac{\overline{r_1}}{S}\left(u_{\mu_{\mathcal{G}(e_k)}}\right) \widetilde{\subseteq} \tilde{B}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e)}}\right)$, since $u_{\mu_{\mathcal{G}(e_k)}}$ is an arbitrary element of $\tilde{B}_{\bar{r}}(\; u_{\mu_{\mathcal{G}}(e)} \;)$ we get $\tilde{B}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e)}}\right)$ is a fuzzy soft open.

Definition 3.11. Let $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ be a $\mathcal{F}_{SS} - BMS$ having at least two $\mathcal{F}_{SS}$-points then $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ is said to be $\mathcal{F}_{SS}-$ Hausdorff-BMS if for any distance points in $\mathcal{F}_{SS}(U)$ if there exists two $\mathcal{F}_{SS}-$ open ball $\tilde{B}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e_1)}}\right)$ and $\tilde{B}_{\bar{r}}\left(u_{\mu_{\mathcal{G}(e_2)}}\right)$ suchthat $\tilde{B}_{\bar{r}}\left(u_{\mu_{G(e_1)}}\right) \cap \tilde{B}_{\bar{r}}\left(u_{\mu_{G(e_2)}}\right) = \widetilde{\Phi}$ for each $u_{\mu_{G(e_1)}}$ $u_{\mu_{G(e_2)}}$

Proposition 3.12. Every $\mathcal{F}_{sS} - BMS\left(\mathcal{F}_{sS}(U), \widetilde{\mathcal{B}}\right)$ is $\mathcal{F}_{sS}-$ Hausdorff-BMS. Proof: Let $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ be a $\mathcal{F}_{SS} - BMS$ having at least two $\mathcal{F}_{SS}-$ points. Let $u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}$ be two $\mathcal{F}_{SS}-$ points in $\mathcal{F}_{SS}(U)$ such that. $\widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) > 0$ Choose any fuzzy soft real number $\bar{r}$ such that

$0 < \bar{r} < \frac{1}{2\,\mathrm{S}} \widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)$, where $\mathrm{S} \geq 1$. Consider two $\mathcal{F}_{SS}-$ open balls

$$\tilde{B}_{\bar{r}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) = \left\{u^{1}_{\mu_{\mathcal{G}(a_1)}} \widetilde{\in} \mathcal{F}_{SS}(U) : \widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(a_1)}}, u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) < \bar{r}\right\}$$

$$\tilde{B}_{\bar{r}}\left(u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) = \left\{u^{2}_{\mu_{\mathcal{G}(a_2)}} \widetilde{\in} \mathcal{F}_{SS}(U) : \widetilde{\mathcal{B}}\left(u^{2}_{\mu_{\mathcal{G}(a_2)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) < \bar{r}\right\}$$

Let $u^{3}_{\mu_{\mathcal{G}(e_3)}} \widetilde{\in} \tilde{B}_{\bar{r}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) \widetilde{\cap} \tilde{B}_{\bar{r}}\left(u^{2}_{\mu_{\mathcal{G}(e_2)}}\right)$ then

$$u^{3}_{\mu_{\mathcal{G}(e_3)}} \widetilde{\in} \tilde{B}_{\bar{r}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}\right) \text{ so } \widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{3}_{\mu_{\mathcal{G}(e_3)}}\right) < \bar{r}$$

$$u^{3}_{\mu_{\mathcal{G}(e_3)}} \widetilde{\in} \tilde{B}_{\bar{r}}\left(u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) \text{ so } \widetilde{\mathcal{B}}\left(u^{3}_{\mu_{\mathcal{G}(e_3)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) < \bar{r}$$

by Condition 4 of a $\mathcal{F}_{SS} - BMS$ produce $\widetilde{\mathcal{B}}\left(u^{1}_{\mu_{\mathcal{G}(e_1)}}, u^{2}_{\mu_{\mathcal{G}(e_2)}}\right) \leq$

$S\left[\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, u^3_{\mu_{\mathcal{G}(e_3)}}\right) + \widetilde{\mathcal{B}}\left(u^3_{\mu_{\mathcal{G}(e_3)}}, u^2_{\mu_{\mathcal{G}(e_2)}}\right)\right] < S[\bar{r} + \bar{r}] = 2S\bar{r}$

Therefore $\bar{r} > \frac{1}{2S}\widetilde{\mathcal{B}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}, u^2_{\mu_{\mathcal{G}(e_2)}}\right)$ which contradicts the hypothesis. So clearly, $\widetilde{\mathcal{B}}_{\bar{r}}\left(u^1_{\mu_{\mathcal{G}(e_1)}}\right) \cap \widetilde{\mathcal{B}}_{\bar{r}}\left(u^2_{\mu_{\mathcal{G}(e_2)}}\right) = \widetilde{\Phi}$ and henc $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$ is $\mathcal{F}_{SS}-$ Hausdorff-BMS.

Theorem 3.13. Suppose that $\left(\mathcal{F}_{sS}(U), \widetilde{\mathcal{B}}\right)$ be $\mathcal{F}_{sS} - BMS$. Then $\left(\mathcal{F}_{sS}(U), \widetilde{\mathcal{B}}\right)$ becomes complete space if for every $\mathcal{F}_{SS}-$ Cauchy sequence in $\mathcal{F}_{SS}(U)$ has a convergent subsequence.

Proof: Let $\left\langle u_{\mu^n_{\mathcal{G}(en)}}\right\rangle$ be a $\mathcal{F}_{SS}$-Cauchy sequence in $\left(\mathcal{F}_{SS}(U), \widetilde{\mathcal{B}}\right)$. We show that if $\left\langle u_{\mu^n_{\mathcal{G}(en)}}\right\rangle$ has a subsequence $\left\langle u^{n_k}\mu_{\mathcal{G}\left(e_{n_k}\right)}\right\rangle$ such that goes to $\mathcal{F}_{SS}$-point $u_{\mu_{\mathcal{G}(e_0)}}$ then the $\mathcal{F}_{SS}$-sequence $\left\langle u^n_{\mu_{\mathcal{G}(en)}}\right\rangle$ itself converges to $u_{\mu_{\mathcal{G}(e_0)}}$

Given $\tilde{\varepsilon} > \tilde{0}$, there exists a $\tilde{\delta} > \tilde{0}$ satisfying the conditions first choose $\mathrm{N} = \mathrm{N}(\tilde{\varepsilon})$ large enough that $\widetilde{\mathcal{B}}\left(u^n_{\mu_{\mathcal{G}(en)}}, u^m_{\mu_{\mathcal{G}(em)}}\right) \leq \frac{\tilde{\delta}}{2S}$, for all $n, m \geq N$ implies $\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}(e_m)}(s)\right| < \frac{\tilde{\varepsilon}}{2S}$ then choose $n_i \geq N$ and $\widetilde{\mathcal{B}}\left(u^{n_i}\mu_{\mathcal{G}\left(e_{n_i}\right)}{}' u_{\mu_{\mathcal{G}(e_0)}}\right) < \frac{\tilde{\delta}}{2S}$ with $\left|\mu_{G\left(e_{n_i}\right)}(s) - \mu_{\mathcal{G}(e_0)}(s)\right| \frac{\tilde{\varepsilon}}{2S}$ using the fact that $n_1 < n_2 < \ldots$ which is monotone sequence exactly increasing sequence of integers and $\left\langle u^{n_i}\mu_{\mathcal{G}\left(e_{n_i}\right)}\right\rangle$ goes to $u_{\mu_{\mathcal{G}(e_0)}}$, thus $n \geq N$ so one get; $\widetilde{\mathcal{B}}\left(u_{\mu^n_{\mathcal{G}(en)}}, u_{\mu_{\mathcal{G}(e_0)}}\right) \leq S\left[\widetilde{\mathcal{B}}\left(u^n_{\mu_{\mathcal{G}(en)}}, u^{n_i}\mu_{\mathcal{G}\left(e_{n_i}\right)}\right) + \widetilde{\mathcal{B}}\left(u^{n_i}\mu_{\mathcal{G}\left(e_{n_i}\right)}, u_{\mu_{\mathcal{G}(e_0)}}\right)\right] \leq S\left[\frac{\tilde{\delta}}{2S} + \frac{\tilde{\delta}}{2S}\right] = \tilde{\delta}$ $\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}(e_0)}(s)\right| \leq S\left[\left|\mu_{\mathcal{G}(e_n)}(s) - \mu_{\mathcal{G}\left(e_{n_i}\right)}(s)\right| + \mid \mu_{\mathcal{G}\left(e_{n_i}\right)}\left(s - \mu_{\mathcal{G}(e_0)}(s) \mid\right]\right.$ $< S\left[\frac{\tilde{\varepsilon}}{2S} + \frac{\tilde{\varepsilon}}{2S}\right] = \tilde{\varepsilon}$.

4. Conclusions

We established soft fuzzy b- metric in this paper, which differs from the concept of $\mathcal{F}_{SS}$-metric found in some sources. Inside this new space, we looked at various topological structures and in theoretical mathematics sciences we can refine our findings.

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