MODELING ACTIVE GROUP DEFENSE MECHANISMS IN PREY-PREDATOR INTERACTIONS

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This article focuses on the study of active group defense in prey, where they defend themselves against attacking predators as a group. The aim of this paper is to create a mathematical model to examine the functional response resulting from active group defense. The model integrates a mechanism involving the joining and separation of prey individuals in response to attacks. The formation of defense clusters occurs as a result, with the attacking predator acting as the central point of cohesion. The study also takes into account the metabolic costs of defense, which can reduce the growth rate of the prey population. The study also investigates the conditions under which the prey can successfully defend itself against the predator and how this affects the stability of the system. Overall, this study provides a deeper understanding of the relationships between predators and prey, and the factors that influence their dynamics. Conventional predator-prey research often overlooks the dynamic coagulation and fragmentation processes involved in collective prey defense. This novel study introduces a unique perspective, shedding light on how prey species unite against predators. The manuscript's originality lies in its exploration of coagulation and fragmentation as crucial to unraveling prey defense strategies. A mathematical model is introduced, delving into these processes and unveiling an unexplored facet of predator-prey interactions. The findings enhance our understanding of collaborative prey defense mechanisms and hold broader applications, amplifying the significance of this research.

Keywords: Defense mechanism, coagulation, fragmentation, clusters, Geritz and Gyllenberg model.

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1. Introduction

The study of prey-predator relationships is an endlessly captivating and critically important area of research that holds far-reaching implications for the intricate workings of ecosystems and the conservation of our planet. Whether through cutting-edge technology or traditional fieldwork, the exploration of prey-predator relationships stands as a fundamental and indispensable endeavor that continues to yield valuable insights into the dynamics of the natural world.[1]

In the vast realm of nature, prey-predator relationships reign supreme as one of the most common and consequential interactions. The underlying mechanisms that govern these relationships have a profound impact on the balance of ecosystems and the survival of species. To comprehend and predict the behavior of the animals involved, scientists have developed prey-predator models, mathematical representations that capture the essence of these intricate relationships [2–4]. In this enlightening article, we embark on a journey through one such model, specifically focusing on the incorporation of defense mechanisms.

Defense mechanisms play a paramount role in shaping the dynamics of prey-predator relationships. Recent research has shed new light on the multifaceted nature of these defense mechanisms, revealing their incredible diversity and sophistication. Among the various forms of defense, they can be broadly categorized into three types: permanent, constitutive defenses (such as carapaces, horns, or crypsis); temporary, inducible defenses; and permanent, reducible defenses. Each type has its own unique attributes and adaptations, enabling prey species to navigate the ever-present threat of predation.[5]

As prey species evolve and adapt to minimize predation, predators, in turn, employ strategies to maximize their capture success. This evolutionary rise gives a captivating arms race, where prey species develop an array of defense mechanisms, and predators continuously refine their tactics to overcome these barriers. Mechanical defenses exemplify the ingenuity of prey species, with examples such as the sturdy shells of turtles that inflict pain upon predators, dissuading further pursuit.[6] Chemical defenses, like those found in millipedes, combine chemical and mechanical deterrents, as these creatures emit noxious substances when threatened. Additionally, prey species utilize body shape and coloration as cunning tools of evasion, adopting camouflage strategies to blend seamlessly with their surroundings. The incredible color-changing abilities of chameleons and the ingenious mimicry displayed by non-venomous insects imitating the warning coloration of harmful species are but a few examples of the diverse repertoire of defense mechanisms at play. Within the realm of mathematical modeling, the Lotka-Volterra model serves as a versatile framework to explore prey-predator dynamics. To incorporate defense mechanisms into this model, a parameter representing the effectiveness of the prey's defenses is introduced. This parameter acts as a scaling factor that modifies the predation rate, accounting for the impact of defense mechanisms on the relationship between prey and predator. By modulating the predation term, the model allows us to investigate the dynamic interplay between prey density, defense effectiveness, and predation rates, ultimately shedding light on the stability of the system. [7, 8]

Delving deeper into the captivating world of defense mechanisms, the model illuminates the incredible adaptations and responses of prey species. Group defense mechanisms, where prey unites and fights back collectively against predators, take center stage in this exploration. By formulating a model that captures the essence of the active group defense, we can unveil the functional response resulting from such behavior. The clusters formed during these defense processes embody the collective strength of the prey, with the predator serving as the primary force that unites them. These clusters exhibit dynamic behavior, capable of both growth and contraction as prey join or leave, and they may disband entirely based on the success or failure of the attack or defense. In essence, this coagulation and fragmentation process represents a broader framework of the well-known Becker-Döring equations, showcasing the richness and complexity of prey-predator interactions [9].

Stability analysis is a crucial aspect of understanding the dynamics of prey-predator systems. The analysis reveals the existence of unique and stable equilibria, even when considering arbitrarily large maximum cluster sizes. These findings provide a deeper understanding of the intricate dynamics at play and underscore the resilience and adaptability of prey species [10].

Beyond the confines of theoretical models, the real world presents additional factors to consider the metabolic costs of defense mechanisms, for instance, can impact the growth rate of prey populations. This model showcases the effect of defense mechanisms on the predation process, demonstrating that predators take more time to consume prey when defense mechanisms are at play. Notably, this effect is more pronounced when prey invest in a large reservoir as opposed to rapid secretion restoration [11]. By exploring longer time scales and stationary states, the model reveals that defense mechanisms can significantly increase the abundance of both predators and prey. However, the investment in defense only proves advantageous when predator density reaches a certain threshold, and the costs of defense remain within manageable limits.[12]

In conclusion, the study of prey-predator relationships unveils a captivating world of interactions that holds paramount importance in ecological understanding and conservation efforts. The intricate adaptations of prey species to minimize predation, coupled with the remarkable strategies employed by predators to maximize their capture success, collectively shape the delicate balance of ecosystems. Incorporating defense mechanisms into mathematical models enriches our comprehension of these relationships, allowing us to survey the impacts of varying parameters on the dynamics and stability. Through continuous research and exploration in this fascinating field, we stand poised to unlock further insights into the awe-inspiring world of prey-predator interactions and their profound significance in the natural realm.

The paper's introduction surveys existing literature on predator-prey dynamics, pinpointing research gaps and advocating for a grasp of collective defense mechanisms. The study's primary aim is to introduce a fresh perspective on predator-prey dynamics, illuminating how prey species collaborate against predators. A significant focus is on coagulation and fragmentation processes driving these strategies.

In Section 1.1, coagulation and fragmentation processes are explored as distinct interactions, pivotal in understanding group dynamics. Parallels are drawn to the Becker-Doring equations and their relevance in homogeneous clusters. This section delves into the intricate transformations within groups. Section 1.2 elaborates on the Geritz and Gyllenberg Model for Coagulation and Fragmentation. This model examines prey defense via coagulation and fragmentation, aligning with Becker-Doring principles. It details group formation, distinguishing collective and individual responses, and introduces the concept of quasi-equilibrium. Section 1.3 delves into the Mathematical Modeling of Prey Cooperation and Active Defense. It introduces an extended model incorporating cluster formation due to predators. The processes of cluster growth, contraction, and dissolution are meticulously explained. Equations governing predator-prey interactions within clusters are elucidated, integrating birth and death processes for realism. The conclusion encapsulates findings on prey defense, highlighting novelty and significance. It emphasizes deepened understanding of predator-prey dynamics and potential applications in ecological and conservation contexts.

1.1. Coagulation and fragmentation processes

Coagulation and fragmentation processes represent a distinct and noteworthy category of interactions observed in various models. These processes elucidate the dynamics of group size, involving the growth and reduction of particle clusters. At their core, the Becker-Doring equations capture the behavior of homogeneous clusters, wherein the groups can incrementally expand or diminish by one unit at a time. These equations encapsulate the remarkable ability of these clusters to undergo continuous transformations, embodying a dynamic and ever-changing landscape of particle interactions.

$$\begin{cases} \dot{n}_1 = -a_1 u_1 n_1 + b_2 n_2 - \sum_{k=1}^{k-1} \left(a_k n_k n_1 + b_{k+1} n_{k+1}\right) \\ \dot{n}_1 = a_1 n_1 n_1 + b_3 n_3 - a_2 n_2 n_1 - b_2 n_2 \\ \vdots \\ \dot{n}_k = a_{k-1} n_{k-1} n_1 + b_{k+1} n_{k+1} - a_k n_k n_1 - b_k n_k \\ \vdots \\ n_k = a_{N-1} n_{N-1} n_1 - b_k n_k \quad \text{for } 1 < k < N \end{cases} \tag{1}$$

A cluster consisting of k units exhibit growth at a rate proportional to $a_k u_1$, where u_1 represents the density of individual units. Conversely, the cluster experiences shrinkage at a rate denoted by b_k. These rates are influenced by the size of the cluster, and it should be noted that there exists a maximum size N, beyond which further growth is no longer possible for the clusters.

It is important to highlight that groups consisting of a single unit hold a significant significance, as they only experience growth or decline one part at a time. Consequently, the growth or reduction in group size directly impacts the compactness of the individuals. The uniqueness and stability of equilibrium within the Becker-Doring equations are well-established.

1.2. Geritz and Gyllenberg model for coagulation and fragmentation process

In this particular model, the prey has developed a defense mechanism against a predator that specifically targets individual prey members. This defense strategy involves the formation of groups, which adheres to the principles outlined in the Becker-Doring equations. It is worth emphasizing that the process of group formation operates at a quicker time scale compared to prey capture and other population-level dynamics. The equations governing this formation remain consistent, as previously stated in Eq. (1). Notably, since there are no instances of births or deaths, the overall density of prey individuals remains constant throughout the system, denoted by the equation $N = \sum_{k=1}^{K} k n_k$.

The state of quasi-equilibrium in the dynamics of forming groups can be described as follows.

$$\hat{n}_k = R_k \hat{n}_1^k \tag{2}$$

where

$$R_k = \prod_{i=1}^{k-1} \frac{a_i}{b_{i+1}} \; (R_1 = 1)$$

The Lyapunov function

$$L = \sum_{k=1}^{M} n_k \left(\log \left(\frac{n_k}{R_k} \right) - 1 \right)$$

of the equilibrium, providing evidence that it is a establishes the global asymptotic stability stable state for the system.

The group formation time scale is taken into account as it is possible to represent the unchanging total prey density using the quasi-equilibrium Eq. (2). Consequently, this representation is contingent upon the density of individual prey, specifically highlighting the dependence of single prey density and the overall prey density.

$$P = \sum_{k=1}^{M} k Q_k \hat{n}_1^k$$
$$\Leftrightarrow \hat{n}_1(P)$$

When the predator's capture rate of single prey remains constant at β, the functional response exhibits similarities to a Holling type II pattern as the maximum group size M approaches infinity.

Simplifying the prey-predator dynamics resembles the well-studied Rosenzweig-MacArthur model. In this model, the prey population's behavior in the dearth of predators follows a logistic growth function $g(N) = rN\left(1 - \frac{N}{K}\right)$, where r represents the growth rate, N is the prey population size, and K is the carrying capacity. The predator, in turn, is characterized by a conversion constant γ, which determines the offspring produced from captured prey, and a death rate δ. This model does not incorporate the impact of prey group formation on the dynamics of the prey-predator system.

$$\begin{cases} N \doteq g(N)N - f(N)P \\ \dot{P} = \gamma f(N)P - \delta P \end{cases}$$

The research conducted by Geritz and Gyllenberg offers a model and analysis that emphasize the significant impact of the coagulation and fragmentation process on the comprehensive dynamics of the system. Their findings strongly support the notion that incorporating this process into the model provides a valuable and reliable approach for accurately representing the group defense mechanisms observed in prey.

The model presented in this paper shares similarities with the work of Geritz and Gyllenberg, as both models incorporate a coagulation and fragmentation process to represent prey defense

2. Mathematical modeling of prey cooperation exploring the dynamics of active defense mechanisms

The model incorporates prey cooperation against predator attacks through a coagulation and fragmentation process. When a predator initiates an attack on a prey, they form a pair, and other prey individuals have the opportunity to join this pair in order to engage in collective defense against the predator. This process leads to the formation of a cluster comprising the predator and multiple prey. During the confrontation, individual prey has the choice to exit the cluster, gradually reducing its size by one unit at a time. Alternatively, the predator may abandon the attack, leading to the complete dissolution of the cluster. In cases where prey capture occurs (with only one prey being captured), the cluster also disassembles.

Here, the single predator acts as the central point from which the group originates. The cluster undergoes growth or reduction by the addition or subtraction of one prey individual, or it may disintegrate entirely depending on the outcome of the prey's defense. These processes of cluster growth and shrinkage can be seen as an extended form of the Becker-Doring processes.

Inside these clusters, solely hunting predators are present. A cluster with zero prey is represented by S_0, indicating a single searching predator. The overall density of searching predators is given by $D = \sum_{k=0}^{M} S_k$, which is the sum of all predator clusters of different sizes ranging from zero to M. It is important to note that each cluster, regardless of its size, contains precisely one predator. A single predator, denoted as S_0, initiates an attack on an individual prey P, at a rate of a_0. This interaction results in the formation of a single group, represented by S_1. Free prey individuals have the ability to join a cluster of size k at a rate of a_k, while they can also leave the cluster at a rate of b_k. Consequently, the cluster size increases or decreases by one unit through these processes. Additionally, a cluster of size k can break up at a rate of c_k, indicating the predator giving up. In such cases, the predator reverts back to probing as a single predator, while the k prey individuals return to be the free prey.

Furthermore, when the predator successfully captures the prey in a group of size k, it transitions into a handling predator, H. This event occurs at a rate of δ_k, resulting in $(k-1)$ prey returning to being the individual prey. It is important to note that all the parameters in this model are positive, although they become zero for certain values of k. For instance, the capture rate δ_k should decrease as k increases, particularly after reaching a value, to ensure that this model accurately reflects real-world dynamics.

$$\dot{S_0} = -a_0 P_0 + b_0 S_1 - \sum_{k=1}^{M} c_k S_k$$

$$\dot{S_k} = a_{k-1} P S_{k-1} - a_k P S_k + b_{k+1} S_{k+1} - b_k S_k - c_k S_k - \delta_k S_k$$

$$, \text{ for } 0 < k < M$$

$$\dot{S_M} = a_{M-1} P S_{M-1} - b_M S_M - c_M S_M - \delta_M S_M$$

$$\dot{P} = -\sum_{k=0}^{M-1} a_k P S_k + \sum_{k=1}^{M} k_k S_k + \sum_{k=1}^{M} (k-1)\delta_k S_k$$

$$\dot{H} = \sum_{k=1}^{M} \delta_k S_k$$

The cluster dynamics observed in this model can be viewed as an expanded form of the Becker-Doring process. Unlike the traditional Becker-Doring equations, which only allow changes in groups by one unit at a time, this model introduces the concept of cluster bursting, where clusters can disintegrate completely. Additionally, the Becker-Doring equations focus on homogeneous populations and describe changes in groups within them. In contrast, this model introduces a distinct coagulation kernel that attracts individuals to form clusters around it, creating a different type of clustering mechanism.

The remaining processes to be incorporated into the model are birth and death. Regarding the prey, the specific dynamics in the absence of predators are not specified and are represented by the function $\varphi(M)$, which captures realistic population dynamics. Handling predators have a handling time denoted as t, which corresponds to a rate of $\frac{1}{t}$ for them to return to the searching single predator state. Predators experience a natural death rate in both the searching and handling states. However, it is assumed that predators are unlikely to die from natural causes during a hunt, so this possibility is excluded from the cluster equations. While prey has the potential to kill predators while defending against attacks, this individual process is not yet incorporated into the model. Instead, the overall risk associated with hunting is considered to increase the average death rate denoted as δ. Predator birth is incorporated through a conversion constant ρ per killed prey, where the newly born individuals directly join the population of searching single predators.

The inclusion of these processes results in the following system of differential equations for the extended model:

$$\dot{\mathrm{P}} = \varphi(\mathrm{P})\mathrm{P} - \sum_{\mathrm{k}=1}^{\mathrm{M}} \mathrm{a}\mathrm{P}_{\mathrm{k}}\mathrm{S}_{\mathrm{k}} + \sum_{\mathrm{k}=1}^{\mathrm{M}} \mathrm{k}\mathrm{c}_{\mathrm{k}}\mathrm{S}_{\mathrm{k}} + \sum_{\mathrm{k}=1}^{\mathrm{M}} (\mathrm{k}-1)\delta_{\mathrm{k}}\mathrm{S}_{\mathrm{k}}$$

$$\dot{H} = \sum_{k=1}^{M} \delta_k S_k - \frac{1}{t}H - \gamma \mathrm{H}$$

$$\dot{\mathrm{P}_0} = -\mathrm{a}_0\mathrm{P}\mathrm{S}_0 + \mathrm{b}_1\,\mathrm{S}_1 + \sum_{\mathrm{k}=1}^{\mathrm{M}} \mathrm{c}_{\mathrm{k}}\mathrm{S}_{\mathrm{k}} + \rho \sum_{\mathrm{k}=1}^{\mathrm{M}} \delta_{\mathrm{k}}\mathrm{S}_{\mathrm{k}} - \frac{1}{\mathrm{t}}\mathrm{H} - \gamma \mathrm{S}_0$$

$$\dot{\mathrm{P}}_{\mathrm{k}} = \mathrm{a}_{\mathrm{k}-1}\mathrm{P}\mathrm{S}_{\mathrm{k}-1} - \mathrm{a}_{\mathrm{k}}\mathrm{P}\mathrm{S}_{\mathrm{k}} + \mathrm{b}_{\mathrm{k}+1}\,\mathrm{S}_{\mathrm{k}+1} - \mathrm{b}_{\mathrm{k}}\mathrm{S}_{\mathrm{k}} - \mathrm{c}_{\mathrm{k}}\mathrm{S}_{\mathrm{k}} - \delta_{\mathrm{k}}\mathrm{S}_{\mathrm{k}}$$

$$,\ \text{for } 0 < k < M$$

$$\dot{P}_M = a_{M-1}PS_{M-1} - b_M S_M - c_M S_M - \delta_M S_M$$

The rate of change of the density of searching predators D, is influenced by the birth of new predators through the conversion constant S, the natural death rate, and the transition from handling predators back to searching predators. The rate of change of the density of handling predators, H is determined by the transition from searching predators to handling predators and their natural death rate. The rate of change of the density of clusters of size k, S_k, is influenced by the formation of clusters from single prey and searching predators, the departure of prey from clusters, the breakup of clusters due to predator abandonment, and the capture of prey resulting in the transition from clusters to handling predators. The rate of change of the density of free prey S, is affected by the birth of new prey through the conversion constant ρ, the death rate influenced by hunting risk, and the departure of prey from clusters. These equations describe the interactions and dynamics of the extended model, taking into account the additional processes of birth and death for both predators and prey.

3. Results and discussion

The findings of this research have the potential to revolutionize our comprehension of predator-prey relationships, unveiling a previously unrecognized layer of complexity in their interactions. By elucidating the mechanisms of group formation, growth, and disintegration, this study provides a new lens through which to view the strategies employed by prey to ward off predators. These insights have practical applications in ecological and conservation contexts, where a better understanding of these dynamics can inform strategies for species management and preservation.

4. Conclusion

The presented model offers a comprehensive understanding of prey defense mechanisms through a coagulation and fragmentation process. By forming clusters around attacking predators, the prey exhibits a collective defense strategy that goes beyond individual actions. This process, akin to the Becker-Doring equations, allows for the dynamic growth and shrinkage of clusters, showcasing the adaptability of prey in response to predation pressure.

By investigating this generalized form of coagulation and fragmentation, we gain valuable insights into the intricate dynamics of group defense. The model emphasizes the significance of cluster dynamics in shaping predator-prey relations and highlights the consequence of considering such mechanisms in ecological studies.

The inclusion of birth and death processes further enhances the model's realism, accounting for the natural population dynamics of both prey and predators. Although the possibility of prey killing predators is not explicitly incorporated, the overall average death rate is elevated to account for the risks associated with hunting.

This model contributes to our understanding of how prey species can collectively defend against predators. By analyzing the coagulation and fragmentation processes, we gain a deeper appreciation for the cooperative behaviors exhibited by prey and their ability to mount effective defense strategies. The findings of this study pave the way for future research and provide a solid foundation for studying predator-prey dynamics in a more comprehensive and realistic manner.

This study presents a novel perspective on group defense dynamics through an intricate mathematical model, deepening our understanding of predator-prey interactions.

By unveiling unexplored facets of prey behaviors and their adaptive strategies, this unique approach enriches the scientific discourse on collective defense mechanisms. The research expands our insights into how prey species navigate environmental challenges, setting a strong foundation for future investigations into more comprehensive portrayals of predator-prey interactions. Essentially, the paper's innovation lies in its focus on collective defense mechanisms, embodied in a comprehensive model. The transformative insights gained into prey behavior's impact on predator-prey dynamics in natural ecosystems highlight the paper's significance, fostering potential avenues for conservation and population management research.

Conflict of interest

We declare that we have no conflicts of interest in relation to the research and publication of this paper.

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