Enhancement and Analysis of ECG signals using Combined Difference Total Variation Optimization

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An Electrocardiogram (ECG) signal representing the heart's electrical behaviour is often corrupted by artefacts that may prevent correct diagnosis and hence need to be reduced for better clinical assessment. The first difference total variation that measures variation between consecutive samples of signals has been useful for reducing artefacts from signals. However, for quasi-stationary signals having a weak signal to noise ratio, the method's performance is not satisfactory. In this paper, the concept of first difference total variation has been utilized to derive combined difference total variation. The algorithm is executed to reduce simulated noise comprising power line interference, baseline wander, and Gaussian noise added to ECG signals. The performance is measured with standard assessment tools, and the results obtained are compared with the other denoising models reported in the recent literature.

Keywords: ECG, First Difference, Total Variation, Majorization-Minimization Optimization

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1. Introduction

Electrocardiography (ECG) carries important clinical information related to the functioning of the heart [1]. However, the characteristics of these waves are often changed by the noises of variable frequency and magnitude. In a few cases, the extent of noises is relatively high that can even prevent accurate interpretation. Therefore, the extraction of true ECG signal from the noisy signal is the first preprocessing stage in ECG signal processing. Noises with different intensity levels viz., Power line interference (PLI), electromyography noises, and baseline wander) were successfully reduced to significant levels. However, electromagnetic noises like electrode pop noise, electrosurgical noise and instrumentation noise which shows a dynamic frequency range is challenging [2–4]. Since the timing pattern and characteristics of ECG signals are clinically significant, they must be retained even after processing. The noise estimation problem entails an error objective criterion specification to be optimized in some chosen functional space [5]. Most noise suppression methods tend to damage the ECG signals, which can be avoided by using the local signal and the noise orthogonalization algorithm [6]. In this article, we propose an algorithm to suppress electromagnetic noises like white Gaussian noise along with PLI and baseline wander noise from ECG signals by considering the difference between adjacent samples of ECG signals.

Total Variation (TV) measures the average difference between the adjacent samples of any signal. It has been discovered that noisy signals have a large TV, i.e., the integral of the absolute gradient of the signals is high [7]. The conventional TV filtering algorithms reduce noise and smoothes away edges to a greater or lesser degree, has been successfully applied to the images having low signal-to-noise ratios for inpainting, restoration, denoising, zooming, and segmentation [8–12]. Even conventional TV algorithms were successfully applied to ECG signals in [13, 14]. This paper presents a novel method to reduce noises from ECG signals by utilizing traditional TV concept differently.

The rest of the paper is organized as follows: in Section 2, we present the TV approach's theoretical background. Further in this section, utilizing the conventional method, we derive the proposed algorithm. The result and discussion are presented in Section 3. Finally, conclusions regarding the presented algorithms are provided in Section 4.

2. Material and Methods

2.1. Total Variation Denoising

The TV of any real-valued function on an interval [a,b] ϵ R is obtained as [15]:

$$TV(x) = \sum_{i=1}^{n} |x(n) - x(n-1)| \tag{1}$$

Since the TV of any signal calculates the error between adjacent sample points, we define this as TV1D using first differences. ECG signals with noise are generally modeled as

$$y(n) = x(n) + w(n) \tag{2}$$

where y(n) is the noisy ECG signal, x(n) is the true ECG signal and w(n) is the noise. In this paper, w(n) consist of PLI noise, baseline wander, and white Gaussian noise. The TV Denoising (TVD) finds an approximation x(n), for a given input signal y(n) with smaller TV than y(n) in the sum of square error estimate as

$$E(x,y) = \frac{1}{2}\sum_{n}(y(n) - x(n))^2 \tag{3}$$

The TVD optimization problem is formulated

$$min\{E(x,y) + \lambda\, |TV(x)|\} \tag{4}$$

with λ known as the regularization parameter to tune. The idea of TV regularization is to penalize the signal's TV, which results in the minimization problem

$$x = min\left\{\frac{1}{2}\, \|y - x\|_2^2 + \lambda\, \|TV(x)\|_1\right\} \tag{5}$$

where $\| \,.\, \|_2$ is the L_2 norm and $\| \,.\, \|_1$ is the L_1 norm.

Solving Eq. 5 for signals like ECG is non-trivial [15]. TV(.) is a convex regularizer used to stabilize and λ is a regularization parameter providing the tradeoff between fidelity of measurements and noise sensitivity and hence it should be carefully chosen. Following the original approach by differentiating Eq. 5 wrt y(n), we can obtain the corresponding Euler Lagrange equation that can be numerically integrated with the original signal x(n) as the initial condition [7].

Given a noisy signal $= [y(0), y(1), \ldots, y(n-1)]^T \epsilon \mathbb{R}^N$, we can efficiently compute the denoised signal:

$$x = [x(0), x(1), \ldots, x(n-1)]^T \epsilon \mathbb{R}^N \tag{6}$$

defined implicitly as the solution to the minimization problem of TVD. Even though the mathematical formulation of the TV using the first difference has been established [7, 15, 16], we reproduce here the complete mathematical formulation to build the necessary formulations for the proposed TV algorithm.

2.1.1. *Total Variation Denoising using Combined Differences*

The TV1D of N discrete point signal x(n), $0 \leq n \leq N-1$ can also be represented as

$$\|TV1D(x))\|_1 = \|F_D x\|_1 \tag{7}$$

The TV1D calculates the denoised signal x(n) by solving the optimization function given by Eq. 5. With the regularization parameter as α for first differences, the same optimization problem can be restated as:

$$x = min\left\{\frac{1}{2}\, \|y - x\|_2^2 + \alpha\, \|F_D x\|_1\right\} \tag{8}$$

The function to be minimized in Eq. 8 is strongly convex, and the solution x(n) to the problem exists, which is unique for any data y(n) [15, 16].

TV1D has been successfully applied to image in for image inpainting, restoration, denoising, zooming, and segmentation [8–11]. Also, TV1D tends to introduce stairs resulting in fewer flat regions within the denoised signal. While TV1D may be appropriate for denoising piecewise constant signals, it is not typically the most straightforward denoising methodology for quasi-stationary ECG signals. For such signals, a higher-order difference can preferably be used. So, TV1D is modified to a TV based on the second difference TV2D and is used to reduce the staircase effect while maintaining the quality of the reconstructed signal.

Expressing the signal x(n) in terms of second difference we have

$$\|TV2D(x)\|_1 = \sum_{n=2}^{N-1} |(x(n) - x(n-1)) - (x(n-1) - x(n-2))| = \sum_{n=2}^{N-1} |(x(n) - 2x(n-1)) + x(n-2)| = \|S_D x\|_1 \tag{9}$$

With the regularization parameter as β for second differences, the same optimization function Eq. 5 can be used to derive the signal x(n) utilizing the TV2D as :

$$x = min \left\{ \frac{1}{2} \|y - x\|_2^2 + \beta \|F_D x\|_1 \right\} \tag{10}$$

TV2D is the L_1 norm of the hessian function and can connect significant gaps. The TV2D approach also finds applications in the image (normal and medical) inpainting, restoration, and destriping of remote sensing. The TV2D method produces more connected results at the cost of blur at the images' edges [8–11, 17, 18]. The TV2D has also been applied to ECG signals [19].

The combination of TV1D and TV2D, henceforth named as (TVCD), computes the signal x(n) by solving the optimization function given by Eq. 5 with a combination of Eqs. 7 and 9. The same optimization function can be reformulated as:

$$x = min \left\{ \frac{1}{2} \|y - x\|_2^2 + \alpha \|F_D x\|_1 + \beta \|S_D x\|_1 \right\} \tag{11}$$

where α and β are regularization parameters balancing the TV1D and TV2D, respectively.

To solve the convex non-smooth optimization problems Eq. 11, we mostly find iterative fixed-point methods in the literature. These methods have rather high computational complexity [20]. Since the L_1 norm is not differentiable, minimization of the objective functions of this kind is difficult. Hence, the majorization-minimization (MM) method is used to obtain a clean x from a noisy y signal.

2.2. Majorization-Minimization Approach

The Majorization-Minimization (MM) algorithm begins by successively minimizing a sequence of upper bounds of the objective function. These upper bounds are tight at the current estimate, and each iteration monotonically drives the objective function downhill. The MM algorithm ensures convergence of convex and non-convex optimization when the upper bounds approximate the objective up to a smooth error [15, 16, 19]. If x and y $\epsilon \mathbb{R}$, f and g be real valued functions on $\mathbb{R}^N$ then the function g majorizes the function f at y if

- g(x) $\geq$ f(x) $\forall$ x
- g(y) = f(y)

While minimizing the objective function f iteratively, let $x^{(k)}$ be the current best minimizer at the k^{th} iteration. A majorizing function g is constructed that majorizes f at $x^{(k)}$. If $x^{(k)}$ minimizes g the procedure is terminated, otherwise a new solution $x^{(k+1)}$ is found by minimizingg, i.e.,

$$f(x^{(k+1)}) \leq g(x^{(k+1)}) \leq g(x^{(k)}) = f(x^{(k)}) \tag{12}$$

Subsequently, a new majorizing function is constructed at $x^{(k+1)}$, and the steps are repeated to produce a decreasing sequence of function values. To derive a majorizer for the objective function given by Eq. 11, the property of quadratic majorizers has been exploited. The function f(x)= | x | has a quadratic majorizer at each x_k except at $x_k = 0$. If $x_k \neq 0$ then the majorizer for f(x) is given by [21]

$$|x| = \frac{1}{2|x_k|} x^2 + \frac{1}{2} |x_k| \tag{13}$$

Therefore, instead of minimizing Eq. 11, the Majorization approach solves a sequence of optimization problems, G_k(x), k=0,1,2. . . , where each function g_k(x), is a majorizer of f(x). The MM algorithm for TV denoising can be obtained by majorizing Eq. 11 by a quadratic function of x of Eq. 13. Then the optimization function can be majorized by a quadratic function, which can in turn be minimized by solving a system of linear equations.

Rewriting Eq. 13 as

$$\frac{1}{2|x_k|} x^2 + \frac{1}{2} |x_k| \geq |x| \tag{14}$$

and substituting v(n) for x and summing over n provides

$$\sum_n \left[\frac{1}{2|v_k(n)|} v^2(n) + \frac{1}{2} |v_k(n)| \right] \geq \sum_n |v(n)| \tag{15}$$

which can be compactly expressed as

$$\frac{1}{2} v^T \Lambda_k^{-1} v + \frac{1}{2} \|v_k\|_1 \geq \|v\|_1 \tag{16}$$

Here, the absolute value of v is applied element-wise. Using Dx=TV(x), and replacing v by Dx and v^T by $x^T D^T$, Eq. 16 can be expressed as

$$\frac{1}{2} x^T D^T \Lambda_k^{-1} D x + \frac{1}{2} \|D x_k\|_1 \geq \|D x\|_1 \tag{17}$$

In Eq. 17, the majorizer of $\|Dx\|_1$=$\|TV(x)\|_1$ is a function of x. Since x_k is the value of x at the previous iteration, so $\|Dx\|_1$ is constant. Also, Λ_k is also not a function of x as it is a diagonal matrix with constant values given as Λ_k=diag(|Dx_k|). Then, a majorizer of the TV cost function given in Eq. 5 can be derived from Eq. 17 by adding $\frac{1}{2}\|y - x\|_2^2$ to both sides

$$\frac{1}{2} \|y - x\|_2^2 + \frac{1}{2} x^T D^T \Lambda_k^{-1} D x + \frac{1}{2} \|D x_k\|_1 \geq \frac{1}{2} \|y - x\|_2^2 + \lambda \|D x\|_1 \tag{18}$$

$$g_k(x) \geq f(x) \tag{19}$$

Therefore, the majorizer g_k(x) is obtained as

$$g_k(x) = \frac{1}{2}\|y-x\|_2^2 + \frac{1}{2}x^T D^T \Lambda_k^{-1} Dx + \frac{1}{2}\|Dx_k\|_1 \tag{20}$$

satisfying $g_k(x_k)$=f(x_k) with Λ_k=diag($|Dx_k|$). So, x_k can be obtained by minimizing g_k(x) and written as

$$x_{k+1} = min(g_k(x)) \tag{21}$$

$$x_{k+1} = \frac{1}{2}\|y-x\|_2^2 + \frac{1}{2}x^T D_T \Lambda_k^{-1} Dx + \frac{1}{2}\|Dx_k\|_1 \tag{22}$$

Thus differentiating Eq. 20 wrt x and equating to zero, we obtain

$$-(y-x) = \lambda D^T \Lambda_k^{-1} Dx = 0 \tag{23}$$

$$y = x + \lambda D^T \Lambda_k^{-1} Dx \tag{24}$$

$$y = (1 + \lambda D^T \Lambda_k^{-1} D)x \tag{25}$$

which gives the minimum value of x. With an initial value, the iterative solution to Eq. 22 is given by

$$x_{k+1} = (I + \lambda D^T \Lambda_k^{-1} D)^{-1} y \tag{26}$$

Some of values of Dx_k in Eq. 26 may go to zero, resulting in certain entries of Λ_k^{-1} of Eq. 26 as infinity. This can be resolved using the matrix inverse lemma and rewriting as

$$(I + \lambda D^T \Lambda_k^{-1} D)^{-1} = I - D^T \left(\frac{1}{\lambda}\Lambda_k + DD^T\right)^{-1} D \tag{27}$$

In Eq. 27 values of right hand side $(.)^{-1}$ will not approach zero. Therefore, after substituting Λ_k=diag($|Dx_k|$) the minimizer Eq. 26 becomes

$$x_{k+1} = y - D^T \left(\frac{1}{\lambda} diag(|Dx_k|) + DD^T\right)^{-1} D \tag{28}$$

which has no effect even if some elements of Dx_k are zero. The matrix $(\frac{1}{\lambda} diag(|Dx_k|) + DD^T)$ in Eq. 28 is a banded matrix helping to solve linear systems efficiently. Therefore, the majorizer function Eq. 11 for TVCD can be expressed as

$$g_{kCD} = \frac{1}{2}\|y-x\|_2^2 + \alpha\frac{1}{2}x^T F_D^T \Lambda_{k1D}^{-1} F_D x + \alpha\frac{1}{2}\|F_D x_k\|_1 + \beta\frac{1}{2}x^T S_D^T \Lambda_{k2D}^{-1} S_D x + \beta\frac{1}{2}\|S_D x_k\|_1 \tag{29}$$

and the minimizer is

$$x_{k+1} = (I + \alpha F_D^T \Lambda_{k1D}^{-1} F_D + \beta S_D^T \Lambda_{k2D}^{-1} S_D)^{-1} y \tag{30}$$

Some of values of $F_D x_k$ and $S_D x_k$ in Eq. 30 may go to zero, resulting in certain entries of Λ_{k1D}^{-1} and Λ_{k2D}^{-1} of Eq. 30 as infinity. To resolve this issue, instead of concurrently optimizing g_{kCD} in Eq. 30, we optimize the function sequentially using MM algorithm. Therefore, to find the minimizer the governing equations are given as:

$$x'_{k+1} = y - F_D^T (\frac{1}{\alpha} diag(F_D x_k) + F_D F_D^T)^{-1} F_D y \tag{31}$$

$$x_{k+1} = y - S_D^T (\frac{1}{\beta} diag(|S_D x'_{k+1}|) + S_D S_D^T)^{-1} S_D y \tag{32}$$

3. Results and Discussion

We have analyzed all algorithms on 26 ECG test signals of 10 seconds, sampled at 360 Hz frequency with 11 bits/sample of the MIT-BIH database [22]. The algorithms have been developed in MATLAB (R2019a) environment on a computer having Intel(R) Core (TM)2 Duo CPU T6570 2.10 GHz, RAM 8 GB and hard disk of 500 GB. Performance of all the methods is assessed through Mean Absolute Deviation (MAD), Root Mean Square Difference (RMSD), Percentage Root Mean Square Difference (PRD), Normalized Percentage Root Mean Square Difference (PRDN), Signal to Noise Ratio (SNR) and Cross-Correlation (CC). Details about these performance tools can be obtained from [23]. MAD, RMSD, PRD, PRDN are negative oriented (representing errors between original and restored signal) tools, and hence lower values are considered better for denoising applications. SNR represents signal strength over noise, and CC shows the similarity between original and denoised signals. So, higher values are considered better. Although the test signals may/may not have noise, for analysis purpose as well as to check the performance of the algorithms, additional noises, viz., PLI; Baseline wander, and White Gaussian noise is added to these signals so as to make SNRs of the noisy signal as 15 dB. In this paper, SNR at three stages, i.e., before the addition of noise SNRi, after the addition of noise SNRs and after application of algorithms SNRr has been considered.

ECG Denoising using Total variation majorization minimization approach

Inputs:**ECG signal** y(i), i = 1,...,N; length of ECG signal N; Regularization parameter α, ; No. of iterations Nit

Output: **Denoised signal**, x = 1,...,M; MAD, RMSD, PRD, PRDN, SNRi, SNRr, CC;

1. Set k=0;
2. x_0=y ;
3. k = k +1;

4. Compute x_{k+1} using Eq. 32;

5. Set $x_k = x_{k+1}$ as the current minimizer of $G_k(x)$ in Eq. 29;

6. If k <Nit go to step (3);

Compute MAD, RMSD, PRD, PRDN, SNRi, SNRr, CC.

3.1. Regularization parameter

The regularization parameter λ >0 balances the regularization and data-fidelity terms. Through proper selection of the regularization parameter, a balance can be obtained to reduce noise level and preserve signal quality. Thus, the choice of the regularization parameter is critical to achieving just the right amount of noise removal. A larger value of the parameter may not be able to denoise signal efficiently, and lower value may produce over smoothing [14–16, 24]. In order to find the optimum values of the regularization parameters, i.e., α and β, we have chosen a set of values, i.e., 0, 0.1, 0.2, 0.4, 0.5, 0.6, 0.8 and 0.9. Since TV is based on the average error between samples of any signal, assessment tools with the error may be used for selection of λ . Out of the MAD, RMSD, PRD, and PRDN, PRDN provides error related to distortion of the true signal [25], so this assessment tool is utilized for fixation of regularization parameters for TVCD. Moreover, PRDN is calculated after removing gain and offset voltage, which is added to ECG signals for storage purposes as DC/noise component has no diagnostic meaning [26]. So, PRDN for different values of α and β are calculated. We have performed the same experiment over all the test signals resulting in similar findings. Variation of PRDN with regularization parameter for record \#109 is illustrated in Table 1, where β=0 refers to the TV1D, and α=0 refers to the TV2D, and any other combination of β and α indicates TVCD.

Following conclusions are drawn from Table 1:

- Lower values of regularization parameters produce better results for TV1D, TV2D, and even for TVCD.
- Performance of TV2D is better than TV1D for the same value of α and β.
- PRDN also depends upon the ratio of $\frac{\beta}{\alpha}$
  - It decreases with a decrease in $\frac{\beta}{\alpha}$ provided β is higher than α.
  - It increases with decrease in $\frac{\beta}{\alpha}$ provided α is higher than β.
- For the same ratio of $\frac{\beta}{\alpha}$ lower values of both α and β provide better results.

From Table 1 and Fig. 1, TV1D provides lowest PRDN as $19X1^{-4}$ at α=0.1 whereas TV2D provides the least PRDN as $18.40X10^{-4}$ at β=0.1. So, here we conclude that lower regularization offers better results. Since PRDN of TV2D is lower than TV1D, so we can also find that TV2D provides better results than TV1D. The lowest PRDN for TVCD is $18.19X10^{-4}$ at α=0.1 and β=0.1 which is lower than TV1D and TV2D and henceforth, α=0.1 and β=0.1 is fixed for TVCD.

We apply the TV algorithms on the test signals of MIT-BIH database and evaluate their performance in terms of MAD, RMSD, PRD, PRDN, SNRi(dB), SNRr(dB), SNRim (Improvement in SNR in dB) and CC.

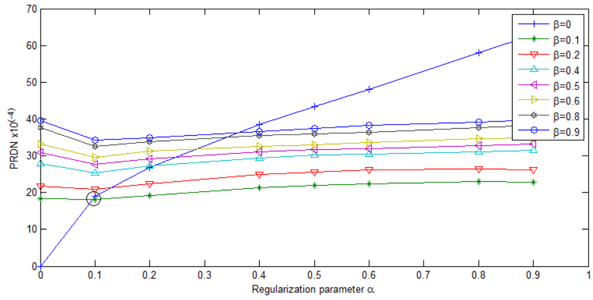

Fig. 1. Effect of regularization parameter (α and β) on PRDN for record #109

3.2. Results for combined difference total variation

Performance tools and their values for the TVCD are tabulated in Table 2. Fig. 2 shows the result of TVCD over record #119. Khamman and Ahmed [27] utilized optimal wavelets to reduce noises from records #100, #101, #102, #105, #110, #113, #117, #119, #205, #209 and #210.

The average performance parameters reported SNRim=32.25 dB, PRD $=39.36X10^{-2}$, PRDN $19.94X10^{-2}$ which are found to be menial than those reported in this paper (SNRim =35.96dB, PRD=0.2853, PRDN= 0.0013).

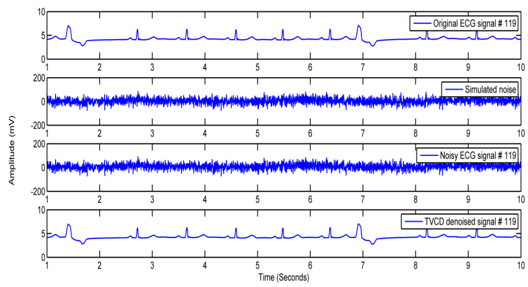

Fig. 2. Original, simulated noise, noisy and TVCD reconstructed signal #109

Table 1. PRDN ($X10^{-4}$) with different values of regularization parameters α and β for record #109.

α \ β	0		0.1		0.2		0.4		0.5		0.6		0.8		0.9
0	0	-	19.01	-	26.77	-	38.51	-	43.43	-	48.10	-	57.95	-	62.99
0.1	18.40	1	18.19	0.5	19.22	0.25	21.42	0.2	21.97	1.67	22.34	0.125	23.01	0.111	22.89
0.2	21.75	2	20.93	1	22.33	0.5	24.87	0.4	25.61	0.33	26.10	0.25	26.38	0.222	26.24
0.4	27.87	4	25.28	2	27.35	1	29.32	0.8	30.16	0.67	30.39	0.5	30.97	0.444	31.47
0.5	30.90	5	27.62	2.5	29.27	1.25	30.99	1	31.66	0.83	31.85	0.625	32.84	0.555	33.24
0.6	33.29	6	29.67	3	31.35	1.5	32.54	1.2	32.97	1	33.57	0.75	34.67	0.667	34.87
0.8	37.56	8	32.56	4	33.86	2	35.54	1.6	36.05	1.33	36.28	1	37.57	0.888	38.37
0.9	39.65	9	34.26	4.5	34.80	2.25	36.57	1.8	37.37	1.5	38.21	1.125	39.16	1	39.80

Table 2. Performance matrix of ECG signals using the TV2D in noisy environment at α=0.1 and β=0.1.

Record	$MADx10^{-4}$	$RMSDx10^{-2}$	PRD	PRDN	SNRi	SNRs	SNRr	SNRim	CC
100	1.55	1.24	0.2589	0.0012	13.78	15	51.74	36.74	0.9975
101	1.77	1.33	0.2764	0.0013	13.62	15	51.17	36.17	0.9978
102	2.01	1.42	0.2883	0.0014	13.77	15	50.80	35.80	0.9968
103	1.48	1.22	0.2486	0.0012	13.92	15	52.09	37.09	0.9992
104	2.05	1.43	0.2927	0.0014	13.78	15	50.67	35.67	0.9987
105	1.31	1.15	0.2342	0.0011	13.95	15	52.61	37.61	0.9993
106	3.59	1.89	0.3828	0.0019	13.99	15	48.34	33.34	0.9988
107	1.84	1.36	0.2778	0.0013	13.62	15	51.13	36.13	0.9999
108	2.34	1.53	0.3178	0.0015	13.73	15	49.96	34.96	0.9956
109	2.57	1.60	0.3391	0.0016	13.58	15	49.39	34.39	0.9991
113	1.93	1.39	0.2845	0.0014	13.72	15	50.92	35.92	0.9995
115	1.54	1.24	0.2668	0.0012	13.26	15	51.48	36.48	0.9992
117	1.84	1.36	0.3166	0.0013	12.45	15	49.99	34.99	0.9983
119	2.06	1.43	0.3338	0.0014	12.71	15	49.53	34.53	0.9996
121	1.20	1.09	0.2509	0.0011	12.99	15	52.01	37.01	0.9988
124	1.49	1.22	0.2809	0.0012	12.71	15	51.03	36.03	0.9995
201	1.20	1.09	0.2213	0.0011	13.78	15	53.10	38.10	0.9985
205	1.82	1.35	0.2836	0.0013	13.66	15	50.95	35.95	0.9974
209	2.39	1.54	0.3094	0.0015	13.97	15	20.19	35.19	0.9980
212	2.14	1.46	0.2945	0.0014	13.93	15	50.62	35.62	0.9990
215	2.63	1.62	0.3254	0.0016	13.99	15	49.75	34.75	0.9977
217	2.27	1.51	0.3002	0.0015	14.25	15	50.45	35.45	0.9997
219	1.77	1.33	0.2940	0.0013	13.18	15	50.63	35.63	0.9996
223	1.21	1.10	0.2409	0.0011	13.06	15	52.36	37.36	0.9996
230	1.45	1.20	0.2483	0.0012	13.60	15	52.10	37.10	0.9995
231	1.60	1.27	0.2513	0.0012	13.99	15	51.99	36.99	0.9991
Average	1.89	1.36	0.2853	0.0013	13.58	-	-	35.96	0.9987

ECG denoising models using linear polynomials and wavelet thresholding techniques have been presented in [28], where authors have claimed higher SNR (9 to 30 dB) with PRD>1. SNRim (upto 6 dB) is enhanced through Wavelet Domain Wiener Filtering for noise-free signals of the different datasets in [29]. ECG denoising method based on bivariate shrinkage function improved SNRim to 6 dB over level-dependent threshold, Visushrink, Sureshrink and BayesShrink for record #103 [30]. Jokic et al. [31] modeled ECG signals using polynomials on the same database and reported the lowest PRD as 6.1 for the ECG record #205. However, with the TVCD method we could model the same signal with PRD as 0.2836. Least square iterative polynomials were also used to model ECG signals in [32]. Here the authors have claimed their computed RMSD of the order of 10^{-2} for record #100, which is better than Butterworth, moving average, FIR filters, and median filters. For all the signals, RMSD attained by the TVCD approach is of order 10^{-3} which is even better than those reported in [33]. CC values are also better than those reported in [21].

4. Conclusions

TV of a signal is the sum of differences between adjacent samples. It is observed that signals having high noise show large TV. So, TV models are used to reduce noise from signals. In this paper, the concept of the first difference TV has been utilized to develop combined difference TV model for reduction of noise. The proposed model is optimized using MM optimization technique. The model is tested on MIT-BIH signals to reduce simulated noise. The performance of the TV denoising model depends upon the selection of the regularization parameter. The lower value of regularization may yield better results at the cost of over smoothening. The results of the TVCD are compared with recent existing models and are found to be superior in terms of standard performance assessment tools.

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