Assessment of the Electric Grid Interdiction Problem Considering Different Network Models

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The vulnerability assessment of power systems consists on finding the set of most critical assets in order to device strategies to make the system more resilient. The electric grid interdiction problem (EGIP), also known as the terrorist threat problem, addresses this issue by modeling the interaction of a disruptive agent and the system operator. The EGIP is usually modeled as a bilevel programming problem. The disruptive agent is placed in the upper-level optimization problem and aims at maximizing the system damage subject to limited destructive resources. The system operator is placed in the lower-level optimization problem and reacts to the attacks minimizing load shedding by redispatching available generation resources. Traditional approaches to the EGIP consider a simplified version of the network by means of a DC model. This allows some advantages from the standpoint of complexity; nevertheless, the effect of reactive power and voltage magnitudes are neglected in this model. An AC modeling of the network is more accurate but implies higher complexity. This paper presents a comparison of these models applied to the EGIP through an Iterated Local Search metaheuristic. Several tests were performed on a benchmark power system to contrast the performance of both models. Results show that using a DC model provides faster results but also reports conservative solutions that do not fully take into account the actual damage inflicted in the network. This might lead the system operator to underestimate the real vulnerably of the system and not carry out effective corrective or protective actions.

KEYWORDS: Vulnerability; interdiction problem; bilevel programming; iterated local search

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1. Introduction

Electric power systems are critical infrastructure that must be protected against both natural occurring outages and intentional attacks. Load shedding and higher operational costs associated to repairing infrastructure are the main effects of malicious attacks to power systems [1]. Traditional approaches to power system security relay on the N-1 and N-2 criterion which basically consists of guaranteeing that the system is able to operate within specified limits after the occurrence of one or two simultaneous contingencies, respectively. These studies require exhaustive simulation and only take into account naturally occurring outages; nevertheless, the electric grid interdiction problem (EGIP), also known as the terrorist threat problem, considers the interaction of a disruptive agent and the system operator. This problem was first proposed in [2] as a max-min attacker-defender problem. On one hand, a disruptive agent decides which set of lines to attack with the aim of maximizing total damage to the power system (measured as load shedding); on the other hand, the system operator aims at minimizing system damage by redispatching available generation resources. This attacker-defender problem can be portrait as a leader-follower Stackelberg game. In this game, the leader moves first, anticipating the reaction of the follower. Then, the follower makes a decision affecting the gains of the leader. From the standpoint of mathematical programing a Stackelberg game can be modelled as a bilevel programming problem. In this case, the leader and follower are positioned in the upper and lower optimization problems, respectively.

Several approaches have been proposed in the specialized literature to solve the EGIP. In [3] the goal of the disruptive agent is to minimize the number of power system components that must be destroyed in order to achieve a given goal on load shedding. Such goal is tempered by the logical assumption that the system operator will implement all feasible corrective actions to minimize system load shedding. The bilevel problem proposed in [3] is transformed into a single-level equivalent by replacing the lower optimization problem by its optimality conditions and linearizing the resulting equations. In [4] the EGIP is solved through a mixed integer linear programming procedure also recasting the original problem into a single-level equivalent and using linearization. In [5] the authors approach the EGIP though a generalized Benders decomposition that allows application in large power systems. In [6] line switching is introduced as an alternative defensive strategy against malicious attacks to the power system. In this case the system operator is able to modify network topology as well as redispatching generation in order to reduce the effects of intentional attacks. In [7] the authors propose a maximum and a minimum vulnerability model to deal with the EGIP. The maximum vulnerability model is based on the research reported in [3] and [4]. In this case, the analysis of vulnerability consists on identifying the maximum level of load shedding attainable with a fixed number of simultaneous line outages. The minimum vulnerability model is defined as the identification of the lowest number of simultaneous line attacks that result in a load shedding greater or equal than a pre-specified threshold. Both models are solved reformulating the lower-level optimization problem as an equivalent set of constraints given by the Karush-Kuhn-Tucker optimality conditions or by using duality theory. In [8] the authors present a vulnerability model that includes short-term and medium-term impacts of possible attacks in power systems. In this case, a cascading outage analysis is performed to emulate a blackout subsequent to specific terrorist attacks. In [9] the authors propose a tri-level expansion planning model considering intentional attacks. The system planner is placed in the first level and looks for an optimal transmission expansion plan to fortify the power network against intentional attacks. In the second level, the attacker tries to maximize damages to the network by devising attack plans that would maximize load shedding. In the third level, the adverse effects of the attacks on the network are minimized by the system operator. In this case, instead of considering a single malicious agent; a cooperative game of multiple virtual attackers is considered. A similar approach, considering a single attacker is also considered in [10] and [11] also within a tri-level expansion plan.

The common denominator of the aforementioned approaches to the EGIP is the use of the DC model to represent the transmission network. This model allows replacing the lower level optimization problem for its optimality conditions; by doing so, the original bi-level problem is turned into a single-level equivalent. The DC model allows representing the network in a simplified fashion that only considers angles and active power injections. However, a more detailed representation of the network can be obtained through an AC modelling. The latter considers not only active but reactive power injections and their effect on voltage magnitudes. This modelling approach to the EGIP is developed in [12] and [13]. In this case, a more detailed modeling of the network results in higher computational burden; however, it is compensated by more reliable results.

This paper presents a performance comparison of AC and DC models applied to the EGIP. In both cases, an Iterated Local Search (ILS) is used to identify the most critical set of elements in terms of the load shedding caused if they are simultaneously attacked. Several tests were performed on the two-area IEEE RTS system. Results show important differences in load shedding of both models, especially when the number of simultaneous attacks increases. The information provided by these models results of paramount importance to system planners and system operators in order to device corrective and protective actions to make the system more resilient not only to natural occurring events but also to intentional attacks.

2. Mathematical modeling

Two different modeling approaches of the network were considered in the EGIP studied in this paper. The first one is the traditional DC model implemented in most vulnerability studies. This model does not take into account reactive power and assumes that all voltages are equal to 1 p.u. In this case, power flows are given as a function of angle differences and the reactance of the lines. The second one is the AC model which considers the effect of reactive power and takes into account the variation and limits of voltage magnitudes. The following hypotheses are considered for both models:

• The disruptive agent is able to anticipate the reaction of the system operator. This is one of the main assumptions of any bilevel programming model. All decisions of the disruptive agent are performed taking into account the reaction of the system operator.

• The disruptive agent has an upper limit on destructive resources that must adequately allocate to device the most effective attack plan in terms of load shedding. For the sake of simplicity every attack plan is considered to be 100% effective.

2.1. DC model of the EGIP

The DC model of the EGIP is given by Eq. (1)-(9) [7]. Eq. (1) corresponds to the objective function of the disruptive agent. In this case δ^{Lin} is a binary interdiction vector. Every position of this vector contains the state of the corresponding line (0 off service; 1 on service). This vector is used to represent the attack plan of the disruptive agent. P_{DS_n} is the load shedding of bus n, and N is the set of total buses. Eq. (2) indicates the limits on destructive recourses M. For the sake of simplicity it is assumed that the cost of destroying any line is the same. Eq. (3) indicates the binary nature of the interdiction vector, where L is the set of lines. Eq. (4) is the objective function of the system operator which aims at minimizing the cost of load shedding plus the cost of redispatching available resources. P_g^{Gen} is the active power delivered by generator g, while c_g and c_{DS_n} represent the costs of generation and load shedding, respectively. Eq. (5) is the mathematical expression of active power flows in DC modeling. In this case, A_{nl} is the bus-line incidence matrix, θ_n is the set of voltage angles, P_l^{Lin} is the power flow in line l, and Z_l is the line impedance. Eq. (6) is the active power balance constraint. Eq. (7) and (8) indicate limits on active power generation and voltage angles, respectively. Also, superscripts min and max indicate minimum and maximum limits of the corresponding variable. Finally, Eq (9) indicates that the load shedding in a given bus must be lower or equal to the demand of that bus. In this case, P_{DS_n} and P_{D_n} are the load shedding and demand at bus n, respectively.

$$\max_{\delta^{Lin}} \sum_n P_{DS_n}; \forall n \epsilon N \tag{1}$$

Subject to:

$$\sum_l (1-\delta_l^{Lin} \leq M; \forall l \epsilon L \tag{2}$$

$$\delta_l^{Lin} \epsilon 0,1; \forall l \epsilon L \tag{3}$$

$$min \sum_g c_g P_g^{Gen} + \sum_n c_{DS_n} P_{DS_n}; \tag{4}$$

Subject to:

$$P_l^{Lin} = \delta_l^{Lin} * \frac{1}{Z_l} \sum_{n \epsilon N} A_{nl} \theta_n; \forall l \epsilon L \tag{5}$$

$$\sum_{g \epsilon G} P_g^{Gen} - \sum_{l \epsilon L} A_{nl} P_l^{Lin} + P_{DS_n}; \forall n \epsilon N \tag{6}$$

$$P_g^{min} \leq P_g^{Gen} \leq P_g^{max}; \forall g \epsilon G \tag{7}$$

$$\theta_n^{min} \leq \theta_n \leq \theta_n^{max}; \forall n \epsilon N \tag{8}$$

$$0 \leq P_{DS_n} \leq P_{D_n}; \forall n \epsilon N \tag{9}$$

The main advantage of the DC model if the fact that the lower-level optimization problem given by Eq. (4)-(9) can be replaced by its optimality conditions turning the bilevel problem into an equivalent single-level optimization problem which can be solved resorting to traditional optimization approaches.

2.2. AC model of the EGIP

The AC modeling of the EGIP is given by Eq. (10)-(27). The upper level optimization problem and the objective function of the system operator described by Eq. (10)-(13) are essentially the same as those given by Eq. (1)-(4) described in the DC model. The main feature of this model is the fact that it takes into account active and reactive power injections which leads to a non-linear lower-level optimization problem. Eq. (14) to (18) indicate upper and lower limits on angles, voltage magnitudes, active and reactive power generation, and apparent power flows, respectively. In this case V_n, Q_g^{Gen} and S_l^{Lin} are the voltage magnitude at bus n, the reactive power supplied by generator g and the apparent power flow in line l, respectively. Eq. (19) and (20) impose limits on active and reactive load shedding, which must be lower than the corresponding demand at bus n. In this case Q_{DS_n} is the reactive load shedding at bus n while Q_{Dn} is the reactive demand at bus n. Eq. (21) and (22) represent the net active and reactive power injected at bus n, respectively; g_{mn} and b_{mn} are real and imaginary entries of the m, n position of the admitance matrix, respectively; θ_{mn} represents the angular difference between nodes m and n. Eq. (23) expresses the active and reactive components of the apparent power flow. Eq. (24) and (25) are the mathematical expressions that define active and reactive power flows, respectively. Note that Eq. (24) and (25) are multiplied by δ_l^{Lin} meaning that there are no power flows on faulted lines. Finally, Eq. (26) and (27) indicate the active and reactive power balance constraints in very node.

$$\max_{\delta^{Lin}} \sum_n P_{DS_n}; \forall n \epsilon N \tag{10}$$

Subject to:

$$\sum_l (1-\delta_l^{Lin}) \leq M; \forall l \epsilon L \tag{11}$$

$$\delta_l^{Lin} \epsilon 0,1; \forall l \epsilon L \tag{12}$$

$$min \sum_g c_g P_g^{Gen} + \sum_n c_{DSn} P_{DS_n}; \tag{13}$$

Subject to:

$$\delta_n^{min} \leq \delta_n \leq \delta_n^{max}; \forall n \epsilon N \tag{14}$$

$$V_n^{min} \leq V_n \leq V_n^{max}; \forall n \epsilon N \tag{15}$$

$$P_g^{min} \leq P_g^{Gen} \leq P_g^{max}; \forall g \epsilon G \tag{16}$$

$$Q_g^{min} \leq Q_g^{Gen} \leq Q_g^{max}; \forall g \epsilon G \tag{17}$$

$$S_l^{min} \leq P_l^{Lin} \leq S_l^{max}; \forall l \epsilon L \tag{18}$$

$$0 \leq P_{DS_n} \leq P_{D_n}; \forall n \epsilon N \tag{19}$$

$$0 \leq Q_{DS_n} \leq Q_{D_n}; \forall n \epsilon N \tag{20}$$

$$P_n = V_n \sum_n V_m [g_{mn} cos(\delta_{mn}) + b_{mn} sin(\delta_{mn})]; \forall n \epsilon N \tag{21}$$

$$Q_n = V_n \sum_n V_m [g_{mn} sin(\delta_{mn}) + b_{mn} cos(\delta_{mn})]; \forall n \epsilon N \tag{22}$$

$$(S_l^{Lin})^2 = (P_l^{Lin})^2 + (Q_l^{Lin})^2; \forall l \epsilon L \tag{23}$$

$$P_l^{Lin} = \delta_l^{Lin} [g_{mn} V V_n^2 + g_{mn} V_m V_n cos(\theta_{mn}) - b_{mn} V_m V_n sin(\theta_{mn})]; \forall l \epsilon L \tag{24}$$

$$Q_l^{Lin} = \delta_l^{Lin} [-b_{mn} V_n^2 + b_{mn} V_m V_n cos(\theta_{mn} - b_{mn} V_m V_n sin(\theta_{mn})]; \forall l \epsilon L \tag{25}$$

$$P_g^{Gen} - P_{D_n} + P_{DS_n} = P_n; \forall n \epsilon N \tag{26}$$

$$Q_g^{Gen} - Q_{D_n} + Q_{DS_n} = Q_n; \forall n \epsilon N \tag{27}$$

The AC model of the EGIP provides a more detailed description of the network. However, the lower-level optimization problem is non-linear (see Eq. (21) and (25)) and therefore, it cannot be replaced by its optimality conditions. This situation makes difficult the use of conventional mathematic programming techniques to solve the EGIP problem, since these techniques would easily get trapped in local optimal solutions given the multi-modal nature of the model. For these type of problems metaheuristic techniques have proven to be more effective. In this case, an ILS was developed to solve both, the AC and DC models of the EGIP.

3. Solution approach

As already mentioned, the AC model of the EGIP is a non-linear, non-convex, multi-model problem that is better handled by metaheuristic techniques than by traditional mathematical programing approaches. In this case, for comparative purposes, an ILS was implemented to solve both models. The implementation of the ILS metaheuristic technique is explained in detail in this section.

3.1. Problem codification

One of the key aspects when implementing a metaheuristic is the representation of the candidate solutions. For the EGIP a candidate solution is represented by a binary interdiction vector denoted as δ_l^{Lin}. The values of this vector indicate the state of the corresponding asset (on or off service). If a given positon of δ_l^{Lin} is cero, it indicates that such line or transformer is out of service. Fig. 1 illustrates an interdiction vector of a power system, where lines L1, L5, L10 and L12 are under attack.

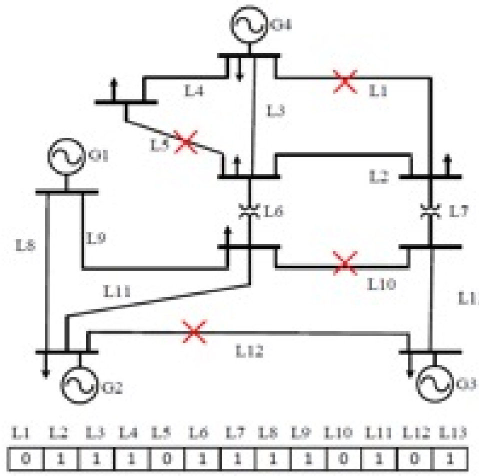

Fig. 1. Example of a power system under attack and its corresponding interdiction vector.

3.2. Objective function evaluation

An initial candidate solution or interdiction vector can be generated in a random or pseudo-random fashion. Every interdiction vector must take into account the limits on destructive resources expressed in both models (see Eq. (2) and (11). Once this is verified, the corresponding load shedding associated to a given interdiction vector can be evaluated. Note that the interdiction vector contains the decision variables of the upper-level optimization problem. These variables then become parameters for the lower-level optimization problem which account for the reaction of the system operator. To solve the lower-level optimization problem, an optimal power flow is executed considering the unavailability of the power system elements affected by the attack plan. In this case, the software Matpower [14] is used to compute the load shedding associated with any given attack plan considering the optimal redispatch executed by the system operator.

3.3. Iterated local search

This metaheuristic is based on successive local optimal search. The problem starts with an initial solution. The algorithm performs a local search in the neighborhood of this solution, and once it has found a local optimum, a perturbation is performed and a new local search is executed. Every time a local optimum is found it is saved. The process is carried out iteratively until a given number of perturbations and local searches are performed. The solution will be the best of the local optimal solutions found. Fig. 2 illustrates an ILS for a minimization problem in which two perturbations and three local searches are applied. An initial solution (blue dot) is proposed; then, the first local search identifies x1 as the initial local optimal solution. A perturbation is then applied and a new local search is performed finding x2 as the new local optimal solution; finally, a third perturbation is applied and x3 is found as the third local optimal solution. The algorithm them selects the minimum of the three local optimal results as the proposed solution. Fig. 2 illustrates a flowchart of the implemented ILS where IV stands for interdiction vector.

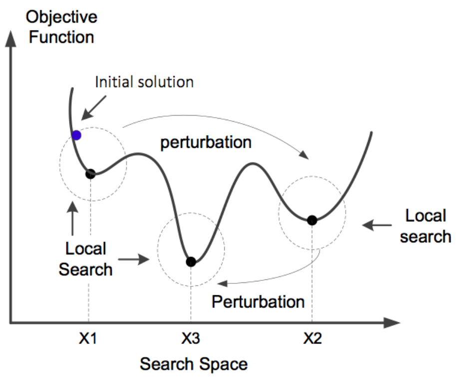

Fig. 2. Illustration of an Iterated Local Search.

The local search implemented within the ILS is performed in two steps. The first step aims at introducing diversification to the problem and is defined by a random and simultaneous variation of two components of the interdiction vector. The new solution is accepted only if it increments the load shedding and does not exceeds the limits on destructive resources. The second step consists on the random, one by one variation of each component of the interdiction vector and is intended to introduce intensification.

4. Tests and results

Several tests were performed on a benchmark power system comprising 48 buses, 79 lines, 62 generators and 34

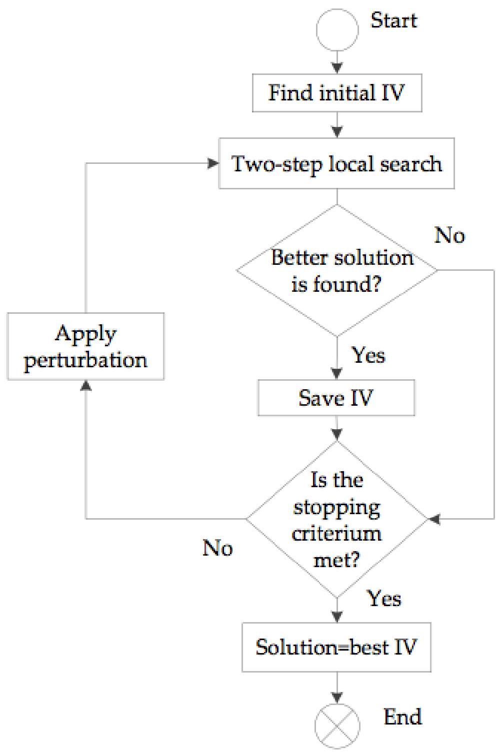

Fig. 3. Flowchart of the implemented ILS.

loads. This power system is based on the IEEE Reliability Test System and considers two areas [15]. The load profile corresponds to a winter weekday at 6:00 pm and adds up (5700 MW). The tests were performed considering an increasing number of destroyed lines, represented as an increase of destructive resources. For the sake of simplicity the cost of attacking any line or transformer is considered to be one monetary unit; being M the number of monetary units available to the disruptive agent in each case. Also, minimum and maximum voltage magnitude limits are considered to be 0.95 and 1.05 pu., respectively. The cost of load shedding was considered to be ten times the cost of the most expensive generator. The ILS was set to run 30 iterations for both intensification and diversification of the two-step local search, and 50 perturbations were carried out for each test. All simulations were carried out on a laptop win an Intel orei-5 processor and 4GW of RAM memory.

Table 1 presents a summary of the obtained results. The fist column indicates the number of destructive resources available to the disruptive agent. The second column indicates the best attack plan found by the ILS. The rest of the columns describe the load shedding obtained using both models and their difference. Tests with the AC model took, in average twice the time as tests with the DC model. Note that when only two lines can be attacked, the algorithm finds two possible options: destroying lines between nodes 111-114, 114-116 or between nodes 211-214, 214-216. Both attack plans result in the same load shedding of 194 MW. This is because in each case bus 114 and 214 are isolated from the rest of the system. These buses have generation; however, it is not enough to supply the local demand. In this case, the same load shedding is obtained for both AC and DC models. When M=4, the solution proposed by the ILS is the combination of the two previously solutions found with M=2, resulting in 368MW of total load shedding and no difference between AC and DC models. This attack plan is depicted in Fig. 4. The red portion of the network highlights the buses with load shedding and the dashed red lines are those under attack.

When M=6 the best attack plan consists on isolating buses 119, 120, 219 and 220, resulting in a total blackout for the demands located in such buses, since they do not have local generation. This attack plan is illustrated in Fig. 5. The dashed red lines are marked as those under attack. As the total demand of buses 119, 120, 219 and 220 is unattended there is no difference between the AC and DC models. So far the best attack plans with M=2, 4 and 6 do not show any difference between the AC and DC models. That is because such plans are are intended to isolate specific portions of the system resulting in total loss of load at specific buses (see Fig. 4 and Fig. 5). Nevertheless, for higher values of M, there are significant differences between both models. Suh differences are up to 11.5% as indicated in Table 1. This is due to the fact that attack plans with higher destructive resources are not directed toward disconnecting specific buses; instead, these attacks are more systemic and aim at introducing a deficit of generation in the network. This is done by destroying the link lines between the main generators located at the upper level of the system from most of the demand, located at the lower level of the system. Fig. 6 depicts this strategy with M=10. Note that no specific buses are disconnected from the system; instead, the attack plan is such that the whole system is split in two areas. The red area has an important deficit in generation; furthermore, reactive power is not enough to keep voltage magnitudes within permissible limits. This forces the system operator to increase load shedding in order to increase voltage magnitudes and preserve the security of the remaining subsystems. Similar situations are presented with higher values of M, in which the difference between load shedding with AC and DC models are significant.

5. Conclusion

This paper presented an assessment of the EGIP considering the AC and DC models of the network. The vulnerability of the power system is addressed from the standpoint of the interaction of a disruptive agent and the system operator, which is formalized as a bilevel programming problem. An ILS metaheuristic was developed to solve both AC and DC models of the EGIP. Results allow concluding that the DC model often reports conservative solutions; especially for those cases in which a disruptive agent with high destructive resources executes a systemic attack. On the other hand, for a disruptive agent with low destructive resources that focusses on specific buses there are no significant differences between AC and DC models. The main advantage of using a metaheuristic for solving the EGIP is the possibility of having a set of high quality solutions instead of a single one. This gives the system operator more information about the most vulnerable elements and provides signals for future reinforcements of the network or stricter surveillance on critical elements. Finally, with the use of a metaheuristic technique instead of a classical mathematical programming approach, both DC and AC models can be solved without resorting to linearization schemes to transform the EGIP into a single-level optimization problem.

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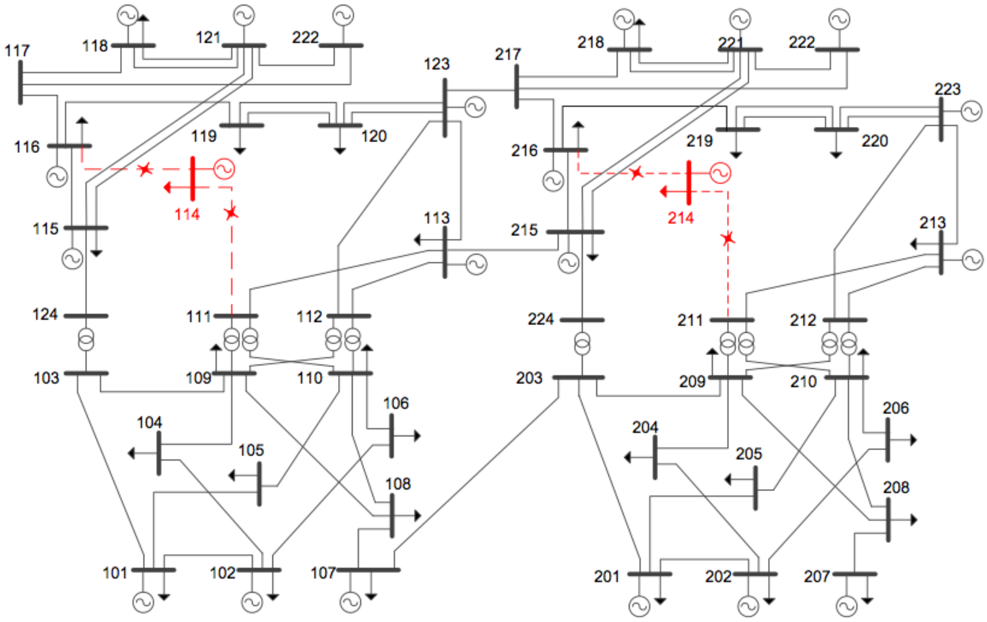

Fig. 4. Illustration of attack plan for M=4.

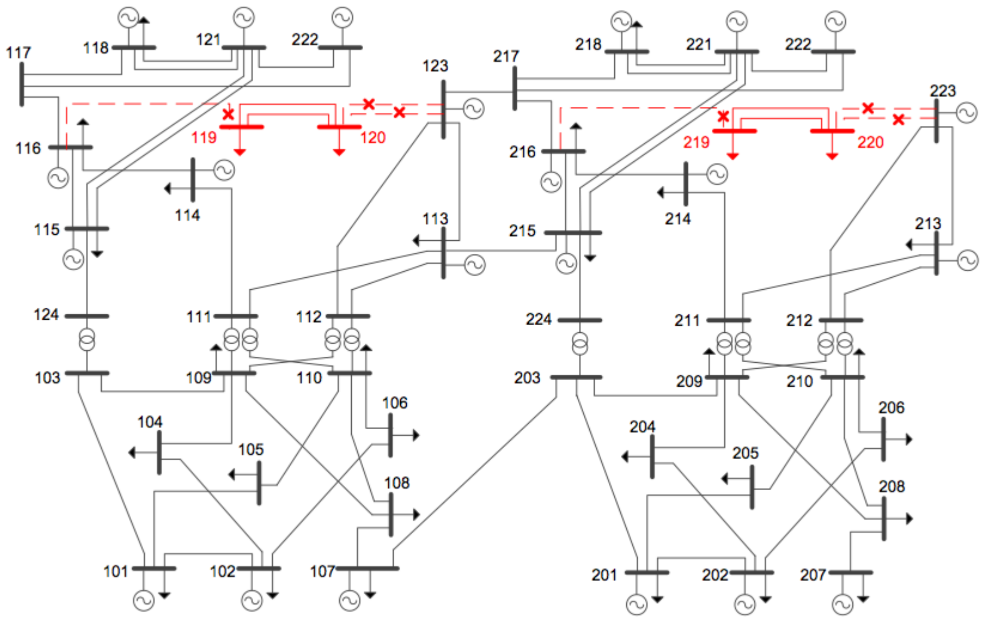

Fig. 5. Illustration of best attack plan for M=6.

Table 1. Worst combination of attacked lines with different destructive resources.

M	Attacked lines	Load checking (MW)		
2	111-114, 114-116	194	194	0
	211-214, 214-216	194	194	0
4	111-114, 114-116, 211-214, 214-216	388	388	0
6	120-123, 120-123, 119-116, 220-223, 220-223, 219-216	618	618	0
8	103-124, 112-123, 113-123, 114-116, 203-224, 212-223, 213-223, 214-216	1032	1119.6	11.5%
10	115-124, 111-114, 111-113, 112-123, 112-113, 215-224, 211-214, 211-213, 212-223, 212-213	1296	1307.5	10.1%
12	103-224, 107-108, 109-112, 110-112, 111-113, 114-116, 203-224, 207-208, 209-212, 210-212, 211-213, 214-216	2034	2048	10.1%

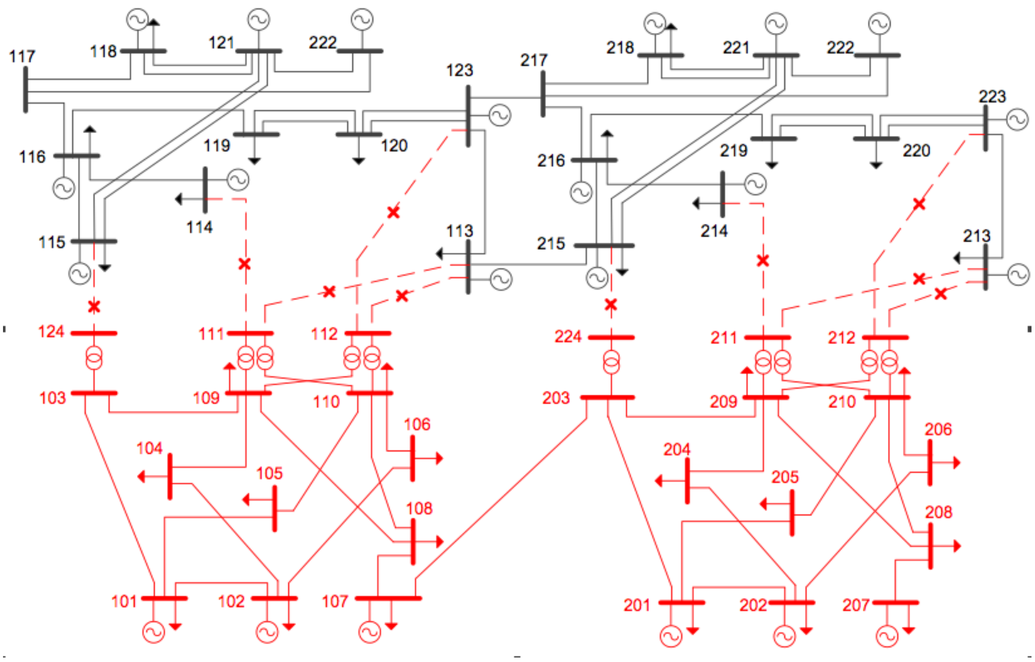

Fig. 6. Illustration of best attack plan for M=10.

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