Experimental Calibration and Head Loss Prediction of Tuned Liquid Column Damper

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Abstract

In this paper, the experimental investigation to systematically calibrate the basic properties of tuned liquid column damper (TLCD) is conducted. Under harmonic excitation, the exact solution of the liquid response in TLCD is firstly derived and it is used in the experimental calibration. Both the free vibration and harmonic forced vibration tests via shake table are performed to obtain the experimental results for calibration. Consequently, an empirical formula to predict head loss coefficients for TLCD is constructed and proposed as a quick reference for designers. The experimental investigation also confirms the fact that the size of liquid mass and the difference of the ratio of horizontal column length versus total column length have no effect on the natural frequency and head loss as well.

Key Words: Tuned Liquid Column Damper, Head Loss Coefficient, Shake Table Tests

1. Introduction

Since Sakai and his co-authors developed the idea of tuned liquid column damper (TLCD) in 1989 for the purpose of structural vibration suppression [1], many successors had employed it in many civil engineering applications to verify its control effectiveness such as those presented in literatures [2–5], *etc*. Such a device indeed has the advantages over other types of energy-dissipating dampers. For instance, its natural frequency is simply determined by the length of the liquid column, and its damping is generated by flow passing through the orifice in the horizontal column. Based on the energy equation in hydraulics, this damping effect is mathematically modeled by the so-called head loss that mainly depends on the orifice size.

As mentioned above, most literatures regarding TLCD emphasize the investigation of its effectiveness on different applications. Some literatures discuss the determination of optimal parameters such as the optimal frequency tuning ratio and optimal head loss coefficient (e.g., [3,6,7], However, so far there is no paper in the literature presenting the relation between the head loss coefficient and the orifice size for TLCD in the experimental manner. From the practical point of view, such information is of great importance because the designer can easily refer to find the suitable orifice size without going through experimentation.

Although the head loss is mainly induced by the flow passing through the orifice in the middle of the horizontal column, there exist other head losses in the flow motion such as those caused by the turn elbow, and even the fluid viscosity on the column wall. Hence, the head loss in TLCD can not be accurately estimated except using experimental calibration. Therefore, the purpose of this paper is to perform experimental investigation to systematically calibrate the basic properties of TLCD, such as the natural frequency and head loss coefficient, with the major objective of proposing an empirical formula for predicting head loss coefficients.

In this paper, the basic theoretical equation of TLCD under direct excitation is firstly derived for the completeness of the paper. Secondly, by employing the equivalent damping in the TLCD equation, the exact solution of the

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liquid surface response under harmonic excitation is presented. This exact solution will be subsequently used in the experimental calibration. Both the free vibration and harmonic forced vibration tests are used to obtain the experimental results. Extensive comparisons are made between the experimental and analytically computed results to calibrate the head loss coefficients. Finally, an empirical formula to predict the head loss coefficient is proposed and suggested for designers as quick references.

2. Formulation

The schematic diagram of a TLCD under the excitation of forced vibration such as on the shake table is shown in Figure 1. Herein, only the TLCD with uniform cross-section is considered. The assumptions in the following are used to derive the equation of motion: (i) the sloshing behavior on the liquid surface is negligible; (ii) the flow is incompressible (i.e., discharge is constant); (iii) the dimension of the cross-section is much smaller than the horizontal length of the TLCD. By means of the energy principle, the equations of motion of the TLCD liquid surface can be derived as described in the following [3,6].

Let x and y denote the displacements of the excitation and liquid surface, respectively; L_h and L_v be the horizontal and vertical column length; ρ be the fluid density; g be the gravity acceleration; $L = 2L_v + L_h$ be the total length of the liquid column; and η be the head loss coefficient. The Lagrange's equation in y coordinate is given as

$$\frac{d}{dt}\left(\frac{\partial\,(T-V)}{\partial\,\dot{y}}\right)-\frac{\partial\,(T-V)}{y}=Q_y \tag{1}$$

in which T and V represent the kinetic and potential energy, respectively, and Q_y represents the non-conservative

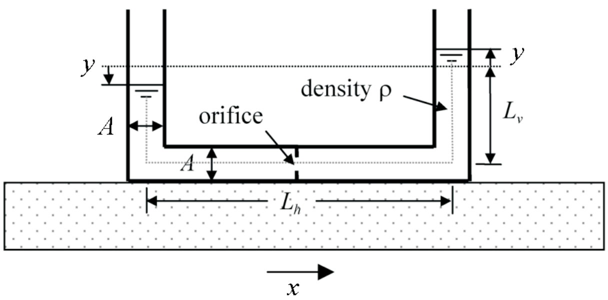

Figure 1. TLCD under the excitation of forced vibration.

force acting in the direction of the y coordinate. With the excitation x given, the kinetic energy can be expressed as

$$T = T_{Left\ Vertical\ Column} + T_{Right\ Vertical\ Column} + T_{Horizontal\ Column} \tag{2}$$

in which

$$\begin{aligned} T_{Left\ Vertical\ Column} &= \frac{1}{2}\rho(L_v - y)A\,(\dot{y}^2+\dot{x}^2) \\ T_{Right\ Vertical\ Column} &= \frac{1}{2}\rho(L_v + y)A\,(\dot{y}^2+\dot{x}^2) \\ T_{Horizontal\ Column} &= \frac{1}{2}\rho L_h A\,(\dot{y}+\dot{x})^2 \end{aligned} \tag{3}$$

The potential energy can be expressed as

$$V = \rho g A\,(2y)(y) - \rho g A\,(y)(y/2)\cdot 2 = \rho g A\,y^2 \tag{4}$$

in which the lower liquid surface in the two vertical columns is taken as the zero potential point. Meanwhile, the non-conservative force Q_y that is basically caused by the head loss can be expressed by

$$Q_y = -\frac{1}{2}\rho\,\dot{y}\,|\dot{y}|\,\eta\,A \tag{5}$$

Therefore, the substitution of Eqs. (2)–(5) into Eq. (1) leads to

$$\rho A L\,\ddot{y} + (1/2)\,\rho A\,\eta\,\dot{y}\,|\,\dot{y}| + 2\rho A g\,y = -\rho A L_h \ddot{x} \tag{6}$$

The head loss can be considered as the overall head loss induced by flow motion in the liquid column, although it is mainly induced by flow passing through the orifice. From Eq. (6), it is observed that the natural frequency of TLCD is $\omega_d = \sqrt{2g/L}$, and accordingly the natural period is $T_d = 2\pi\sqrt{L/2g}$.

2.1 Nondimensionalization

For conciseness of analysis and presentation, the equation of motion of liquid surface in Eq. (6) is nondimensionalized before further derivation. Thus, the resulting form of the nondimensionalized Eq. (6) is obtained as

$$\hat{y}'' + \frac{1}{2}\,p\,\eta\,\hat{y}'\,|\,\hat{y}'\,| + 4\pi^2\hat{y} = -\,p\hat{x}'' \tag{7}$$

in which the nondimensional variables are defined as $\hat{x} = x / L_h$; $\hat{y} = y / L_h$; $\hat{t} = t / T_d$; $p = L_h/L$ is the ratio of horizontal length versus total length of the liquid column. Note that in Eq. (7), the notation prime (′) represents the differentiation with respect to the nondimensional time $\hat{t}$.

2.2 Equivalent Damping for TLCD under Harmonic Excitation

Under a harmonic excitation with a frequency ω, an equivalent viscous damping in the form of $(4/3\pi)\rho A\eta\varphi_y \omega\dot{y}$ (φ_y is the amplitude of y) can be used to replace the damping term $(1/2)\rho A\eta|\dot{y}|\dot{y}$ in Eq. (6) [3] for convenience of computation and analysis. Such an expression can be easily obtained by equating the energy dissipated between these two damping expressions in a harmonic cycle. A concise derivation is described as follows for completeness. Let the harmonic response of y be expressed by $y = \varphi_y \sin \omega t$ in which φ_y is the amplitude of y, the damping force can be written as $F_d = (1/2)\rho A\eta|\dot{y}|\dot{y} = (1/2)\rho A\eta\varphi_y^2\omega^2 \cos\omega t \cdot |\cos\omega t|$. The hysteretic loop of such a damping force in a full cycle is plotted in Figure 2 for illustration. In the first quarter cycle, the relation between y and F_d can be obtained as

$$\frac{F_d}{(1/2)\,\rho\, A\, \eta\, \varphi_y^2\, \omega^2} = 1 - \left(\frac{y}{\varphi_y}\right)^2 \tag{8}$$

Hence, the dissipated energy by the damping force in the quarter cycle is the area under the F_d curve from 0 to φ_y. By integration, the resulting dissipation energy in a full cycle becomes $(4/3)\rho A\eta\omega^2\varphi_y^3$. Equating this to the dissipated energy $\pi C_{eq}\omega\varphi_y^2$ caused by the equivalent viscous damping force $C_{eq}\,\dot{y}$ leads to

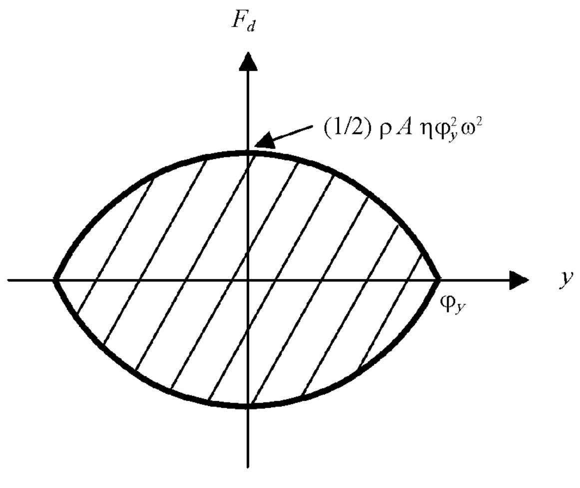

Figure 2. Hysteretic loop of damping force in TLCD.

$$C_{eq} = (4/3\pi)\,\rho A\,\eta\omega\,\varphi_y \tag{9}$$

Consequently, such a nondimensionalized equation is written as

$$\hat{y}'' + (8/3)\,p\,\eta\varphi_{\hat{y}}\,k\,\hat{y}' + 4\pi^2\hat{y} = -p\hat{x}'' \tag{10}$$

in which $k = \omega/\omega_d$ is the non-dimensional excitation frequency; and $\varphi_{\hat{y}} = \varphi_y / L_h$ is the non-dimensional amplitude of y.

2.3 Exact Solution of Liquid Surface Response

By the use of Eq. (10), the exact response of the liquid surface y under harmonic excitation can be derived as follows. First of all, let the nondimensional excitation displacement $\hat{x}$ and $\hat{y}$ in Eq. (10) be substituted by $\frac{D}{L_h}e^{i2\pi k\hat{t}}$ and $A_{\hat{y}}e^{i2\pi k\hat{t}}$, respectively, in which D is the amplitude of excitation displacement, $k = \omega/\omega_d$ is the non-dimensional excitation frequency and $A_{\hat{y}}$ is the complex amplitude of $\hat{y}$. Therefore the complex amplitude of $\hat{y}$, i.e., $A_{\hat{y}}$, can be easily solved in terms of k, $\varphi_{\hat{y}}$ and $e^{i2\pi k\hat{t}}$ as

$$A_{\hat{y}} = \frac{2\pi\,\gamma\,k^2}{2\pi(1-k^2) \;+\; i\;(8/3)\,\eta\,p\,\varphi_{\hat{y}}\,k^2}\;;\;\gamma = p\,D/L_h \tag{11}$$

Because of the relation $\varphi_{\hat{y}} = |A_{\hat{y}}|$, by taking the absolute value on both sides, the amplitude of response of $\hat{y}$ can be solved and expressed as

$$\varphi_{\hat{y}} = \frac{\sqrt{-2\pi^2(1-k^2)^2 + \left(4\pi^4(1-k^2)^4 + k^8(8/3\;\eta p)^2\;4\pi^2\gamma^2\right)^{1/2}}}{k^2(8/3\;\eta p)} \tag{12}$$

The formula of $\varphi_{\hat{y}}$ in Eq. (12) will be used for the experimental calibration for head loss coefficients in the later section. From Eq. (12), it is noted that the amplitude $\varphi_{\hat{y}}$ is not linearly dependent on the excitation amplitude D. This is due to the nonlinearity caused by the damping term.

Although Eq. (12) is obtained by using the equivalent damping form, the accuracy of such an approximation can be examined by numerical simulation. As shown in Figure 3 is the comparison of $\varphi_{\hat{y}}$ for the case with parameters $p = 0.5$, $D = 4$ cm, $L_h = 115$ cm and $\eta = 3.55$. The solid curve is the result obtained from Eq. (12). The points with marks are the results from direct integration of Eq. (7) by means of the Runge-Kutta method. As observed from Figure 3, it is demonstrated that the approximation of using the equivalent damping is remarkably accurate.

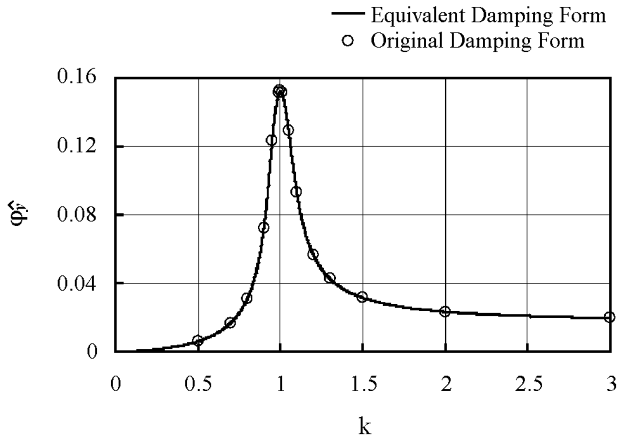

Figure 3. Comparison of the amplitude of liquid displacement using different damping forms.

3. TLCD Designs for Experimental Calibration

In order to systematically investigate the TLCD basic properties, two design factors are concerned. One is the size of the liquid mass, the other is the ratio of horizontal column length versus total column length, i.e. p. Although these two factors are not considered influential in theory because of the assumptions, their effects should be examined experimentally from the practical point of view. Therefore, four differently configured groups of

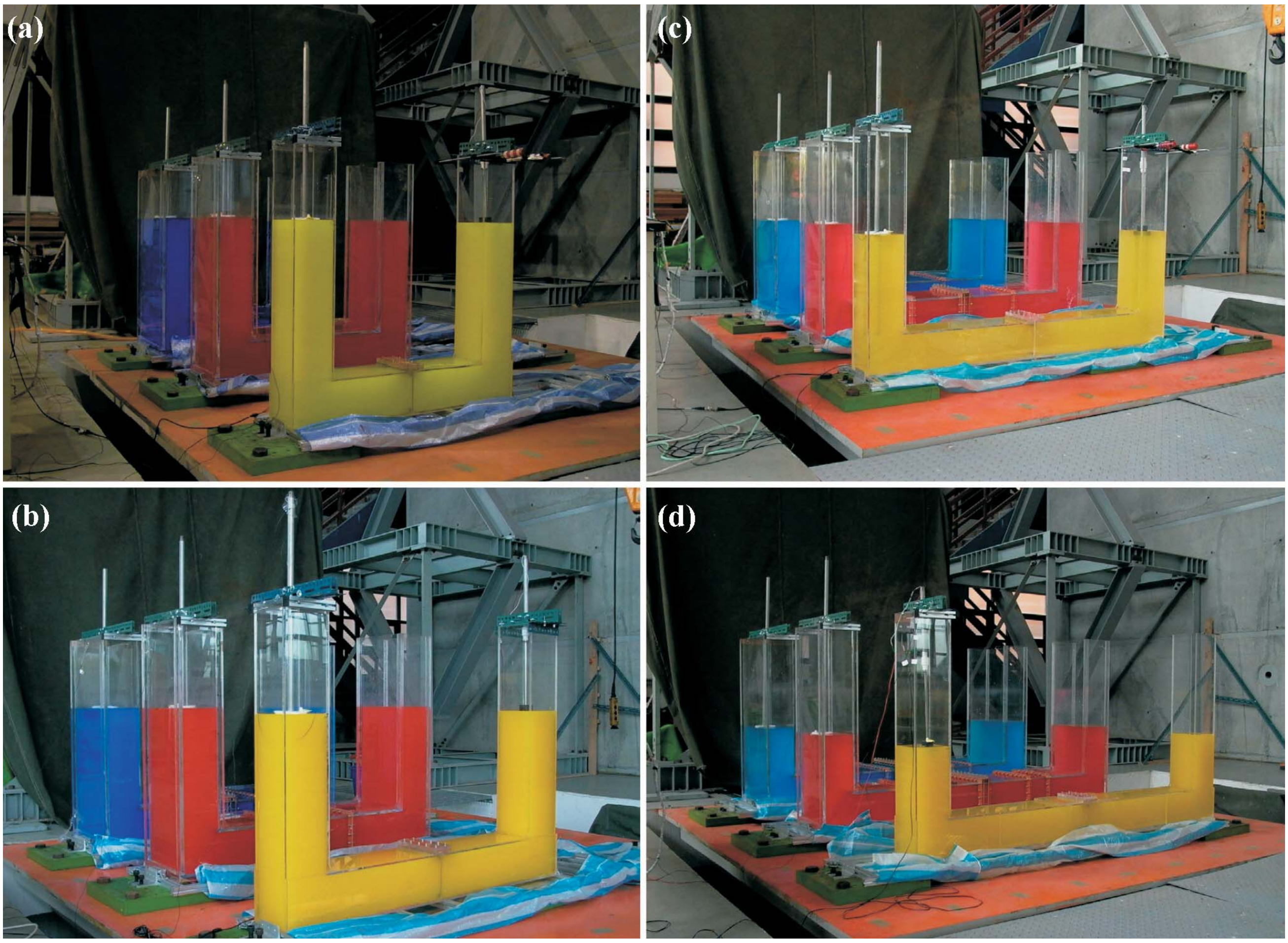

Figure 4. Four configured groups of tlcds on the shake table: (a) Configured Group I; (b) Configured Group II; (c) Configured Group III; (d) Configured Group IV.

TLCDs with uniform cross-section as shown in Figure 4 are constructed. The design of rectangular cross-section and sharp-edged elbow is due to the simplicity in manufacturing. To examine if the size of liquid mass has effects on the basic properties, each configured group contains three TLCDs that each has a cross-section area of 15 cm × 15 cm, 30 cm × 15 cm and 45 cm × 15 cm, respectively. To examine if the difference of p is important on the basic properties, each configured group is designed with a different value of p. Since the damping of TLCD is mainly produced by the energy dissipating mechanism while the flow passes through the orifice located in the middle of the horizontal column, four different orifice areas with area blocking ratios (ψ) of 20%, 40%, 60% and 80%, respectively, are used in each configured group to calibrate the corresponding head loss coefficient η. The detail dimensions of each configured group are listed in Table 1, in which D represents the amplitude of the excitation displacement while harmonic forced vibration tests are tested.

4. Experimental Setup for Calibration

The basic properties of TLCD, i.e., the natural frequency and head loss coefficient, are experimentally calibrated using both free vibration and harmonic forced vibration tests via the shake table in the structural laboratory of Department of Construction Engineering, National Kaohsiung First University of Science and Technology (see Figure 4). Firstly, the four configured groups of TLCD are sequentially placed on the shake table. The natural frequencies of TLCD are calibrated for each group by recording the response of the free vibration tests, in which the liquid surface movement is excited by driving the shake table at a frequency close to the resonance frequency and then suddenly stopping its motion. Secondly, to calibrate the head loss coefficient, forced harmonic vibration tests are performed for each group by driving the shake table at various frequencies, and hence the liquid response is recorded accordingly by changing different blocking ratios in the orifice once at a time.

Table 1. Dimensions of configured groups of TLCD

	Configured Group			
	I	II	III	IV
L_h (cm)	85	115	145	175
L_v (cm)	63.75	57.5	48.33	37.5
A (cm^2)	15 × 15 30 × 15 45 × 15	15 × 15 30 × 15 45 × 15	15 × 15 30 × 15 45 × 15	15 × 15 30 × 15 45 × 15
Blocking Ratio ψ (%)	20, 40, 60, 80	20, 40, 60, 80	20, 40, 60, 80	20, 40, 60, 80
$p = L_h/L$	0.4	0.5	0.6	0.7
$L = L_h$+2 L_v (cm)	212.5	230	241.67	250
$\omega_d = (2g/L)^{1/2}$ (rad/sec) (Predicted)	0.4836 × 2π	0.4648 × 2π	0.4535 × 2π	0.4459 × 2π
D	4 cm	4 cm	4 cm	4 cm

Table 2. Calibrated results of TLCD property tests

	Configured Group			
	I	II	III	IV
Natural Frequency ω_d (rad/sec) (Error w.r.t. Predicted)	0.4923 × 2π (1.8%)	0.4727 × 2π (1.7%)	0.4595 × 2π (1.3%)	0.4516 × 2π (1.3%)
Head Loss Coefficient η				
Blocking Ratio ψ = 20%	3.96	3.55	3.40	3.40
Blocking Ratio ψ = 40%	6.10	5.80	5.70	5.55
Blocking Ratio ψ = 60%	12.80	12.40	12.50	12.00
Blocking Ratio ψ = 80%	54.50	54.00	59.00	56.00

5. Experimental Results

5.1 Calibration of Natural Frequency

As shown in the upper part of Table 2 is the natural frequencies measured for each group. From the result, it is found that both the size of liquid mass and the difference of p value have no effect on the natural frequency. The calibrated results in Table 2 conclude that the analytical natural frequency $\omega_d = (2g/L)^{1/2}/2\pi$ (Hz) is reliable because the errors between the measured and predicted frequencies are as small as less than 2% (see the values in parentheses).

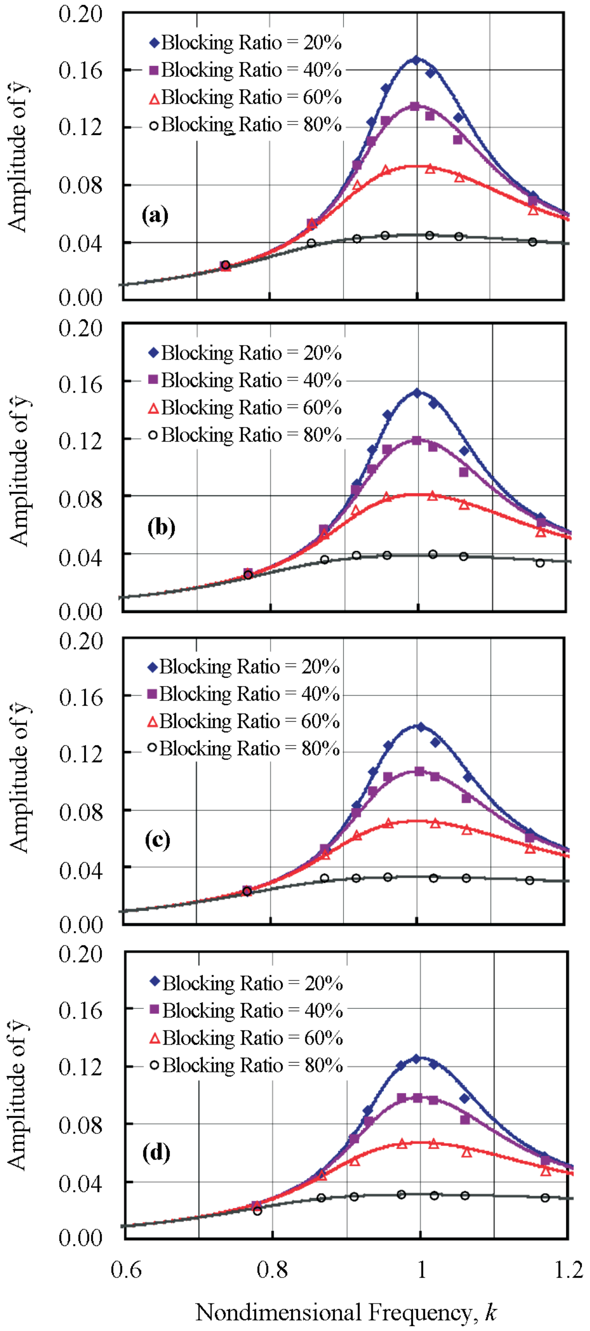

Figure 5. Experimental results of $\hat{y}$ amplitudes for each configured group of TLCD in forced vibration tests: (a) Configured Group I; (b) Configured Group II; (c) Configured Group III; (d) Configured Group IV.

5.2 Calibration of Head Loss Coefficient

The head loss coefficients are calibrated by comparing the measured amplitude responses of the liquid displacement and those from the analytical formula as described in Eq. (12). For each configured group, by adjusting the head loss coefficient η in Eq. (12) to proper values, the amplitude of $\hat{y}$ versus k is plotted together with the experimental data to fit each other. In Figure 5, the fitted results are denoted by the solid curves, while the points with marks are the measured results. The head loss coefficients thus calibrated are listed in the lower part of Table 2. As observed from these results, it is found that the head loss coefficient is neither affected by different size of liquid mass nor by different configurations (different p). It is only significantly affected by the blocking ratio ψ of the area in the orifice.

By employing the fitting techniques for several rounds of trial and error, an empirical formula for predicting the head loss coefficient calibrated can be expressed as

$$\eta = (-0.6\psi + 2.1\psi^{0.1})^{1.6}(1-\psi)^{-2} \quad (13)$$

The illustration of the curve in Eq. (13) and the experimentally calibrated data of η is shown in Figure 6. It

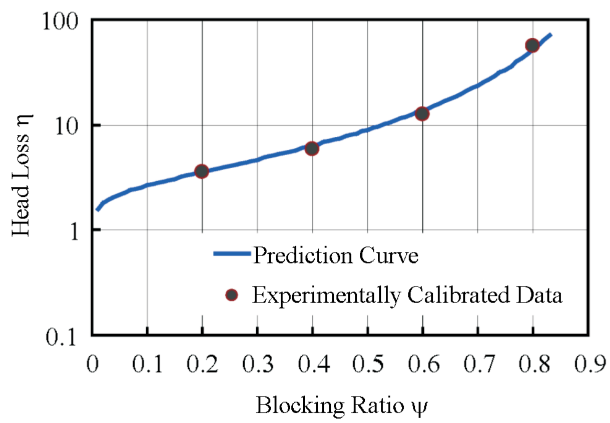

Figure 6. Comparison of head loss coefficients.

is suggested that the prediction formula of η in Eq. (13) shall provide a valuable and practical reference for a TLCD design.

6. Conclusions

The extensive calibration results of the properties of tuned liquid column damper using free vibration and harmonic forced vibration tests have been presented and the comparisons are made with those by analytical formulas. The comparisons indicate that the analytical formula for natural frequencies is practically valid. The head loss coefficients are successfully calibrated, and an empirical formula for predicting the overall head loss in a tuned liquid column damper with sharp-edged elbow is finally proposed and suggested as the quick reference for designers. The experimental investigation also confirms the fact that the size of liquid mass and the different ratio of horizontal column length versus total column length have no effect on the natural frequency and head loss as well.

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