Magnetohydrodynamic Convection Effects with Viscous and Ohmic Dissipation in a Vertical Channel Partially Filled by a Porous Medium

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Abstract

The fully developed MHD mixed convection flow in a vertical channel, partially filled with clear fluid and partially filled with a fluid-saturated porous medium, is investigated by taking into account viscous and Ohmic dissipations. The bounding impermeable walls of the channel are maintained at different constant temperatures. The isothermal-isoflux and isoflux-isothermal temperature conditions on the walls have also been considered. The flow in the porous medium is modeled by the Brinkman equation and the Boussinesq approximation holds. The velocity and temperature distributions in the composite channel are obtained numerically by shooting technique with fourth order Runge-Kutta method and analytically by perturbation series method. These results have been depicted graphically, compared and discussed.

Key Words: Mixed MHD Convection, Vertical Channel, Viscous and Ohmic Dissipation, Permeability of the Porous Medium, Perturbation Series Method

1. Introduction

The study of viscous fluid flow through and across porous media in the presence of a magnetic field, has become increasingly, for many researchers, an attractive field for its widespread engineering applications. It has been the important subject of many recent research papers, for example see Raptis et al. [1], Chamkha [2], Geindreau and Auriault [3], Chauahn and Jain [4], Prasad and Reddy [5], Chauhan and Rastogi [6], Chauhan and Agrawal [7], Pal and Mondal [8], and Pal and Talukdar [9].

Interest in the study of mixed convection flow in porous and non-porous vertical channels, is motivated by the importance of it in a wide range of engineering applications, such as in many situations like moisture migration through air which is contained in fibrous insulations, cooling of electronic instruments and devices, electrochemical processes, and in solar energy collectors. Tao [10] investigated the fully developed mixed convection with uniform wall temperature in a vertical channel. Several researchers devoted their interest to study free and forced convection in vertical channels with symmetric/asymmetric heating or prescribed uniform heat fluxes on the channel walls, such as, Aung et al. [11], Burch et al. [12], Aung and Worku [13−15]. These authors examined the laminar convection flow in the entrance or the fully developed region in the channel neglecting the effect of viscous dissipation. They also studied the flow reversal in the channel for various combinations of thermal boundary conditions. Barletta [16, 17] and Zanchini [18] presented analyses on mixed convection in the fully developed region of a vertical channel by considering the effects of viscous dissipation. These authors have taken the thermal boundary conditions as uniform equal/different temperatures on the walls, or uniform temperature on one wall and uniform heat flux on the other wall, or uniform heat fluxes on the both walls. In these papers, solutions are obtained by means of a perturbation series method. Barletta [16] and

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Zanchini [18] expressed the solution of governing equations as a power series in a non-dimensional parameter which is proportional to *Br* (Brinkman number). This number is a measure of the viscous heating. However, Barletta [17] expressed solution as a power series in a parameter which is the ratio of *Gr* (Grashof number) and Re (Reynolds number). In fact, he has taken the forced convection flow with viscous dissipation as the base heat transfer process and buoyancy effect is evaluated by the above mentioned power series. Barletta [19] also examined the mixed convection flow in vertical channel with isothermal-isoflux boundary conditions, taking into account the viscous heating effect. Umavathi and Malashetty [20] examined the mixed convection MHD flow in a vertical channel in the presence of viscous and Ohmic heating. They obtained the temperature and velocity fields by employing perturbation series method and compared the results with the solution obtained numerically by a finite difference scheme. Viscous and Ohmic dissipations have been discussed by Babaelahi et al. [21] in viscoelastic MHD boundary layer flow. The steady mixed convection in a homogeneous porous layer which is bounded by two impermeable vertical plates is studied numerically with finite wall heat source by Lai et al. [22]. Porous medium is modeled by Darcy's equation with the Boussinesq approximation, while in energy equation viscous dissipation term is dropped. The mixed convection in a vertical channel filled by a porous medium is studied by Ingham et al. [23] with viscous heating effect. Umavathi et al. [24,25] examined numerically and analytically, mixed convection in a vertical channel filled with a porous medium using Brinkman-Forchheimer model. The fully developed mixed convection flow is investigated by Barletta et al. [26] in a vertical channel filled by a porous material, using by Darcy law with Boussinesq approximation. The viscous heating effect is also taken into account, and the mechanical/thermal characteristics of the flow are investigated numerically and analytically. Weidman and Medina [27] investigated the porous medium convection between two vertical walls where flow in the porous medium is modeled by the Brinkman equation.

In thermal natural convection in a porous medium the fluid flow is driven due to buoyancy forces. These forces result from density variations because of temperature gradients in the fluid. For such flows, in the fluid-saturated porous medium, heat is transported by both diffusion and convection, however viscous dissipation effect normally is negligible in these flows. But imposing forced convection viscous heating may becomes significant in altering these flow and temperature field. Practical applications such as electronic cooling, packed bed thermal storage, building insulation, heat transfer from hair covered skin, solidification of concentrated alloys, and packed bed catalytic and nuclear reactors motivate these studies. Many of above mentioned systems are complicated and involve coupled flow and heat transfer processes, and in channels the convection heat transfer is modified by the use of porous layers. Several researchers investigated mixed convection in vertical channels partially filled with a porous medium, for example Chang and Chang [28], Al-Nimr and Haddad [29], and Al-Nimr and Khadrawi [30].

The aim of this investigation is to study the fully developed mixed convection in a vertical channel partially filled by a porous medium in the presence of a magnetic field and by taking into account the viscous and Ohmic heating effects. Flow in the porous medium is modeled by the Brinkman equation modified for the magnetohydrodynamics. Velocity and temperature fields are obtained by means of perturbation series method. Keeping the ratio of the Grashof number and the Reynolds number fixed, the solution is expressed as a power series in a non-dimensional parameter proportional to the Brinkman number. Numerical solution is also obtained by shooting technique with fourth-order Runge-Kutta method.

2. Formulation of the Problem

2.1 Asymmetric Heating

We consider steady, laminar, electrically conducting viscous fluid flow in a vertical channel of width 'h', partially filled with a porous medium of thickness '$\overline{a}$', in the presence of a magnetic field of strength B_0 applied in the normal direction across the channel. Cartesian coordinate system is taken such that the X-axis is parallel, but in the opposite direction to the gravitational field, Y-axis is in transverse direction and the origin is such that the channel impermeable walls are at $Y = 0$ and $Y = h$. These channel walls are kept at constant temperatures T_1 and T_2 respectively with $T_2 > T_1$. The hydromagnetic fully developed flow in the channel is assumed, so the only non-vanishing component of the velocity field is in the X

direction (longitudinal component). Both velocity and temperature of the fluid depend on Y only. The longitudinal pressure gradient, $-\partial P/\partial X = A$, is constant and $\partial P/\partial Y = 0$, where $P = p + \rho_0 gX$, which is the hydromagnetic pressure. ρ_0 is the fluid density at the temperature T_0, which is reference temperature of the Boussinesq approximation and following Barletta and Zanchini [31] it is taken as $T_0 = \frac{1}{2}(T_1 + T_2)$. The fluid density ρ is taken to depend on temperature T by the equation of state:

$$\rho = \rho_0(1-\beta(T-T_0)) \tag{1}$$

The field, for velocity and temperature, is divided into two regions. Region-I is porous region ($0 \le Y \le a$), and Region-II is clear fluid region ($a \le Y \le h$). Let $\overline{U}$, $\overline{T}$ and $\overline{u}$, $\overline{t}$ denote dimensional velocity components in X-direction and temperatures, in the region-I and region-II respectively. A schematic diagram of the problem is shown in Figure 1.

The Brinkman equation modified for magnetohydrodynamics is taken to govern the flow in porous medium (region-I) and in the thermal energy equation for a porous medium the viscous dissipation term is taken following Al-Hadhrami et al. [32]. Thus the governing equations for the porous region (I) in the vertical parallel-plate channel are given by

$$g\beta(\overline{T}-T_0) - \frac{1}{\rho_0}\frac{\partial P}{\partial X} + \phi_1 \upsilon \frac{d^2\overline{U}}{dY^2} - \left(\frac{\sigma_e B_0^2}{\rho_0} + \frac{\upsilon}{K}\right)\overline{U} = 0 \tag{2}$$

$$\phi_2 \alpha \frac{d^2\overline{T}}{dY^2} + \frac{\phi_1 \upsilon}{C_p}\left(\frac{d\overline{U}}{dY}\right)^2 + \left(\frac{\sigma_e B_0^2}{\rho_0 C_p} + \frac{\upsilon}{C_p K}\right)\overline{U}^2 = 0 \tag{3}$$

The governing momentum and energy equations for the clear fluid region (II) in the vertical parallel-plate channel, following Umavathi and Malashetty [20] are given by

$$g\beta(\overline{t}-T_0) - \frac{1}{\rho_0}\frac{\partial P}{\partial X} + \upsilon \frac{d^2\overline{u}}{dY^2} - \frac{\sigma_e B_0^2}{\rho_0}\overline{u} = 0 \tag{4}$$

$$\alpha \frac{d^2\overline{t}}{dY^2} + \frac{\upsilon}{C_p}\left(\frac{d\overline{u}}{dY}\right)^2 + \frac{\sigma_e B_0^2}{\rho_0 C_p}\overline{u}^2 = 0 \tag{5}$$

where g is the acceleration due to gravity, β is the thermal expansion coefficient, $\phi_1 = \overline{\mu}/\mu$ is the viscosity ratio, $\overline{\mu}$ is the effective viscosity in porous medium, μ is the clear fluid viscosity, $\upsilon = \mu/\rho_0$ is the kinematic viscosity, σ_e is the electrical conductivity, K is the permeability of porous medium, $\alpha = k/\rho_0 C_p$ is the thermal diffusivity, $\phi_2 = \overline{k}/k$ is the thermal conductivity ratio, $\overline{k}$ is the effective thermal conductivity in fluid-saturated porous medium, k is the thermal conductivity of the clear fluid, C_p is the specific heat at constant pressure, p is the pressure and $P = p + \rho_0 gX$ is the modified pressure.

The boundary conditions are given by

$$\begin{aligned} &\text{at } Y=0 && \overline{U}=0, && \overline{T}=T_1, \\ &\text{at } Y=\overline{a} && \overline{U}=\overline{u}, && \overline{T}=\overline{t} \\ & && \phi_1\frac{d\overline{U}}{dY}=\frac{d\overline{u}}{dY}, && \phi_2\frac{d\overline{T}}{dY}=\frac{d\overline{t}}{dY}, \\ &\text{at } Y=h && \overline{u}=0, && \overline{t}=T_2 \end{aligned} \tag{6}$$

3. Method of Solution

For the solution of the velocity field, $\overline{T}$ is eliminated from equation (2), using energy balance equation (3), to obtain

$$\frac{d^4\overline{U}}{dY^4} - \left(\frac{\sigma_e B_0^2}{\phi_1\rho_0\upsilon} + \frac{1}{\phi_1 K}\right)\frac{d^2\overline{U}}{dY^2} - \frac{g\beta}{\phi_2\alpha C_p}\left(\frac{d\overline{U}}{dY}\right)^2 - \left(\frac{\sigma_e B_0^2}{\rho_0 C_p}\frac{g\beta}{\phi_1\phi_2\alpha\upsilon} + \frac{1}{C_p K}\frac{g\beta}{\phi_1\phi_2\alpha}\right)\overline{U}^2 = 0 \tag{7}$$

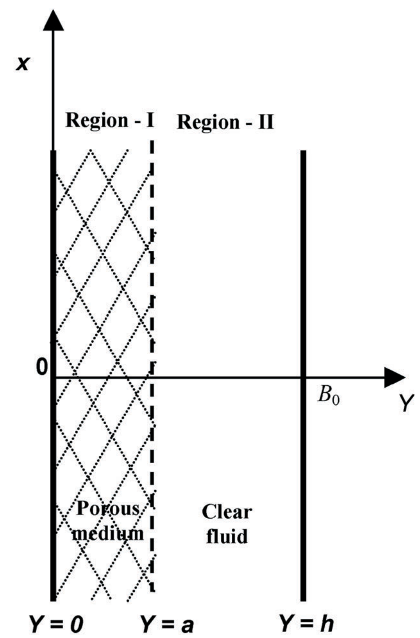

Figure 1. Schematic diagram.

Similarly, equations (4) and (5) give

$$\frac{d^4\overline{u}}{dY^4}-\left(\frac{\sigma_e B_0^2}{\rho_0\upsilon}\right)\frac{d^2\overline{u}}{dY^2}-\frac{g\beta}{\alpha C_p}\left(\frac{d\overline{u}}{dY}\right)^2-\left(\frac{\sigma_e B_0^2}{\rho_0 C_p}\frac{g\beta}{\alpha\upsilon}\right)\overline{u}^2=0 \quad (8)$$

Now, we introduce the following non-dimensional quantities:

$$y=\frac{Y}{h},\quad U=\frac{\overline{U}}{U^*},\quad u=\frac{\overline{u}}{U^*},\quad T=\frac{\overline{T}-T_0}{T_2-T_1},\quad t=\frac{\overline{t}-T_0}{T_2-T_1} \quad (9)$$

where, the reference velocity U^* is given by $U^* = Ah^2/48\mu$.

Using above non-dimensional quantities, the equations (7) and (8) are reduced to

$$\frac{d^4U}{dy^4}-\frac{1}{\phi_1}(M^2+\sigma^2)\frac{d^2U}{dy^2}-\frac{Br}{\phi_2}\frac{Gr}{\mathrm{Re}}\left(\frac{dU}{dy}\right)^2-\frac{Br}{\phi_1\phi_2}\frac{Gr}{\mathrm{Re}}(M^2+\sigma^2)U^2=0 \quad (10)$$

$$\frac{d^4u}{dy^4}-M^2\frac{d^2u}{dy^2}-Br\frac{Gr}{\mathrm{Re}}\left(\frac{du}{dy}\right)^2-Br\frac{Gr}{\mathrm{Re}}M^2u^2=0 \quad (11)$$

where, $Gr=\frac{g\beta(T_2-T_1)h^3}{\upsilon^2}$ is the Grashof number; $\mathrm{Re}=\frac{U^*h}{\upsilon}$, the Reynolds number; $Br=\frac{\upsilon U^{*2}}{(T_2-T_1)\alpha C_p}$, the Brinkman number; $M=B_0h\sqrt{\frac{\sigma_e}{\mu}}$, the Hartmann number; and $\sigma=\frac{h}{\sqrt{K}}$, the permeability parameter.

From equations (2) and (4), using the non-dimensional quantities (9), we obtain

$$T=-\frac{\mathrm{Re}}{Gr}\left[48+\phi_1\frac{d^2U}{dy^2}-(M^2+\sigma^2)U\right] \quad (12)$$

$$\text{and } t=-\frac{\mathrm{Re}}{Gr}\left[48+\frac{d^2u}{dy^2}-M^2u\right] \quad (13)$$

where (Gr/Re) is the mixed convection parameter.

Using the non-dimensional quantities (9), the boundary conditions (6) on U, u, T, t and those induced by the equations (2), (4), (12) and (13) are given by

$$\text{at } y=0 \quad U=0,\quad \frac{d^2U}{dy^2}=\frac{1}{\phi_1}\left(-48+\frac{Gr}{2\,\mathrm{Re}}\right)$$

$$\text{at } y=a \quad U=u,\quad \phi_1\frac{dU}{dy}=\frac{du}{dy},$$

$$\phi_1\frac{d^2U}{dy^2}-(M^2+\sigma^2)U=\frac{d^2u}{dy^2}-M^2u, \quad (14)$$

$$\phi_2\left[\phi_1\frac{d^3U}{dy^3}-(M^2+\sigma^2)\frac{dU}{dy}\right]=\frac{d^3u}{dy^3}-M^2\frac{du}{dy},$$

$$\text{at } y=1 \quad u=0,\quad \frac{d^2u}{dy^2}=-\left(48+\frac{Gr}{2\,\mathrm{Re}}\right)$$

where, $a=\frac{\overline{a}}{h}$.

The present problem can be solved analytically for the cases of either negligible viscous dissipation ($\varepsilon = 0$) or negligible buoyancy forces ($Gr/\mathrm{Re} = 0$). But, in general, the system (2–6) or (10–11, 14), is a nonlinear boundary value problem and does not possess an exact solution. Therefore, the system (10–11, 14) is solved for small values of ε (< 1) by perturbation series method and for $\varepsilon > 1$, the boundary value problem (2–6) is solved, following Conte and Boor [33] numerically by using double shooting technique with the fourth-order Runge-Kutta method.

3.1 Perturbation Method

Let us assume the perturbation expansions for a fixed value of $(Gr/\mathrm{Re}) \neq 0$,

$$U(y)=\varepsilon^0U_0(y)+\varepsilon^1U_1(y)+\varepsilon^2U_2(y)+.... \quad (15)$$

$$u(y)=\varepsilon^0u_0(y)+\varepsilon^1u_1(y)+\varepsilon^2u_2(y)+.... \quad (16)$$

where perturbation parameter $\varepsilon = Br\,(Gr/\mathrm{Re}) < 1$.

Substituting (15) and (16) in the equations (10), (11) and the boundary conditions (14), and comparing the coefficients of ε^0, ε^1, … on both sides in the resulting equations, we obtain the sets of ordinary differential equations for the zeroth order solution, first order solution, etc. On solving these sets of equations under the corresponding boundary conditions, we obtain the solution, neglecting higher order terms ε^n ($n \geq 2$), as

$$U(y)=U_0(y)+\varepsilon U_1(y)=C_1+C_2y+C_3\cosh(M^*y)+C_4\sinh(M^*y)+\varepsilon\begin{pmatrix}C_9+C_{10}y+C_{11}\cosh(M^*y)\\+C_{12}\sinh(M^*y)+F(y)\end{pmatrix} \quad (17)$$

and $u(y)=u_0(y)+\varepsilon u_1(y)=C_5+C_6 y+C_7\cosh(My)$

$$+C_8\sinh(My)+\varepsilon\begin{pmatrix}C_{13}+C_{14}y+C_{15}\cosh(My)\\ +C_{16}\sinh(My)+f(y)\end{pmatrix} \quad (18)$$

where,

$$F(y)=F_1y^2+F_2y^3+F_3y^4+F_4\cosh(2M^*y)$$
$$+F_5\sinh(2M^*y)+F_6y\cosh(M^*y)+F_7y\sinh(M^*y)$$
$$+F_8y^2\cosh(M^*y)+F_9y^2\sinh(M^*y),$$

$$f(y)=F_{10}y^2+F_{11}y^3+F_{12}y^4+F_{13}\cosh(2My)$$
$$+F_{14}\sinh(2My)+F_{15}y\cosh(My)+F_{16}y\sinh(My)$$
$$+F_{17}y^2\cosh(My)+F_{18}y^2\sinh(My),$$

$M^*=[(M^2+\sigma^2)/\phi_1]^{\frac{1}{2}}$, and constants of integration C_1, C_2, …., C_{16} are obtained by the corresponding boundary conditions. Here, F_1, F_2, …. F_{18} are also constants in terms of C_1, C_2,…., C_8 and these constants are not reported for the sake of brevity.

By using (12) and (13) the dimensionless temperature field is obtained as

$$T=-(\mathrm{Re}/Gr)[48-\phi_1M^{*2}(C_1+C_2y)$$
$$+\varepsilon\{-\phi_1M^{*2}(C_9+C_{10}y+F(y))+\phi_1F''(y)\}] \quad (19)$$

$$t=-(\mathrm{Re}/Gr)[48-M^2(C_5+C_6y)$$
$$+\varepsilon\{-M^2(C_{13}+C_{14}y+f(y))+f''(y)\}] \quad (20)$$

The Nusselt number at left wall ($Y=0$) is given by

$$Nu_0=\frac{h}{T_2-T_1}\left(\frac{d\overline{T}}{dY}\right)_{Y=0}=\left(\frac{dT}{dy}\right)_{y=0}$$
$$=(\mathrm{Re}/Gr)[\phi_1M^{*2}C_2+\varepsilon\{\phi_1M^{*2}(C_{10}+F'(0))-\phi_1F'''(0)\}] \quad (21)$$

The Nusselt number at right wall ($Y=h$) is given by

$$Nu_1=\frac{h}{T_2-T_1}\left(\frac{d\overline{T}}{dY}\right)_{Y=h}=\left(\frac{dT}{dy}\right)_{y=1} \quad (22)$$
$$=(\mathrm{Re}/Gr)[M^2C_6+\varepsilon\{M^2(C_{14}+f'(1))-f'''(1)\}]$$

3.2 Particular Cases

(i) when $\sigma\to 0$, $\phi_1=\phi_2=1$, the result is in agreement with that obtained by Umavathi and Malashetty [26].

(ii) When $\varepsilon=0$, $\sigma=0$, $\phi_1=\phi_2=1$ and the results are in agreement with Aung and Worku [14].

4. Mixed Convection Problem with Isothermal-Isoflux Walls

The boundary conditions for this case are given by

$$\text{at } Y=0 \qquad \overline{T}=T_1,$$
$$\text{at } Y=\overline{a} \qquad \overline{T}=\overline{t} \qquad \phi_2\frac{d\overline{T}}{dY}=\frac{d\overline{t}}{dY}, \quad (23)$$
$$\text{at } Y=h \qquad -k\frac{\partial\overline{t}}{\partial Y}=q_2$$

We introduce the following additional non-dimensional quantities for this case,

$$T=\frac{\overline{T}-T_0}{q_2h/k},\quad t=\frac{\overline{t}-T_0}{q_2h/k},\quad Gr=\frac{g\beta q_2\upsilon^2h^4}{k} \quad (24)$$

Using the non-dimensional quantities in the governing equations and above conditions and solving in a similar manner as explained in the isothermal-isothermal case, we obtain the solution for the velocity and temperature fields and are not reported here for the sake of brevity.

5. Mixed Convection Problem with Isoflux-Isothermal Walls

The boundary conditions for this case are given by

$$\text{at } Y=0 \qquad -k\frac{\partial\overline{T}}{\partial Y}=q_1,$$
$$\text{at } Y=\overline{a} \qquad \overline{T}=\overline{t} \qquad \phi_2\frac{d\overline{T}}{dY}=\frac{d\overline{t}}{dY}, \quad (25)$$
$$\text{at } Y=h \qquad \overline{t}=T_2$$

We introduce the following additional non-dimensional quantities for this case,

$$T=\frac{\overline{T}-T_0}{q_1h/k},\quad t=\frac{\overline{t}-T_0}{q_1h/k_1},\quad Gr=\frac{g\beta q_1\upsilon^2h^4}{k} \quad (26)$$

Using the non-dimensional quantities in the governing equations and above conditions and solving in a similar manner as explained in the isothermal-isothermal case, we obtain the solution for the velocity and temperature fields and are not reported here for the sake of brevity.

6. Numerical Method

In many practical applications, it is found that the values of buoyancy force are usually large and in such cases the perturbation series method is not suitable for the computation of results. However, the analytical solution obtained by this method are useful in some laboratory situations for small values of Grashof number, and also in choosing initial guesses in numerical techniques, such as in shooting method, etc. The system of equations (2–6), is a non-linear boundary value problem. In fact, the governing momentum and energy equations (2–5) are coupled non-linear differential equations. These equations after making in dimensionless form are solved by using double shooting technique (Conte and Boor [33]) along with Runge-Kutta fourth order method, subject to the corresponding non-dimensional boundary conditions. First of all, these higher order non-linear coupled differential equations are decomposed into system of first order linear differential equations. The shooting method involves choosing initial guesses for the concerned derivatives, in such a manner that end boundary conditions are satisfied within a prescribed numerical tolerance value. In this study, it is chosen as 10^{-6} which is suitable for computational purposes. With the help of shooting technique as explained by Conte and Boor [33] $U^{'}(0)$ and $T^{'}(0)$ are approximated. Hence the system is reduced to an initial value problem, which is solved using Runge-Kutta fourth order method. While computing the values of $U^{'}(0)$ and $T^{'}(0)$, care has been taken to shoot in steps and the obtained shoots are improved in stages. While solving the system of equations the step size is kept 0.005. Thus numerical solutions are obtained for several values of the pertinent parameters M, ϕ_1, ϕ_2, Gr/Re, ε, σ and plotted in graphs and tabulated.

7. Discussion

This study discusses the fully developed laminar MHD mixed convection in a vertical channel where a porous layer is attached to the left wall

The flow phenomenon encountered in the channel is a result of interplay of the driving pressure gradient, Lorentz force and buoyancy force on the one hand, and the non-linear effects of heat generation due to viscous and Ohmic dissipations on the other hand. For asymmetric heating the velocity and temperature distributions depend on perturbation parameter ε. When Gr/Re is positive there is a buoyancy force assisted flow which is in upward direction, on the other hand when $Gr/\mathrm{Re} > 0$ there is a buoyancy opposed flow (downward). Although the sign of the perturbation parameter ε and that of the mixed convection parameter Gr/Re are constrained to be same, their absolute values, however, are independent (Barletta [16]).

The effects of the parameters ε, Gr/Re, σ, ϕ_1, and M on the velocity distribution are shown in Figure 2 it is seen that the numerical and analytical results are in very good agreement for the small values of the perturbation parameter ε. It is observed that the velocity increases with the increase in the perturbation parameter ε for positive Gr/Re in both regions (porous and clear fluid), because generation of energy by dissipation forces rise the fluid temperature causing an increase in the buoyancy force, which in turn increases the flow in upward direction. This flow in the upward direction is enhanced in the channel by increasing the permeability K of the porous layer (i.e. by decreasing the value of the parameter σ). It is due to the fact that the Darcian resistance to flow through porous medium is inversely proportional to the permeability K of the porous material, hence a higher K value will generate a lesser Darcian resistance to the flow and the flow is therefore accelerated. However, increase in the values of the viscosities ratio $\phi_1(=\overline{\mu}/\mu)$ or the Hartmann number M, reduces flow in the channel. It is known that when Hartmann number increases, it will also increase the Lorentz force which opposes the flow, therefore in the channel flow is decelerated with the rise in M values.

For higher values of the perturbation ($\varepsilon > 1$) and mixed convection parameter ($Gr/\mathrm{Re} > 0$), the velocity distribution obtained by the numerical method is plotted in Figure 3. Two sets of velocity profiles for $\varepsilon = 0$ and $\varepsilon = 2$ are depicted for various values of the mixed convection parameter Gr/Re. It is observed from this figure that for large values of Gr/Re a flow reversal occurs, in the vertical channel with asymmetric heating. It is seen that the flow reversal takes place when the value of the mixed convection parameter Gr/Re surpasses a critical value. Positive value of this parameter causes an increase in the velocity near the right wall (hot wall) and hence a decrease in the velocity is attained near the left wall (cold wall). It is noticed that flow reversal shown in this figure always occurs near the left wall

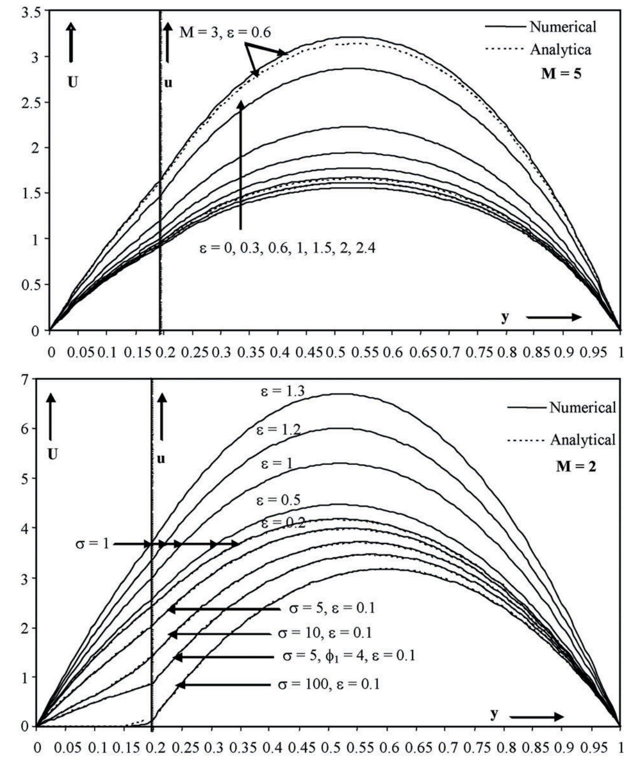

Figure 2. Velocity versus y (Asymmetric heating) for Gr/Re = 1, $\phi_1 = 1.25$, $\phi_2 = 1.5$.

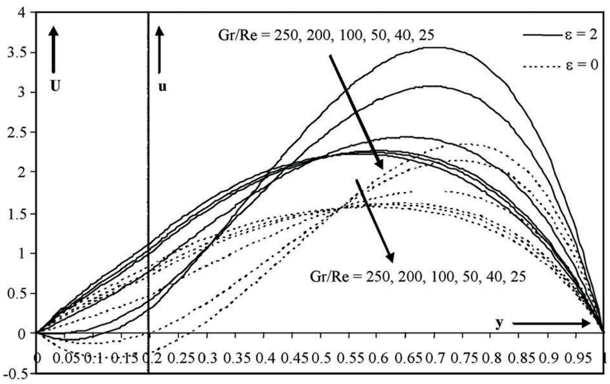

Figure 3. Velocity versus y (Asymmetric heating) for ϕ_1 = 1.25, $\phi_2 = 1.5$, $\sigma = 5$, $M = 5$.

(cold) due to the effect of low temperature distribution in this region. It is also noticed that the maximum value of the velocity near the right wall is seen to be greater than the numerical value of the minimum velocity near the left wall for various values of ε and Gr/Re. Further flow reversal increases with the increase in the values of the parameter Gr/Re that is mixed convection parameter will make the flow reversal wider and deeper. This fact has been reported experimentally by Gau et al. [34] also. However, the parameter ε tends to reduce the flow reversal that is the flow reversal decreases with the increase in viscous in viscous dissipation. It is also reported by Barletta [16].

Figure 4, shows the plots of temperature field t versus y for various values of the parameters ε, σ, ϕ_1, M, and Gr/Re for the case of asymmetric heating of the walls. The comparison of analytical and numerical results are also shown and it is found that then agreement for small ε. It is observed that when $\varepsilon = 0$ (negligible viscous dissipation) the temperature profile in the porous region as well as in the clear fluid region, is a straight line since the heat transfer in this case will be through conduction only. However, the convection regime dominates for $\varepsilon \neq 0$ and the Figure 4, reveals that the temperature in the channel increases with the rise of the value of ε due to viscous heating. Further temperature enhances in the channel by the permeability K of the porous medium (or by decreasing σ) whereas it is suppressed by the parameters ϕ_1, M, and Gr/Re. For higher values of Gr/Re obtained numerically and also plotted in this figure.

Figure 5 illustrates the effects of the various parameters on temperature with isothermal-isoflux wall conditions. It is found that the temperature at the wall with constant heat flux increases as the permeability parameter K or ε increases, however, the other parameters such as ϕ_1, M, Gr/Re have reverse effect. The temperature profiles with isoflux-isothermal wall conditions and the observed results are similar to that for the isothermal-isoflux wall conditions and the corresponding figures are not reported here.

Figure 6 displays the rate of heat transfer for isothermal-isothermal, and isoflux-isothermal wall temperatures. The Nusselt numbers (Nu_0 and Nu_1) have been computed at both walls, for various values of the pertinent parameters. It is of practical interest to know the amount of rate of heat transfer between the channel walls and the fluid. For the case of isothermal-isothermal wall conditions, the figure shows that the rate of heat transfer Nu_0 at the left wall of the channel, increases with the rise of the parameter ε. The rate of heat transfer is further enhanced at the left wall when the permeability K of the porous medium is increased. Whereas reverse effect is observed with parameters M, ϕ_1 and Gr/Re. For the case of isothermal-isothermal wall conditions, the rate of heat transfer, Nu_1, at the right wall decreases as ε increases and becomes zero at certain value of ε, then changes

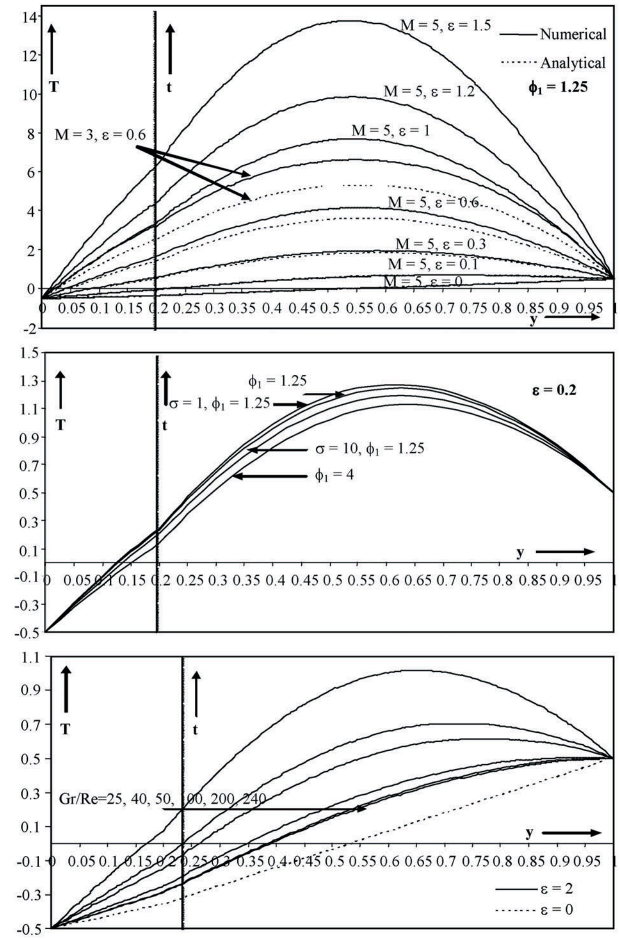

Figure 4. Temperature versus y (Asymmetric heating) for $\sigma = 5$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $M = 5$.

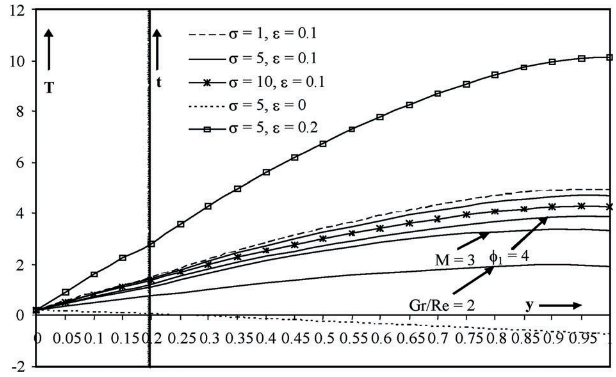

Figure 5. Temperature versus y (Isothermal-Isoflux) for $Gr/\mathrm{Re} = 1$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $M = 2$.

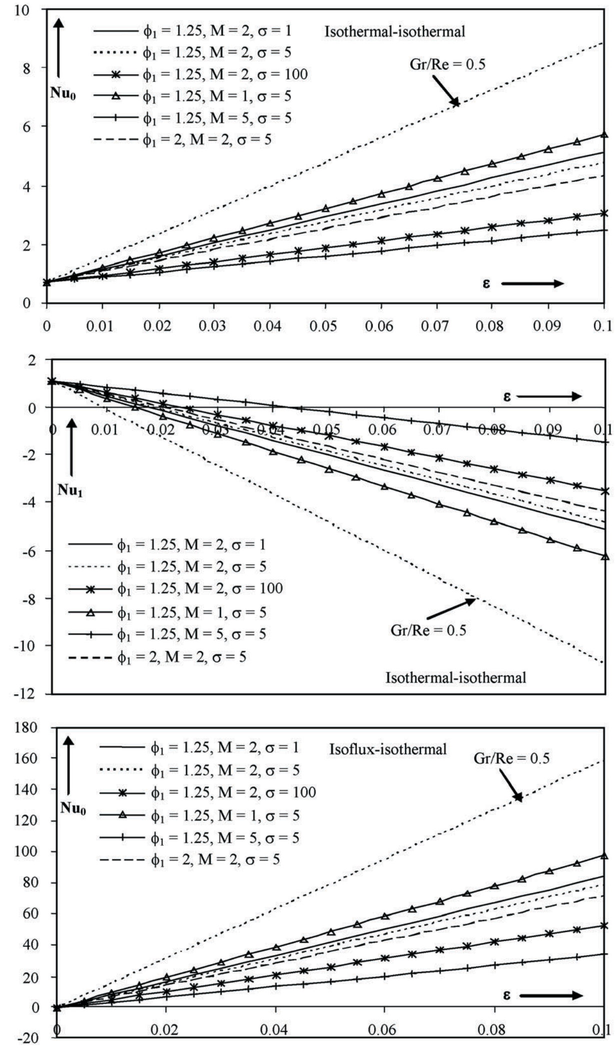

Figure 6. Nusselt number versus ε for $\phi_2 = 1.5$ and $Gr/\mathrm{Re} = 1$.

sign. This sign change of Nu_1 is due to the change of direction of heat flow at the right wall when viscous heating is sufficiently relevant. This critical value of ε is further increased by M or ϕ_1 or Gr/Re or σ. But for the case of isoflux-isothermal wall conditions, it is found that the absolute value of the rate of heat transfer Nu_1 at the right wall increases by increasing ε or K and decreases by M or ϕ_1 or Gr/Re.

Table 1 depicts the values of Nusselt numbers at the left and right wall for higher values Gr/Re and ε. These results are obtained numerically Nusselt numbers are also obtained numerically for the no-porous and no-magnetic field case and the results are in excellent agreement with that of Barletta [16]. It also validates our numerical code.

8. Conclusion

The analytical and numerical results are obtained for velocity distribution, temperature distribution, Nusselt numbers and are in very good agreement for small values of ε.

The viscous friction tends to increase the buoyancy

Table 1. Nusselt number Nu_0 and Nu_1 for asymmetric heating

For $\varepsilon = 2$, $M = 5$, $\sigma = 5$, $\phi_1 = 1.25$ and $\phi_2 = 1.5$		
Gr/Re	Nu_0	Nu_1
1	67.7208	-95.68998
25	3.1944	-3.043622
40	2.205334	-1.622146
50	1.881994	-1.157523
100	1.272415	-0.287463
250	1.065508	-0.044622

For $M = 0$, $\sigma = 0$, $a = 0$, $\phi_1 = \phi_2 = 1$				
ε	Barletta [16]		Present study	
	Nu_0	Nu_1	Nu_0	Nu_1
0	2.000	2.000	2.000000	2.000000
0.5	2.048	1.918	2.048499	1.917902
1	2.099	1.834	2.098714	1.833538
2	2.205	1.657	2.204741	1.657458
3	2.319	1.471	2.319081	1.470521
4	2.443	1.271	2.442949	1.271221

force which in turn increases the velocity in the channel. Velocity further increases by increasing the permeability K of the porous layer attached the left wall, while, it decreases with the increase in ϕ_1 or M.

Temperature in the channel increases by increasing the value of ε or the permeability K, while it decreases by ϕ_1, M, and Gr/Re.

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