

Mixed Convection Flow of Couple Stress Fluid in a Non-Darcy Porous Medium with Soret and Dufour Effects

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Abstract

An analysis is presented to investigate the Soret and Dufour effects on the mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a non-Darcy porous medium saturated with couple stress fluid. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations and then solved numerically. Special cases of the present investigation are compared with previously published work and found to be in good agreement. Profiles of dimensionless velocity, temperature and concentration are shown graphically for different parameters entering into the analysis. Also the effects of the pertinent parameters on the rates of heat and mass transfer in terms of the local Nusselt and Sherwood numbers are discussed.

Key Words: Mixed Convection, Non-Darcy Porous Medium, Couple Stress Fluid, Soret and Dufour Effect

1. Introduction

A situation where both the forced and free convection effects are of comparable order is called mixed or combined convection. The analysis of mixed convection boundary layer flow along a vertical plate embedded in a fluid saturated porous media has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Several authors have studied the problem of mixed convection about different surface geometries. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years.

The inertia effect is expected to be important at a higher flow rate and it can be accounted through the

addition of a velocity squared term in the momentum equation, which is known as the Forchheimer's extension of the Darcy's law. A detailed review of convective heat transfer in Darcy and non-Darcy porous medium can be found in the book by Nield and Bejan [1].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret

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effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H_2 , He) and of medium molecular weight (N_2 , air). The Dufour effect was recently found to be of order of considerable magnitude such that it cannot be neglected [2]. Kafoussias [3] presented the local similarity solution for combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. Dursunkaya and Worek [4] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [5] presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Postelnicu [6] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam and Rahman [7] have investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Both free and forced convection boundary layer flows with Soret and Dufour have been addressed by Abreu [8]. The effect of Soret and Dufour parameters on free convection heat and mass transfers from a vertical surface in a doubly stratified Darcian porous medium has been reported by Lakshmi Narayana and Murthy [9]. Recently, Srinivasacharya and RamReddy [10] studied the Soret and Dufour effects on mixed convection in a non-Darcy porous medium saturated with micropolar fluid numerically using Keller Box method.

Different models have been proposed to explain the behaviour of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes [11] have distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. The spin field due to microrotation of freely suspended particles set up an antisymmetric stress, known as couple-stress, and thus forming couple-stress fluid. These fluids are capable of describing various types of lubricants, blood, sus-

pension fluids etc. The study of couple-stress fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions etc. Application of the couple stress model to biomechanics problems has been proposed in the study of peristaltic transport by Srivastava [12], Shehawey and Mekheimer [13] and blood flow in the microcirculation by Dulal Pal et al. [14]. In lubrication problems many authors have investigated the couple stress effects on different lubrication problems (Chiang et al. [15], Naduvinamani et al. [16], Jian et al. [17], Lu et al. [18]). Recently Srinivasacharya and Kaladhar [19] studied the Hall and ion-slip effects on mixed convective flow of couple stress fluid in a vertical channel.

The objective of this paper is to investigate the Dufour and Soret effects on steady mixed convective and mass transfer flow past a semi-infinite vertical flat plate in couple stress fluid. The Keller-box method [20] is employed to solve the nonlinear problem. The effects of couple stress fluid parameter, non-Darcy parameter; Soret and Dufour numbers are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2. Mathematical Formulation

Consider a two-dimensional mixed convective heat and mass transfer along a semi-infinite vertical plate embedded in a free stream of couple stress fluid saturated non-Darcy porous medium. The free stream velocity which is parallel to the vertical plate with velocity is u_∞ , temperature T_∞ and concentrations C_∞ . Assume that the fluid and the porous medium have constant physical properties. The fluid flow is moderate and the permeability of the medium is low so that the Forchheimer flow model is applicable and the boundary effect is neglected. The fluid and the porous medium are in local thermo dynamical equilibrium. Choose the coordinate system such that x-axis is along the vertical plate and y-axis normal to the plate. The plate is maintained at a uniform wall temperature T_w and concentration C_w . These values are assumed to be greater than the ambient temperature T_∞ and concentration C_∞ at any arbitrary reference point in the medium (inside the boundary layer). In addition, the Soret and Dufour effects are considered [21]. The flow configuration and the coordinates system are shown in Figure 1.

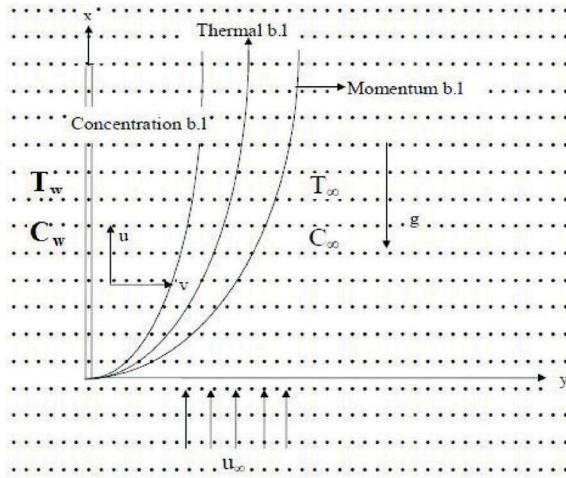


Figure 1. Physical model and coordinate system.

Assuming that the Boussinesq and boundary-layer approximations hold and using the Darcy-Forchheimer model and Dupuit-Forchheimer relationship [1], the governing equations for the couple stress fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\rho}{\varepsilon^2} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\mu}{\varepsilon} \frac{\partial^2 u}{\partial y^2} + \rho g (\beta_T (T - T_\infty) + \beta_C (C - C_\infty)) - \eta_1 \frac{\partial^4 u}{\partial y^4} + \frac{\mu}{K_p} (u_\infty - u) + \frac{\rho b}{K_p} (u_\infty^2 - u^2) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u, v are the velocity components in the x and y directions respectively, μ is the coefficient of viscosity, g is the acceleration due to gravity, ρ is the density, b is the Forchheimer constant, K_p is the permeability, ε is the porosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, α is the thermal diffusivity, D is the mass diffusivity, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio and η_1 is the couple stress fluid parameter. The last two terms on the right hand side of equation (2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively. The

last terms on the right-hand side of the energy equation (3) and concentration equation (4) signifies the Dufour (diffusion-thermo) effect and the Soret (thermal-diffusion) effect, respectively.

The boundary conditions are

$$u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \tag{5a}$$

$$v_x = u_y \quad \text{at } y = 0 \quad \text{and } y \rightarrow \infty \tag{5b}$$

$$u = u_\infty, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty \tag{5c}$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively, where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively, (5a) corresponds to the classical no-slip condition from viscous fluid dynamics and wall conditions of heat, mass transfer. The boundary condition equation (5b) implies that the Vorticity at the boundary is equal to zero.

In view of the continuity equation (1), we introduce the stream function by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Substituting Eq. (6) in Eqs. (2)–(4) and then using the following local similarity transformations

$$\eta = \frac{y}{x} \text{Re}_x^{1/2}, \quad u = u_\infty f'(\eta), \quad v = -\frac{1}{2} \sqrt{\frac{u_\infty v}{x}} [f(\eta) - \eta f'(\eta)],$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

we get the following nonlinear system of differential equations.

$$\frac{1}{\varepsilon} f''' + \frac{1}{2\varepsilon^2} ff'' + g_s \theta + g_c \phi + \frac{1}{Da \text{Re}_x} (1 - f')$$

$$+ \frac{Fs}{Da} (1 - f'^2) - C_\alpha f^{(v)} = 0 \tag{8}$$

$$\frac{1}{\text{Pr}} \theta'' + \frac{1}{2} f \theta' + D_f \phi'' = 0 \tag{9}$$

$$\frac{1}{Sc} \phi'' + \frac{1}{2} f \phi' + S_r \theta'' = 0 \tag{10}$$

Boundary conditions (5) in terms of f, θ and ϕ become

$$f = 0, f' = 0, f'' = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \quad (11a)$$

$$f' = 0, f'' = 0, \theta = 0, \phi = 0 \text{ as } \eta \rightarrow \infty \quad (11b)$$

where primes denote differentiation with respect to η , $Sc = \frac{\nu}{D}$ is the Schmidt number, $Pr = \frac{\nu}{\alpha}$ is the Prandtl

number, $Re_x = \frac{u_\infty x}{\nu}$ is the local Reynolds number, $S_r =$

$\frac{DK_T(T_W - T_\infty)}{\nu T_m(C_W - C_\infty)}$ is the Soret number, $D_f =$

$\frac{DK_T(C_W - C_\infty)}{\nu C_S C_P(T_W - T_\infty)}$ is the Dufour number, $Gr_x =$

$\frac{g\beta_T(T_W - T_\infty)x^3}{\nu^2}$ is the local temperature Grashof num-

ber, $Gc_x = \frac{g\beta_c(C_W - C_\infty)x^3}{\nu^2}$ is the local mass Grashof

number, $C_\alpha = \frac{\eta_1}{\mu x^2} Re_x$ is the local couple stress para-

meter, $g_s = \frac{Gr_x}{Re_x^2}$ is the temperature buoyancy parameter,

$g_c = \frac{Gc_x}{Re_x^2}$ is the mass buoyancy parameter, $Fs = \frac{b}{x}$ is the

Forchheimer number and $Da = \frac{K_p}{x^2}$ is the Darcy number.

If $D_f = 0$, $S_r = 0$, $Da \rightarrow \infty$ and $\varepsilon = 1$, the problem reduces to mixed convection heat and mass transfer on a semi-infinite vertical plate with uniform wall temperature and concentration in a couple stress fluid without Soret and Dufour effects. In the limit, as $C_\alpha \rightarrow 0$, the governing equations (8)–(10) reduce to the corresponding equations for a mixed convection heat and mass transfer in a viscous fluids. Hence, the case of combined free-forced convective and mass transfer flow past a semi-infinite vertical plate of Kafoussias [5] can be obtained by taking $D_f = 0$, $S_r = 0$, $Da \rightarrow \infty$, $\varepsilon = 1$, $C_\alpha \rightarrow 0$.

The heat and mass transfers from the plate are given by

$$q_w = -k \left[\frac{\partial T}{\partial y} \right]_{y=0} \quad (12a)$$

$$q_m = -D \left[\frac{\partial C}{\partial y} \right]_{y=0} \quad (12b)$$

The non dimensional local Nusselt number $Nu_x =$

$\frac{xq_w}{k(T_w - T_\infty)}$ and local Sherwood number $Sh_x =$

$\frac{xq_m}{D(C_w - C_\infty)}$ are given by

$$\frac{Nu_x}{Re_x^{1/2}} = -\theta'(0), \quad \frac{Sh_x}{Re_x^{1/2}} = -\phi'(0) \quad (13)$$

3. Results and Discussion

The flow equation (8) together with the energy and concentration equations (9) and (10), constitute non-linear non homogeneous differential equations for which closed-form solutions cannot be obtained. Hence the governing equations (8)–(10) are solved numerically using the Keller-box implicit method [18]. The method has the following four main steps:

- I. Reduce the system of Eqs. (8) to (10) to a first order system;
- II. Write the difference equations using central differences;
- III. Linearize the resulting algebraic equations by Newtons method and write them in matrix-vector form;
- IV. Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been proven to be adequate and give accurate results for boundary layer equations. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity approaches 1 and temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta = 0.001$, $\Delta\eta = 0.01$ and $\Delta\eta = 0.05$ and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of η at ∞ of 6 and a grid size of η as 0.01.

In the absence of Soret number S_r and Dufour number D_f with $Da \rightarrow \infty$, $\varepsilon = 1$, $C_\alpha = 0$, $Pr = 0.73$, $Sc = 0.24$ for different values of buoyancy parameters g_s and g_c , the results have been compared with the case Kafoussias [5] and found that they are in good agreement, as shown in Table 1.

In order to study the effects of couple stress fluid parameter C_α , non-Darcy parameter Fs , Soret number S_r and Dufour number D_f explicitly, computations were carried out for the cases of $Pr = 0.71$ (air), $Sc = 0.22$ (hydro-

gen at 25 °C and 1 atmosphere pressure), $g_s = 1.0$, $g_c = 0.1$, $\varepsilon = 0.3$ and $Da = 1.0$. The values of Soret number S_r and Dufour number D_f are chosen in such a way that their product is constant according to their definition provided that the mean temperature T_m is kept constant [5]. These values are used throughout the computations, unless otherwise indicated.

Figure 2 displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f with $C_\alpha = 1.0$ and $Fs = 0.5$. It can be observed from this figure that the velocity of the fluid increases with the increase of Dufour number (or decrease of Soret number). Either a decrease in concentration difference or an increase in temperature difference leads to an increase in the value of the Dufour parameter (D_f). Hence, decreasing the Dufour parameter (D_f) decreases the velocity of the fluid i.e., the lowest peak of the reverse flow velocity corresponds to the highest Soret number and lowest Dufour number. The dimensionless temperature for different values of Soret number S_r and Dufour number D_f

with $C_\alpha = 1.0$ and $Fs = 0.5$ is shown in Figure 3. It is clear that the temperature of the fluid decreases with the decrease of Dufour number (or increase of Soret number). The parameter S_r (Soret number) does not enter directly into the energy equation and the Dufour effect enhances the mass fluxes and lowers the heat fluxes. Therefore increasing D_f value strongly heats the flow. Figure 4 demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f with $C_\alpha = 1.0$ and $Fs = 0.5$. It is seen that an increase in Soret number (or decrease of Dufour number) due to the contribution of temperature gradients to species diffusion, increases the concentration.

The dimensionless velocity component for different values of Forchheimer number Fs with $D_f = 0.03$, $S_r = 2.0$ and $C_\alpha = 1.0$, is depicted in Figure 5. It shows the effects of Forchheimer (inertial porous) number on the velocity. In the absence of Forchheimer number (i.e., when $Fs =$

Table 1. Comparison of local Nusselt number (Nu_x)

g_s	g_c	Kafoussias [5] (Nu_x)	Present (Nu_x)
0.10	0.05	0.3296	0.3296
0.10	0.10	0.3404	0.3404
0.10	0.20	0.3589	0.3589
1.00	0.05	0.4129	0.4129
1.00	0.10	0.4179	0.4179
1.00	0.20	0.4274	0.4274
10.0	0.05	0.6449	0.6450
10.0	0.10	0.6461	0.6462
10.0	0.20	0.6487	0.6488

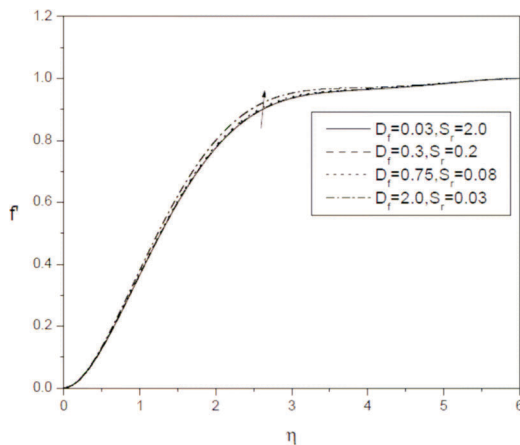


Figure 2. Velocity profile for different values of D_f , S_r at $C_\alpha = 1.0$, $Fs = 0.5$.

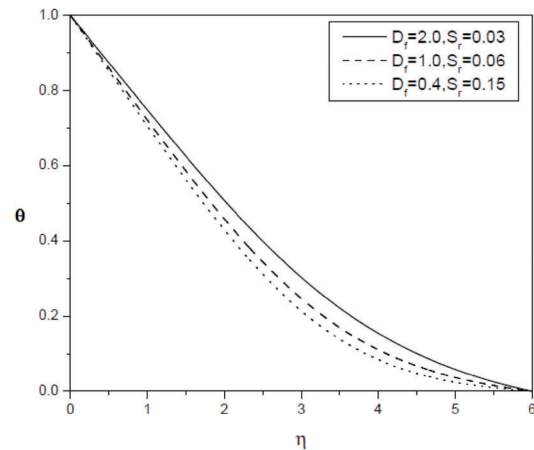


Figure 3. Temperature profile for different values of D_f , S_r at $C_\alpha = 1.0$, $Fs = 0.5$.

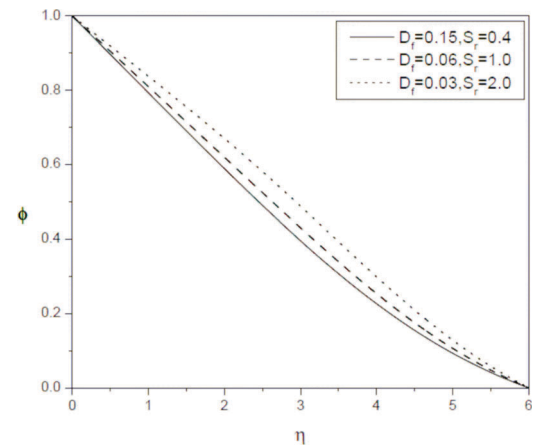


Figure 4. Concentration profile for different values of D_f , S_r at $C_\alpha = 1.0$, $Fs = 0.5$.

0), the present investigation reduces to a mixed convection heat and mass transfer in a couple stress fluid saturated porous medium with Soret and Dufour effects. It is observed from Figure 5 that velocity of the fluid decreases with increase in the value of the non-Darcy parameter F_s . The increase in non-Darcy parameter implies that the porous medium is offering more resistance to the fluid flow. This results in reduction of the velocity profile. The dimensionless temperature for different values of Forchheimer number F_s for $D_f = 0.03$, $S_r = 2.0$ and $C_\alpha = 1.0$, is displayed in Figure 6. An increase in Forchheimer number F_s , increase temperature values, since as the fluid is decelerated, energy is dissipated as heat and serves to increase temperatures. As such the temperature is minimized for the lowest value of F_s and maximized for the highest value of F_s as shown in Figure 6. Figure 7 demonstrates the dimensionless concentration for different values of Forchheimer number with $D_f = 0.03$, $S_r =$

2.0 and $C_\alpha = 1.0$. It is clear that the concentration of the fluid increases with the increase of Forchheimer number. The increase in non-Darcy parameter reduces the intensity of the flow and increases the thermal and concentration boundary layer thickness.

In Figures 8–10, the effects of the couple stress parameter C_α at a given x -location on the dimensionless velocity, temperature and concentration profiles are presented for fixed values of $D_f = 0.03$, $S_r = 2.0$ and $F_s = 0.5$. As C_α increases, it can be observed from Figure 8 that the maximum velocity decreases in amplitude and the location of the maximum velocity moves far away from the wall. This happens because of the rotational field of the velocity generated in couple stress fluid. It is clear from Figure 9 that the temperature increases with the increase of couple stress fluid parameter C_α . It can be seen from Figure 10 that the concentration of the fluid increases with the increase of couple stress fluid parameter C_α .

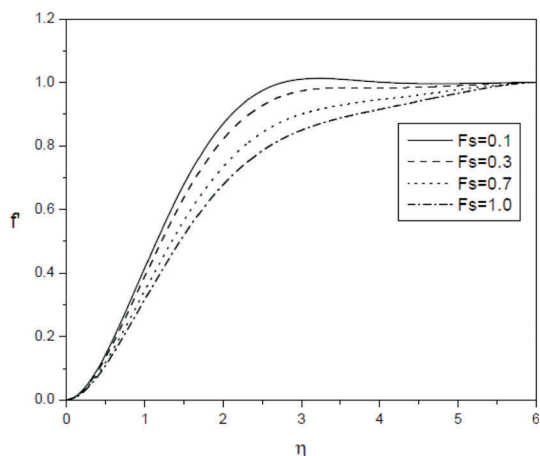


Figure 5. Velocity profile for different Values of F_s at $C_\alpha = 1.0$, $D_f = 0.03$, $S_r = 2.0$.

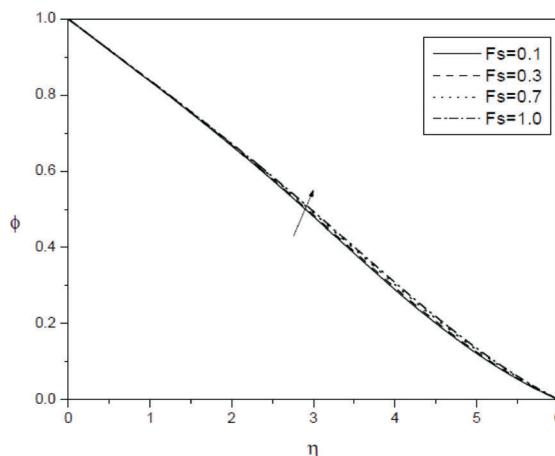


Figure 7. Concentration profile for different values of F_s at $C_\alpha = 1.0$, $D_f = 0.03$, $S_r = 2.0$.

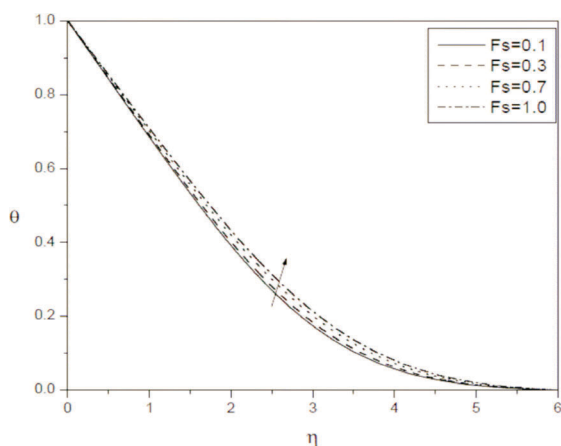


Figure 6. Temperature profile for different values of F_s at $C_\alpha = 1.0$, $D_f = 0.03$, $S_r = 2.0$.

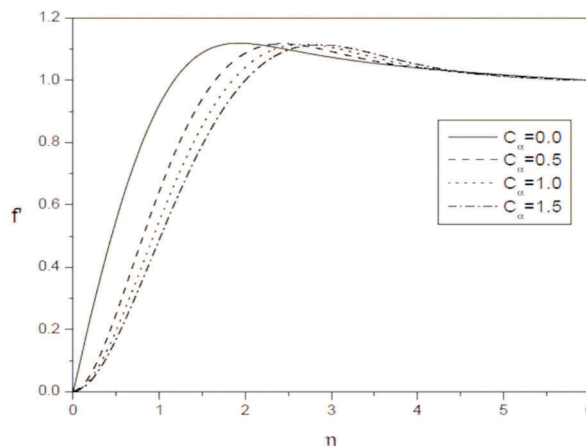


Figure 8. Velocity profile for different values of C_α at $F_s = 0.5$, $D_f = 0.03$, $S_r = 2.0$.

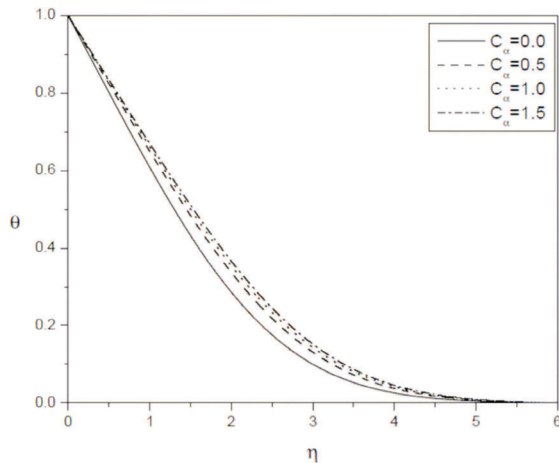


Figure 9. Temperature profile for different values of C_α at $Fs = 0.5, D_f = 0.03, S_r = 2.0$.

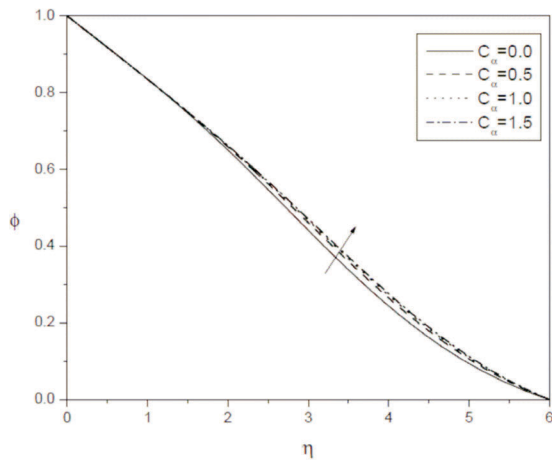


Figure 10. Concentration profile for different values of C_α at $Fs = 0.5, D_f = 0.03, S_r = 2.0$.

The variations of $-\theta'(0)$ and $-\phi'(0)$, which are proportional to the rate of heat and mass transfers are shown in Table 2 for different values of the couple stress parameter with fixed Forchheimer, Soret and Dufour numbers. From this table, it is observed that the value of the heat transfer rate decreases and the mass transfer rate increases with an increase in C_α . From this data, it is observed that the value of the heat transfer rate decreases and the mass transfer rate increases with an increase in C_α . The heat transfer is lower and mass transfer is higher in the couple stress fluid compared to that of Newtonian fluid. Hence, the presence of couple stresses in the fluids decreases the heat transfer coefficient and increases the mass transfer coefficient, which may be beneficial in flow, temperature and concentration control of polymer processing. Also, it can be observed from this table that, for fixed values of C_α, S_r and D_f , the heat and mass trans-

Table 2. Effect of couple stress parameter, Forchheimer, Prandtl, Soret and Dufour numbers on heat and mass transfer coefficients

C_α	Fs	S_r	D_f	Pr	$-\theta'(0)$	$-\phi'(0)$
0.1	0.50	2.00	0.03	0.71	0.374589	0.164133
0.3	0.50	2.00	0.03	0.71	0.360891	0.164571
0.5	0.50	2.00	0.03	0.71	0.352836	0.164718
0.7	0.50	2.00	0.03	0.71	0.346929	0.164776
1.0	0.50	2.00	0.03	0.71	0.340169	0.164793
1.0	0.10	2.00	0.03	0.71	0.325004	0.163627
1.0	0.30	2.00	0.03	0.71	0.333254	0.164221
1.0	0.70	2.00	0.03	0.71	0.346088	0.165336
1.0	1.00	2.00	0.03	0.71	0.353572	0.166095
1.0	0.50	2.00	0.03	0.10	0.199076	0.222374
1.0	0.50	2.00	0.03	0.50	0.301763	0.180097
1.0	0.50	2.00	0.03	1.00	0.382300	0.148076
1.0	0.50	2.00	0.03	5.00	0.642275	0.041496
1.0	0.50	2.00	0.03	0.71	0.340169	0.164793
1.0	0.50	1.60	0.0375	0.71	0.339868	0.178521
1.0	0.50	1.20	0.05	0.71	0.339402	0.192245
1.0	0.50	1.00	0.06	0.71	0.339045	0.199107
1.0	0.50	0.80	0.075	0.71	0.338523	0.205969
1.0	0.50	0.50	0.12	0.71	0.336999	0.216273
1.0	0.50	0.20	0.30	0.71	0.331013	0.226657
1.0	0.50	0.10	0.60	0.71	0.321056	0.230264

fer coefficients are increases with the increasing values of Forchheimer number Fs . Hence, the inertial effects in couple stress fluid saturated non-Darcy porous medium increase the heat and mass transfer coefficients. The effect of increasing the value of Pr is to decrease the mass transfer rate but increase heat transfer rate with fixed values of C_α, Fs, S_r and D_f . Finally, the effects of Dufour and Soret number on the rate of heat and mass transfer are shown in this table. The behavior of these parameters is self-evident from the Table 2 and hence is not discussed for brevity.

4. Conclusion

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in a non-Darcy couple stress fluid over a vertical plate with uniform wall temperature and concentration conditions in the presence of Soret and Dufour effects is considered. Using the similarity variables, the governing equations are transformed into a set of non-similar parabolic equations where numerical solution has been presented for different values of parameters. The numerical results indicate that the

rate of heat and mass transfers in the couple stress fluid are lower compared to that of the Newtonian fluid. The higher values of the Forchheimer number F_s indicate lower velocity, the rate of heat and mass transfers but higher wall temperature and wall concentration distributions. Increasing the Prandtl number substantially decreases the mass transfer rate and the increase the rate of heat transfer. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects. The presence of couple stresses in the fluid decreases the velocity and increases temperature and concentration.

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