Unsteady Hydromagnetic Couette Flow within a Porous Channel

G. S. Seth*, Md. S. Ansari and R. Nandkeolyar

Department of Applied Mathematics, Indian School of Mines, Dhanbad-826004, India

Abstract

Unsteady MHD Couette Flow of a viscous incompressible electrically conducting fluid, in the presence of a transverse magnetic field, between two parallel porous plates is studied. The fluid flow within the channel is induced due to the impulsive and uniformly accelerated motion of the lower plate of the channel. The magnetic lines of force are assumed to be fixed relative to the moving plate. Laplace transform technique is applied to obtain the solution for the velocity field. The expression for the non-dimensional shear stress at the lower plate is also derived. The asymptotic solution valid for large time t is obtained to gain some physical insight into the flow pattern. The numerical results for the velocity is depicted graphically for various values of magnetic parameter M^2, suction/injection parameter S and time t whereas numerical values of the shear stress at the lower plate are presented in tabular form for different values of M^2, S and t.

Key Words: MHD Couette Flow, Suction/Injection, Hartmann Boundary Layer

1. Introduction

The governing equations for the flow of viscous incompressible electrically conducting fluid in the presence of magnetic field are, in general, non-linear and, therefore, simplified models are considered in literature with a view to study some specific aspects of the fluid flow features. Of these models, the one corresponding to the MHD Couette flow is known to lead to the equations for which analytical solution can be obtained [1,2]. Hayat et al. [3] discussed three unidirectional non-linear flows (Couette, Poiseuille and generalized Couette flows) of an MHD Oldroyd 8-constant fluid. The study of unsteady MHD Couette flow is of considerable importance from practical point of view because fluid transients may be expected in MHD pumps, MHD generators, accelerators, flow meters and nuclear reactors. Keeping in view this fact, Katagiri [4] investigated MHD Couette flow of a viscous incompressible electrically conducting fluid in the presence of a uniform transverse magnetic field when the fluid flow is induced due to the impulsive motion of one of the plates. Muhuri [5] studied this problem in a porous channel when the fluid flow is induced due to the accelerated motion of one of the plates. Katagiri [4] and Muhuri [5] presented their analysis by considering that the magnetic lines of force are fixed relative to the fluid. Singh and Kumar [6] considered the problem studied by Katagiri [4] and Muhuri [5] in a non-porous channel when the magnetic lines of force are fixed relative to the moving plate. Khan et al. [7] investigated MHD flow of a generalized Oldroyd-B fluid in a porous space taking Hall current into account whereas Khan et al. [8] also considered MHD transient flows of an Oldroyd-B fluid in a channel of rectangular cross-section in a porous medium. Hayat et al. [9] studied the influence of Hall current and heat transfer on the steady MHD flows of a generalized Burgers' fluid between two eccentric rotating infinite disks of different temperatures. In this case the fluid flow is induced due to a pull with constant velocities of the disks. Khan et al. [10]

*Corresponding author. E-mail: gsseth_ism@yahoo.com

considered the effects of variable suction and heat transfer on the oscillatory magnetohydrodynamic flow of a non-Newtonian fluid through a porous medium with slip at the wall. Khan et al. [11] obtained exact solutions of accelerated flows for a Burgers' fluid induced by the accelerating plate by considering two cases of interest viz. (i) constantly accelerating flow and (ii) variable accelerating flow whereas Khan et al. [12] also studied some accelerated flows for generalized Oldroyd-B fluid. They obtained exact solutions for the flows induced due to constantly accelerating plate and variable accelerating plate by means of discrete Laplace transform technique. Khan et al. [13] also presented an analysis of transient oscillatory and constantly accelerated magnetohydrodynamic flow of an Oldroyd-B fluid in a porous medium. The study of MHD flow in a porous channel may find applications in petroleum and mineral industries, polymer technology, designing of cooling systems with liquid metals, geothermal reservoirs, underground energy transports, MHD generators, pumps, flow meters, accelerators and in purification of crude oil etc. Taking into account this fact Muhuri [5], Reddy and Bathaiah [14], Prasad Rao et al. [15], Abbas et al. [16] and Hayat et al. [17,18] investigated MHD flow within a parallel plate channel with porous boundaries under different conditions.

The aim of the present paper is to study unsteady MHD Couette flow problem, considered by Singh and Kumar [6] when the fluid flow is confined to porous boundaries. Two cases of interest are considered viz (i) impulsive movement of the lower plate and (ii) uniformly accelerated movement of the lower plate. Solution in both the cases is obtained with the help of Laplace transform technique. The expressions for the shear stress at the moving plate is also derived. Asymptotic solution for large value of time t is obtained to gain some physical insight into the flow pattern. It is found for large t in both the cases, that the fluid flow is in quasi-steady state. The steady state flow is confined to the modified Hartmann boundary layer of thickness $O(1/(\alpha + S/2))$ and can be viewed as classical Hartmann boundary layer modified by suction/injection. It may be mentioned here that the fluid motion attains final steady state quicker in the case of impulsive movement than in the case of an accelerated movement of the plate of the channel. The numerical results for the velocity field, computed with the help of MATLAB software from the analytical solution, is depicted graphically versus y for various values of magnetic parameter M^2, suction/injection parameter S and time t. It is found that the magnetic field and time have accelerating influence on the velocity field in both the cases whereas suction exerts retarding influence and injection has accelerating influence on the fluid velocity. The numerical values of the non-dimensional shear stress at the moving plate are computed with the help of MATLAB software and are presented in tabular form for various values of M^2, S and t. It is observed that the magnetic field, time and injection reduce shear stress at the moving plate in both the cases while suction increases it at the moving plate.

2. Formulation of the Problem and Its Solution

Consider the unsteady flow of a viscous incompressible electrically conducting fluid between two parallel porous plates $y' = 0$ and $y' = h$ of infinite length, in x' and z' directions, in the presence of a uniform transverse magnetic field H_0 applied parallel to y' axis (see Figure 1). Initially (i.e. when time $t' \leq 0$), fluid and the plates of the channel are assumed to be at rest. When time $t' > 0$ the lower plate ($y' = 0$) starts moving with time dependent velocity $U_0 t'^n$ (U_0 being a constant and n a positive integer) in x' direction while the upper plate ($y' = h$) is kept fixed. The fluid suction/injection takes place through the porous walls of the channel with uniform velocity V_0 which is greater than zero for suction and is less than zero for injection.

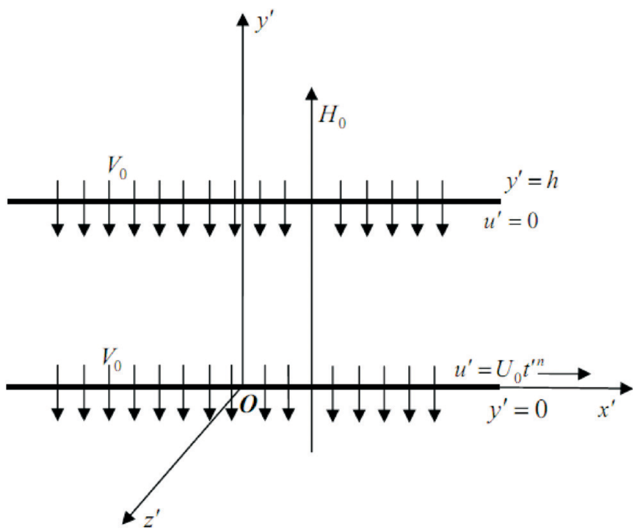

Figure 1. Physical model of the problem.

It is assumed that no applied or polarization voltages exist (i.e. $\vec{E} = 0$, $\vec{E}$ being electric field). This corresponds to the case where no energy is being added or extracted from the fluid by electrical means [19]. Since magnetic Reynolds number is very small for metallic liquids and partially ionized fluids so the induced magnetic field can be neglected in comparison to the applied one [20]. Therefore the fluid velocity $\vec{q}$ and magnetic field $\vec{H}$ is given by

$$\vec{q} \equiv (u', V_0, 0),\ \vec{H} \equiv (0, H_0, 0) \tag{1}$$

Under the above assumptions the equations of motion for viscous incompressible electrically conducting fluid reduce to

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu_e^{\,2} H_0^{\,2}}{\rho} u' \tag{2}$$

where ρ, σ, μ_e and υ are, respectively, fluid density, electrical conductivity, magnetic permeability and kinematic coefficient of viscosity.

The initial and boundary conditions are

$$\left.\begin{aligned} &u' = 0;\ 0 \le y' \le h \text{ and } t' \le 0 \\ &u' = U_0 t'^n \ \text{ at } y' = 0; t' > 0 \\ &u' = 0 \quad\ \text{ at } y' = h; t' > 0 \end{aligned}\right\} \tag{3}$$

Equation (2) is valid when the magnetic field is fixed relative to the fluid. On the other hand, when the magnetic field is fixed with respect to the moving plate [21], the equation (2) is replaced by

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu_e^{\,2} H_0^{\,2}}{\rho} (u' - U_0 t'^n) \tag{4}$$

Now we have to find the solution of equation (4) subject to the initial and boundary conditions (3). For this purpose we have considered two cases of interest viz. (i) impulsive movement of the lower plate (i.e. $n = 0$) and (ii) uniformly accelerated movement of the lower plate (i.e. $n = 1$).

Case I. Impulsive Movement of the Lower Plate

Setting $n = 0$ in equation (4) and introducing the non-dimensional variables

$$y = y'/h,\ u = u'h/\upsilon \text{ and } t = t'\upsilon/h^2 \tag{5}$$

the equation (4), in non-dimensional form, become

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 (u - R_e) \tag{6}$$

where $S = V_0 h/\upsilon$ is the suction/injection parameter, $M^2 = \sigma\mu_e^2 H_0^2 h^2/\rho\upsilon$ is the square of Hartmann number and $R_e = U_0 h/\upsilon$ is the Reynolds number.

The initial and boundary conditions (3), in non-dimensional form, reduce to

$$u = 0;\ \ 0 \le y \le 1 \text{ and } t \le 0 \tag{7a}$$

$$u = R_e \ \text{ at } y = 0;\ t > 0 \tag{7b}$$

$$u = 0 \text{ at } y = 1;\ t > 0 \tag{7c}$$

Using Laplace transform, the equation (6) with the help of (7a) reduces to

$$\frac{d^2 \bar{u}}{dy^2} - S\frac{d\bar{u}}{dy} - (p + M^2)\bar{u} = -\frac{M^2 R_e}{p} \tag{8}$$

where $\bar{u}(y, p) = \int_0^\infty e^{-pt} u(y, t)dt$; $(p > 0)$, p being Laplace transform parameter.

The boundary conditions (7b) and (7c) with the help of Laplace transform become

$$\bar{u} = R_e/p \ \text{ at } y = 0 \text{ and } \bar{u} = 0 \text{ at } y = 1 \tag{9}$$

The solution of equation (8) subject to the boundary conditions (9) is given by

$$\bar{u}_i = \sum_{k=0}^{\infty}\left[(-1)^k \frac{1}{(p+M^2)}\left\{e^{-\frac{aS}{2}} e^{-a\sqrt{p+\alpha^2}} + e^{-\frac{bS}{2}} e^{-b\sqrt{p+\alpha^2}}\right\} - \frac{1}{p}\left\{e^{-\frac{cS}{2}} e^{-c\sqrt{p+\alpha^2}} + e^{-\frac{dS}{2}} e^{-d\sqrt{p+\alpha^2}}\right\}\right] + \frac{M^2}{p(p+M^2)} \tag{10}$$

where $\bar{u}_i = \bar{u}(y, p)/R_e$, $\alpha = (M^2 + S^2/4)^{1/2}$, $a = k + y$, $b = 1 + k - y$, $c = 1 + 2k - y$ and $d = 1 + 2k + y$.

Taking inverse Laplace transform of equation (10), the solution for velocity field is expressed in the following form (McLachlan [22])

$$u_i=\sum_{k=0}^{\infty}\left[(-1)^k\frac{e^{-M^2t}}{2}\left\{erfc\left(\frac{a}{2\sqrt{t}}+\frac{S}{2}\sqrt{t}\right)+e^{-aS}erfc\left(\frac{a}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)+erfc\left(\frac{b}{2\sqrt{t}}+\frac{S}{2}\sqrt{t}\right)+e^{-bS}erfc\left(\frac{b}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)\right\}-\frac{1}{2}\left\{e^{-c\left(\frac{S}{2}-\alpha\right)}\times erfc\left(\frac{c}{2\sqrt{t}}+\alpha\sqrt{t}\right)+e^{-c\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{c}{2\sqrt{t}}-\alpha\sqrt{t}\right)-e^{-d\left(\frac{S}{2}-\alpha\right)}\times erfc\left(\frac{d}{2\sqrt{t}}+\alpha\sqrt{t}\right)-e^{-d\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{d}{2\sqrt{t}}-\alpha\sqrt{t}\right)\right\}\right]+1-e^{-M^2t} \tag{11}$$

where $u_i = u(y, t)/R_e$.

Equation (11) represents the solution for unsteady MHD Couette flow within porous channel when the fluid flow is induced due to the impulsive movement of the lower plate of the channel. It clearly brings out the contribution due to the initial Couette flow, the final steady Hartmann boundary layer and the decaying oscillations excited by the interaction of the magnetic field and initial impulsive motion. In the absence of suction or injection ($S = 0$) it agrees with the solution obtained by Singh and Kumar [6]. Furthermore, in the absence of magnetic field ($M^2 = 0$) and suction or injection ($S = 0$) the solution (11) reduces to that of Schlichting [23].

When time t is large, using the asymptotic expression of $erfc(x)$ i.e.

$$erfc(x)\approx\frac{\exp(-x^2)}{\sqrt{\pi}x}\quad\text{as } x\to\infty$$

together with $erfc(-x) = 2 - erfc(x)$, the solution (11) may be represented in the following form as

$$u_i=\sum_{k=0}^{\infty}\left[e^{-d\left(\frac{S}{2}+\alpha\right)}-e^{-c\left(\frac{S}{2}+\alpha\right)}+2c\sqrt{\frac{t}{\pi}}e^{-\left(\frac{cS}{2}+\frac{c^2}{4t}+\alpha^2t\right)}-2d\sqrt{\frac{t}{\pi}}e^{-\left(\frac{dS}{2}+\frac{d^2}{4t}+\alpha^2t\right)}+(-1)^k e^{-M^2t}\left(e^{-aS}+e^{-bS}-2a\sqrt{\frac{t}{\pi}}\frac{e^{-\left(\frac{aS}{2}+\frac{a^2}{4t}+\frac{S^2}{4}t\right)}}{S^2t^2-a^2}-2b\sqrt{\frac{t}{\pi}}\times\frac{e^{-\left(\frac{bS}{2}+\frac{b^2}{4t}+\frac{S^2}{4}t\right)}}{S^2t^2-b^2}\right)\right]+1-e^{-M^2t} \tag{12}$$

The expression (12) reveals that the velocity field is in quasi steady state. The steady state flow is confined within a boundary layer of thickness $O(1/(S/2 + \alpha))$, which may be recognized as modified Hartmann boundary layer and can be viewed as classical Hartmann boundary layer modified by suction/injection. The thickness of this boundary layer decreases with increase in either M^2 or S or both. The unsteady state flow in the flow-field damp out effectively in dimensionless time of order $O(1/\alpha^2)$ when the final steady state is developed. The time of decay is less than the case when there is no suction/injection of the fluid.

Shear stress at the moving plate

The non-dimensional shear stress at the lower plate ($y = 0$) is given by

$$\tau_i\big|_{y=0}=\sum_{k=0}^{\infty}\left[(-1)^{k+1}\frac{e^{-M^2t}}{2}\left\{\left(1-e^{-\left(\frac{c'}{4t}+\frac{S}{2}\right)}\right)\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{kS}{2}+\frac{k^2}{4t}+\frac{S^2}{4}t\right)}+S\left(e^{-kS}erfc\left(\frac{k}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)-e^{-b'S}erfc\left(\frac{b'}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)\right)\right\}-\left\{\left(\frac{S}{2}-\alpha\right)e^{-c'\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{c'}{2\sqrt{t}}+\alpha\sqrt{t}\right)+\left(\frac{S}{2}+\alpha\right)e^{-c'\left(\frac{S}{2}+\alpha\right)}\times erfc\left(\frac{c'}{2\sqrt{t}}-\alpha\sqrt{t}\right)+\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{c'S}{2}+\frac{c'^2}{4t}+\alpha^2t\right)}\right\}\right] \tag{13}$$

where $b' = 1 + k$ and $c' = 1 + 2k$.

Case II. Uniformly Accelerated Movement of the Lower Plate

Substituting $n = 1$ in equation (4) and using (5), the equation (4), in non-dimensional form, reduces to

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-M^2(u-Rt) \tag{14}$$

where $R = U_0h^3/\upsilon^2$ is non-dimensional parameter.

The initial and boundary conditions (3), in non-dimensional form, are

$$u=0;\quad 0\le y\le 1 \text{ and } t\le 0 \tag{15a}$$

$$u=Rt \text{ at } y=0;\quad t>0 \tag{15b}$$

$$u=0 \text{ at } y=1;\quad t>0 \tag{15c}$$

Using Laplace transform, the equation (14) with the help of (15a) reduces to

$$\frac{d^2\bar{u}}{dy^2} - S\frac{d\bar{u}}{dy} - (p+M^2)\bar{u} = -\frac{RM^2}{p^2} \tag{16}$$

The boundary conditions (15b) and (15c) reduce to

$$\bar{u} = R/p^2 \text{ at } y=0 \text{ and } \bar{u} = 0 \text{ at } y=1 \tag{17}$$

The solution of equation (16) subject to the boundary conditions (17) is given by

$$\begin{aligned}\bar{u}_a = \sum_{k=0}^{\infty}\Bigg[(-1)^k\Bigg\{&\frac{1}{pM^2}\left(e^{-\frac{aS}{2}}e^{-a\sqrt{p+\alpha^2}} + e^{-\frac{bS}{2}}e^{-b\sqrt{p+\alpha^2}}\right)\\ &-\frac{1}{M^2(p+M^2)}\left(e^{-\frac{aS}{2}}e^{-a\sqrt{p+\alpha^2}} + e^{-\frac{bS}{2}}e^{-b\sqrt{p+\alpha^2}}\right)\Bigg\}\\ &-\frac{1}{p^2}\left(e^{-\frac{cS}{2}}e^{-c\sqrt{p+\alpha^2}} - e^{-\frac{dS}{2}}e^{-d\sqrt{p+\alpha^2}}\right)\Bigg] + \frac{M^2}{p^2(p+M^2)}\end{aligned} \tag{18}$$

where $\bar{u}_a = \bar{u}(y, p) / R$.

Taking inverse Laplace transform of equation (18), the solution for velocity field is presented in the following form (McLachlan [22])

$$\begin{aligned}u_a = \sum_{k=0}^{\infty}\Bigg[&\frac{(-1)^k}{2M^2}\Bigg\{\Bigg(e^{-a\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{a}{2\sqrt{t}}+\alpha\sqrt{t}\right)+e^{-a\left(\frac{S}{2}+\alpha\right)}\\ &\times erfc\left(\frac{a}{2\sqrt{t}}-\alpha\sqrt{t}\right)+e^{-b\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{b}{2\sqrt{t}}+\alpha\sqrt{t}\right)\\ &+e^{-b\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{b}{2\sqrt{t}}-\alpha\sqrt{t}\right)\Bigg)-e^{-M^2t}\Bigg(erfc\left(\frac{a}{2\sqrt{t}}+\frac{S}{2}\sqrt{t}\right)\\ &+e^{-aS}erfc\left(\frac{a}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)+erfc\left(\frac{b}{2\sqrt{t}}+\frac{S}{2}\sqrt{t}\right)+e^{-bS}\\ &\times erfc\left(\frac{b}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)\Bigg)\Bigg\}-\frac{\sqrt{t}}{2\alpha}\Bigg\{\left(\frac{c}{2\sqrt{t}}+\alpha\sqrt{t}\right)e^{-c\left(\frac{S}{2}-\alpha\right)}\\ &\times erfc\left(\frac{c}{2\sqrt{t}}+\alpha\sqrt{t}\right)-\left(\frac{c}{2\sqrt{t}}-\alpha\sqrt{t}\right)e^{-c\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{c}{2\sqrt{t}}-\alpha\sqrt{t}\right)\\ &-\left(\frac{d}{2\sqrt{t}}+\alpha\sqrt{t}\right)e^{-d\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{d}{2\sqrt{t}}+\alpha\sqrt{t}\right)+\left(\frac{d}{2\sqrt{t}}-\alpha\sqrt{t}\right)\\ &\times e^{-d\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{d}{2\sqrt{t}}-\alpha\sqrt{t}\right)\Bigg\}\Bigg]+\frac{1}{M^2}\left(e^{-M^2t}-1\right)+t\end{aligned} \tag{19}$$

where $u_a = u(y, t) / R$.

Equation (19) represents the solution of unsteady hydromagnetic Couette flow within a porous channel when the fluid flow is induced due to the uniformly accelerated movement of the lower plate of channel. In the absence of suction/injection the solution (19) is in agreement with the solution obtained by Singh and Kumar [6]. Further more, in the absence of the magnetic field and suction/injection the solution (19) reduces to the one given by Schlichting [23].

When time t is large, the solution (19) in asymptotic form becomes

$$\begin{aligned}u_a = \sum_{k=0}^{\infty}\Bigg[&\Bigg\{\frac{(-1)^k}{M^2}\left(e^{-a\left(\frac{S}{2}+\alpha\right)}+e^{-b\left(\frac{S}{2}+\alpha\right)}\right)+\frac{c}{2\alpha}e^{-c\left(\frac{S}{2}+\alpha\right)}-\frac{d}{2\alpha}e^{-d\left(\frac{S}{2}+\alpha\right)}\Bigg\}\\ &-\frac{(-1)^k}{M^2}\Bigg\{2a\sqrt{\frac{t}{\pi}}\frac{e^{-\left(\frac{aS}{2}+\frac{a^2}{4t}+\alpha^2t\right)}}{4\alpha^2t^2-a^2}+2b\sqrt{\frac{t}{\pi}}\frac{e^{-\left(\frac{bS}{2}+\frac{b^2}{4t}+\alpha^2t\right)}}{4\alpha^2t^2-b^2}+e^{-M^2t}\\ &\times\left(e^{-aS}+e^{-bS}-2a\sqrt{\frac{t}{\pi}}\frac{e^{-\left(\frac{aS}{2}+\frac{a^2}{4t}+\frac{S^2}{4}t\right)}}{S^2t^2-a^2}-2b\sqrt{\frac{t}{\pi}}\frac{e^{-\left(\frac{bS}{2}+\frac{b^2}{4t}+\frac{S^2}{4}t\right)}}{S^2t^2-b^2}\right)\Bigg\}\\ &-t\left(e^{-c\left(\frac{S}{2}+\alpha\right)}-e^{-d\left(\frac{S}{2}+\alpha\right)}\right)\Bigg]+\frac{1}{M^2}\left(e^{-M^2t}-1\right)+t\end{aligned} \tag{20}$$

The solution (20) clearly represents the final steady state flow along with unsteady state flow. The steady state flow is confined within the modified Hartmann boundary layer of thickness $O(1/(S/2 + \alpha))$, which is similar to that of Case I. The fluid velocity u_a increases with the increase in time t. Thus we may conclude that the fluid motion attains final steady state quicker in the case of impulsive movement than in the case of an accelerated movement of the plate of the channel.

Shear stress at the moving plate

The non-dimensional shear stress at the lower plate ($y = 0$) is given by

$$\begin{aligned}\tau_a\big|_{y=0} = \sum_{k=0}^{\infty}\Bigg[&\frac{(-1)^{k+1}}{2M^2}\Bigg\{\left(erfc\left(\frac{k}{2\sqrt{t}}+\alpha\sqrt{t}\right)-e^{-\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{1+k}{2\sqrt{t}}+\alpha\sqrt{t}\right)\right)\\ &\times\left(\frac{S}{2}-\alpha\right)e^{-k\left(\frac{S}{2}-\alpha\right)}+\left(1-e^{-\left(\frac{c'}{4t}+\frac{S}{2}\right)}\right)\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{kS}{2}+\frac{k^2}{4t}+\alpha^2t\right)}\end{aligned}$$

$$+\left(erfc\left(\frac{k}{2\sqrt{t}}-\alpha\sqrt{t}\right)-e^{-\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{1+k}{2\sqrt{t}}-\alpha\sqrt{t}\right)\right)$$
$$\times\left(\frac{S}{2}+\alpha\right)e^{-k\left(\frac{S}{2}+\alpha\right)}-e^{-M^2t}\left[\left(1-e^{-\left(\frac{c'}{4t}+\frac{S}{2}\right)}\right)\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{kS}{2}+\frac{k^2}{4t}+\frac{S^2}{4}t\right)}\right.$$
$$\left.+\left(erfc\left(\frac{k}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)-e^{-S}erfc\left(\frac{1+k}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\right)\right)Se^{-kS}\right]\Bigg\}$$
$$-\Bigg\{\left(\frac{S}{2}-\alpha\right)\left(\frac{c'}{2\alpha}+t\right)e^{-c'\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{c'}{2\sqrt{t}}+\alpha\sqrt{t}\right)$$
$$-\left(\frac{S}{2}+\alpha\right)\left(\frac{c'}{2\alpha}-t\right)e^{-c'\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{c'}{2\sqrt{t}}-\alpha\sqrt{t}\right)$$
$$+2\sqrt{\frac{t}{\pi}}e^{-\left(\frac{c'S}{2}+\frac{c'^2}{4t}+\alpha^2t\right)}-\frac{1}{2\alpha}e^{-c'\left(\frac{S}{2}-\alpha\right)}erfc\left(\frac{c'}{2\sqrt{t}}+\alpha\sqrt{t}\right)$$
$$+\frac{1}{2\alpha}e^{-c'\left(\frac{S}{2}+\alpha\right)}erfc\left(\frac{c'}{2\sqrt{t}}-\alpha\sqrt{t}\right)\Bigg\}\Bigg] \quad (21)$$

3. Results and Discussion

To study the effects of magnetic field, time and suction/injection on the flow-field the numerical results of the fluid velocity, computed with the help of MATLAB software from the analytical solution in both the cases viz. (i) impulsive movement of the lower plate and (ii) uniformly accelerated movement of the lower plate, is depicted graphically versus y for various values of M^2, S and t in Figures 2 to 7. It is evident from Figures 2 to 5 that the velocities u_i and u_a increase with the increase in either M^2 *or* t throughout the channel. Thus we conclude that the magnetic field and time have accelerating influence on the fluid flow in both the cases. Figures 6 and 7 reveal that the fluid velocity in both the cases decreases with the increase in suction parameter $S(>0)$ and increases with the increase in injection parameter $S(<0)$.

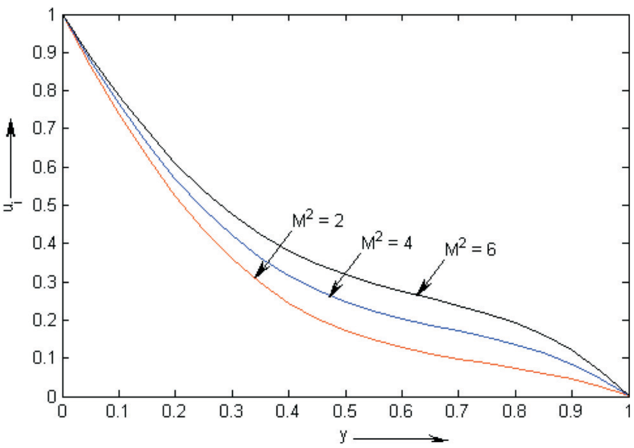

Figure 2. Velocity profiles when $S = 1$ and $t = 0.05$.

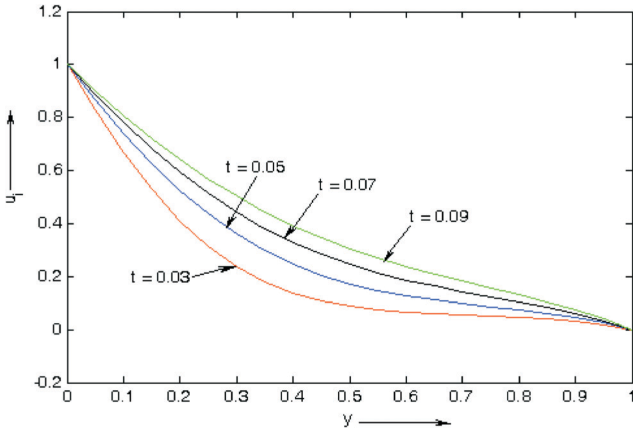

Figure 3. Velocity profiles when $S = 1$ and $M^2 = 2$.

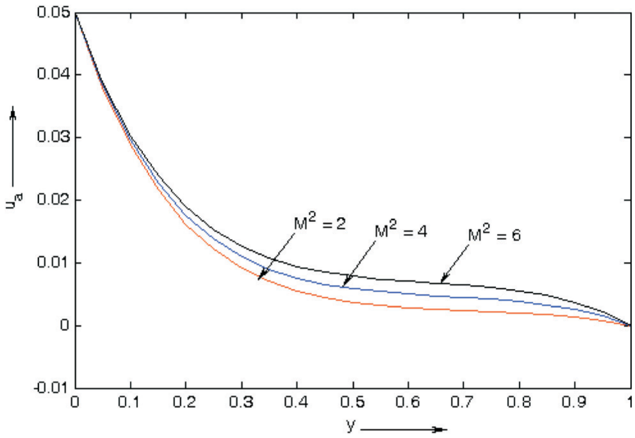

Figure 4. Velocity profiles when $S = 1$ and $t = 0.05$.

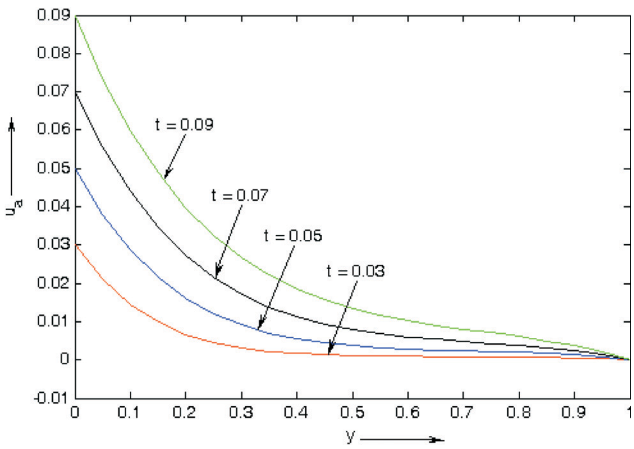

Figure 5. Velocity profiles when $S = 1$ and $M^2 = 2$.

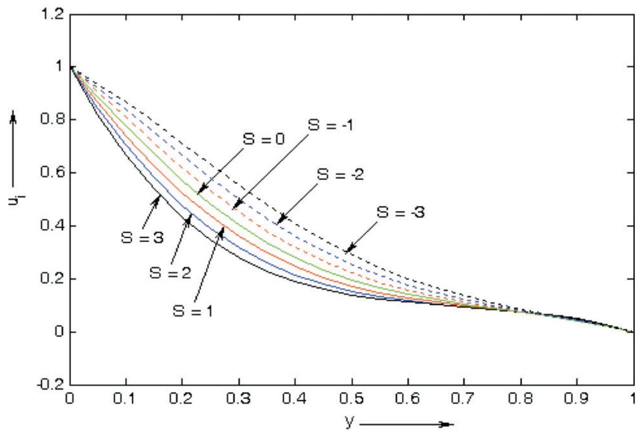

Figure 6. Velocity profiles when $M^2 = 2$ and $t = 0.05$.

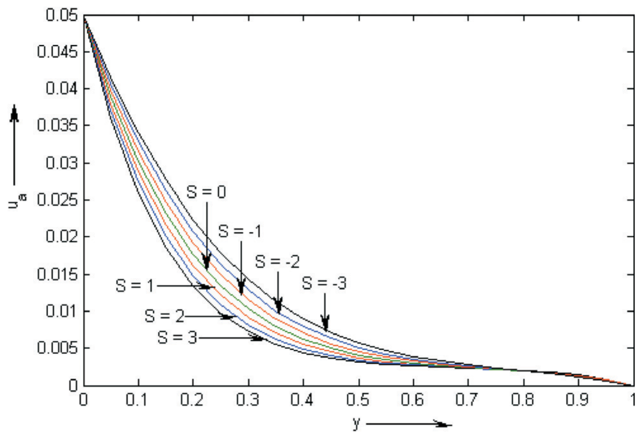

Figure 7. Velocity profiles when $M^2 = 2$ and $t = 0.05$.

Thus we conclude that the suction exerts retarding influence on the fluid velocity whereas injection has accelerating influence on it.

The numerical values of non-dimensional shear stress at the lower plate, computed with the help of MATLAB software, are presented in Tables 1 to 4 for various values of M^2, S and t. It is found from Tables 1 to 4 that the non-dimensional shear stress τ_i and τ_a at the lower plate decrease with the increase in either M^2 or t whereas it increase with the increase in suction parameter $S(> 0)$ and decrease with the increase in injection parameter $S(< 0)$. This implies that the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases it at the lower plate.

Table 1. Shear stress $-\tau_i$ at the lower plate when $t = 0.05$

$S_{\rightarrow}$ $M^2\downarrow$	-3	-2	-1	1	2	3
2	1.1797	1.4926	1.8599	2.7642	3.3013	3.8925
4	1.0691	1.3516	1.6836	2.5015	2.9873	3.5222
6	0.9689	1.2240	1.5240	2.2637	2.7032	3.1871

Table 2. Shear stress $-\tau_i$ at the lower plate when $M^2 = 2$

$S_{\rightarrow}$ $t\downarrow$	-3	-2	-1	1	2	3
0.03	1.8598	2.2175	2.6198	3.5615	4.1010	4.6851
0.05	1.1797	1.4926	1.8599	2.7642	3.3013	3.8925
0.07	0.8448	1.1199	1.4563	2.3228	2.8527	3.4432
0.09	0.6551	0.8954	1.2023	2.0295	2.5490	3.1336

Table 3. Shear stress $-\tau_a$ at the lower plate when t = 0.05

$S_{\rightarrow}$ $M^2\downarrow$	-3	-2	-1	1	2	3
2	0.1816	0.2005	0.2214	0.2689	0.2957	0.3243
4	0.1768	0.1949	0.2147	0.2601	0.2855	0.3127
6	0.1723	0.1895	0.2085	0.2517	0.2759	0.3018

Table 4. Shear stress $-\tau_a$ at the lower plate when $M^2 = 2$

$S_{\rightarrow}$ $t\downarrow$	-3	-2	-1	1	2	3
0.03	0.1521	0.1644	0.1775	0.2066	0.2226	0.2395
0.05	0.1816	0.2005	0.2214	0.2689	0.2957	0.3243
0.07	0.2015	0.2263	0.2542	0.3195	0.3569	0.3974
0.09	0.2163	0.2463	0.2806	0.3628	0.4107	0.4630

Acknowledgement

We are grateful to Prof. A. K. Singh, Department of Mathematics, BHU, Varanasi, India for his valuable advice in the preparation of this paper.

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Manuscript Received: Nov. 27, 2009
Accepted: May 6, 2010